A Quest for Provable Security against Side-Channel Attacks

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AFRICACRYPT 2022

July 19, 2022, Fes, Morocco

CryptoExperts

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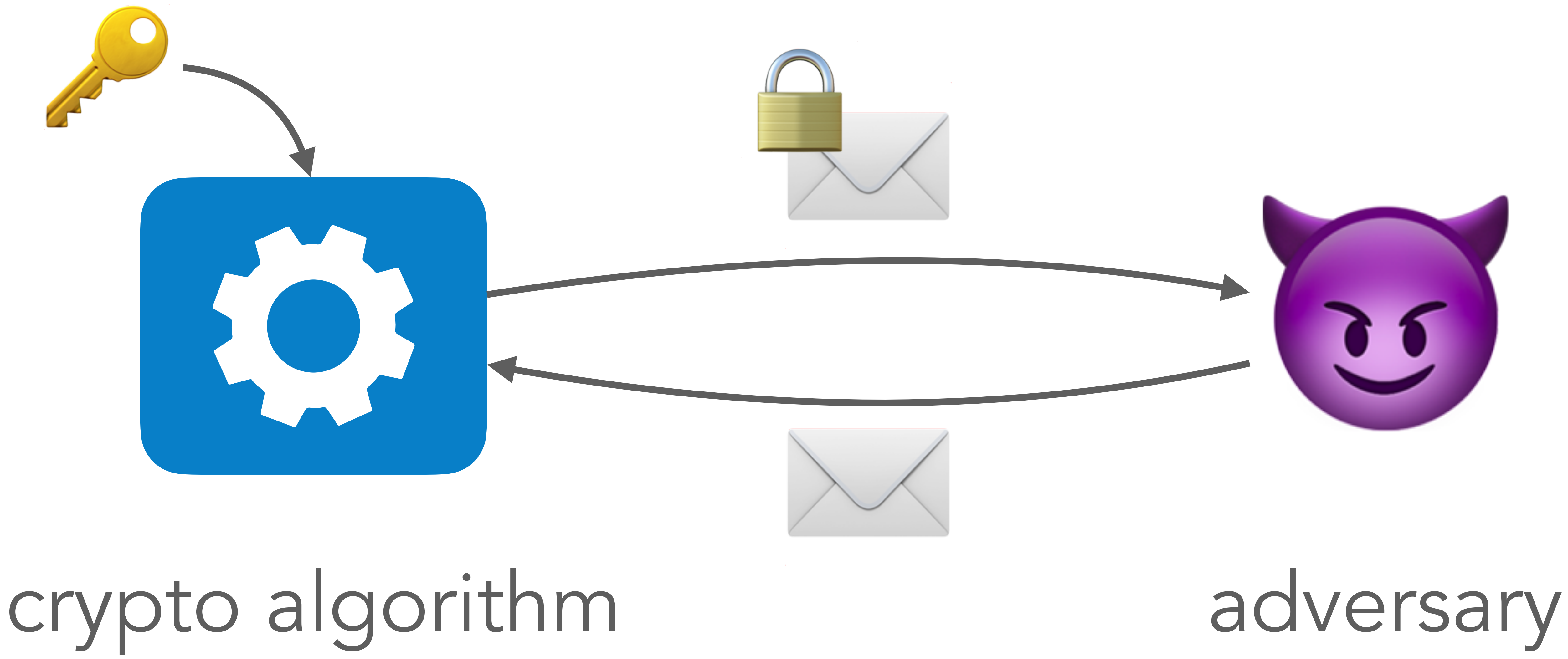
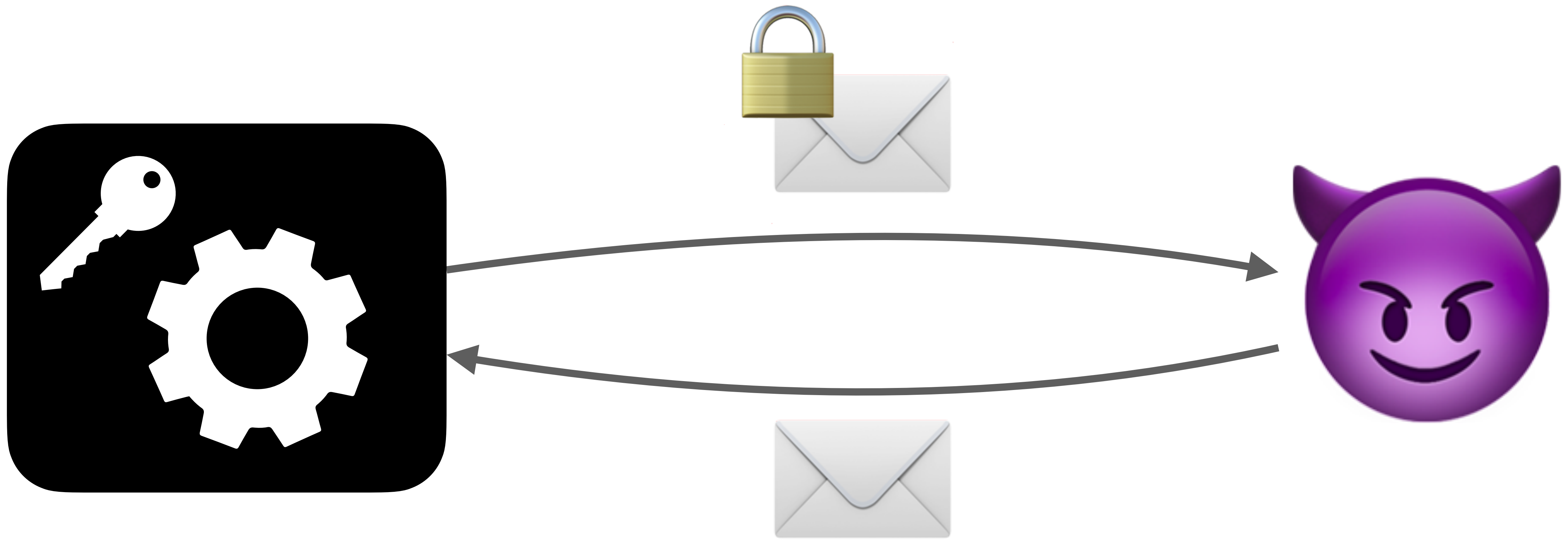
Introduction

Provable security



Provable security

Provable security

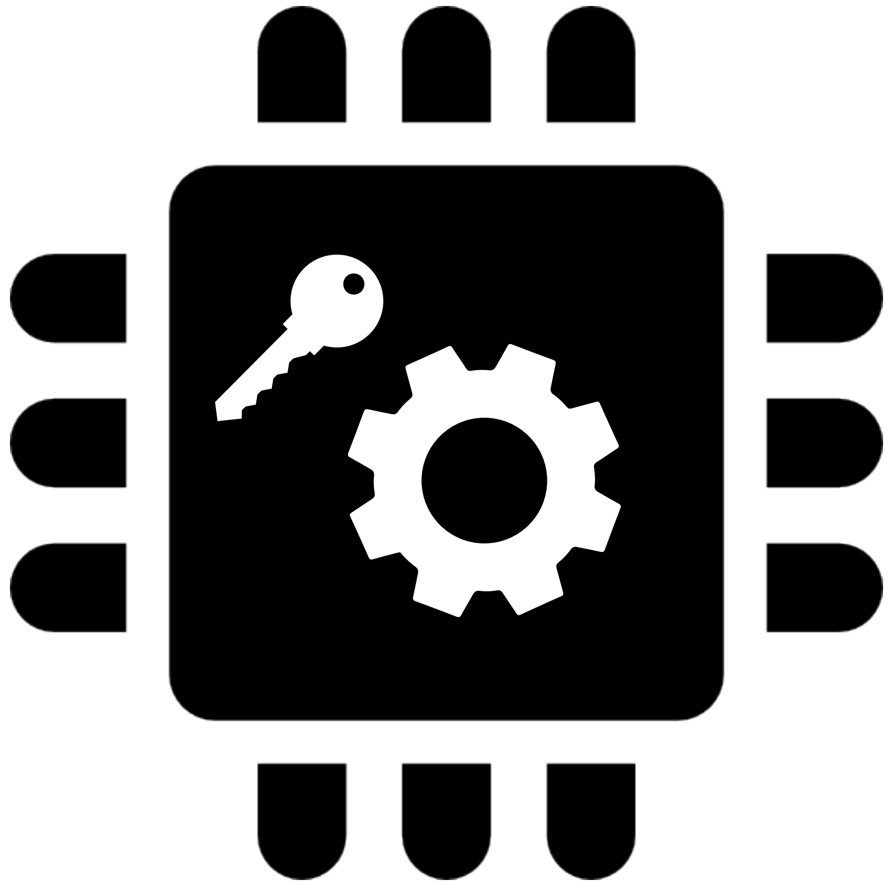


The "black-box model"

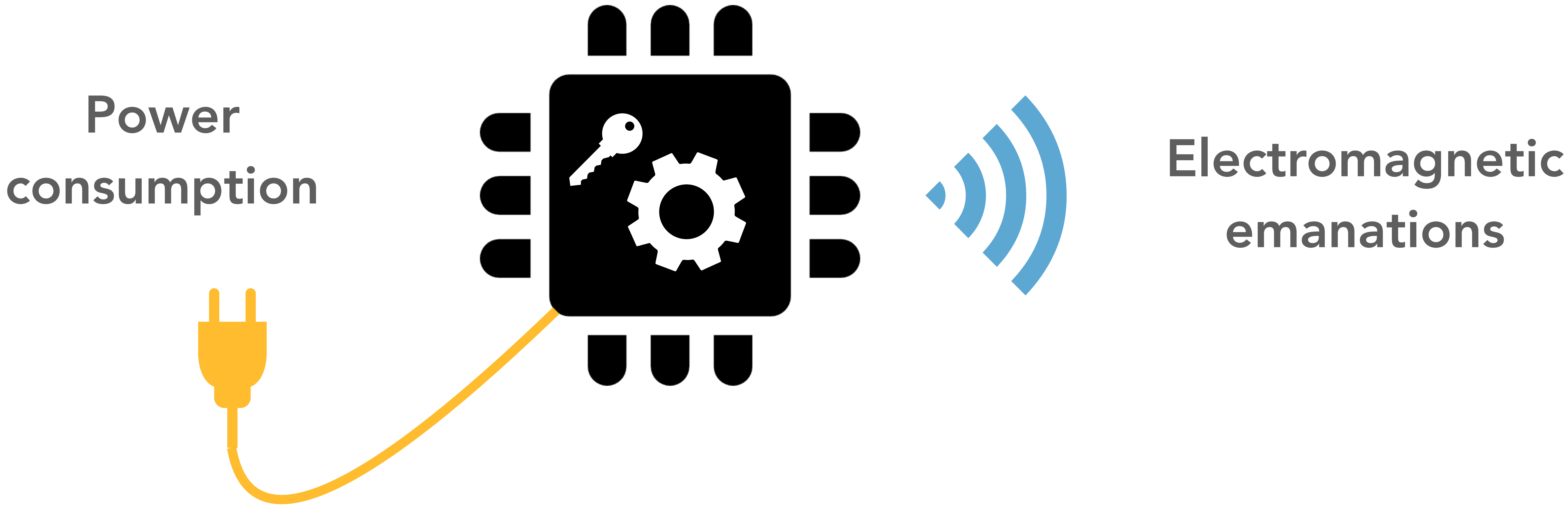
😈 needs unaffordable computing power to recover 🔑

security proof

Side-channel attacks



Side-channel attacks



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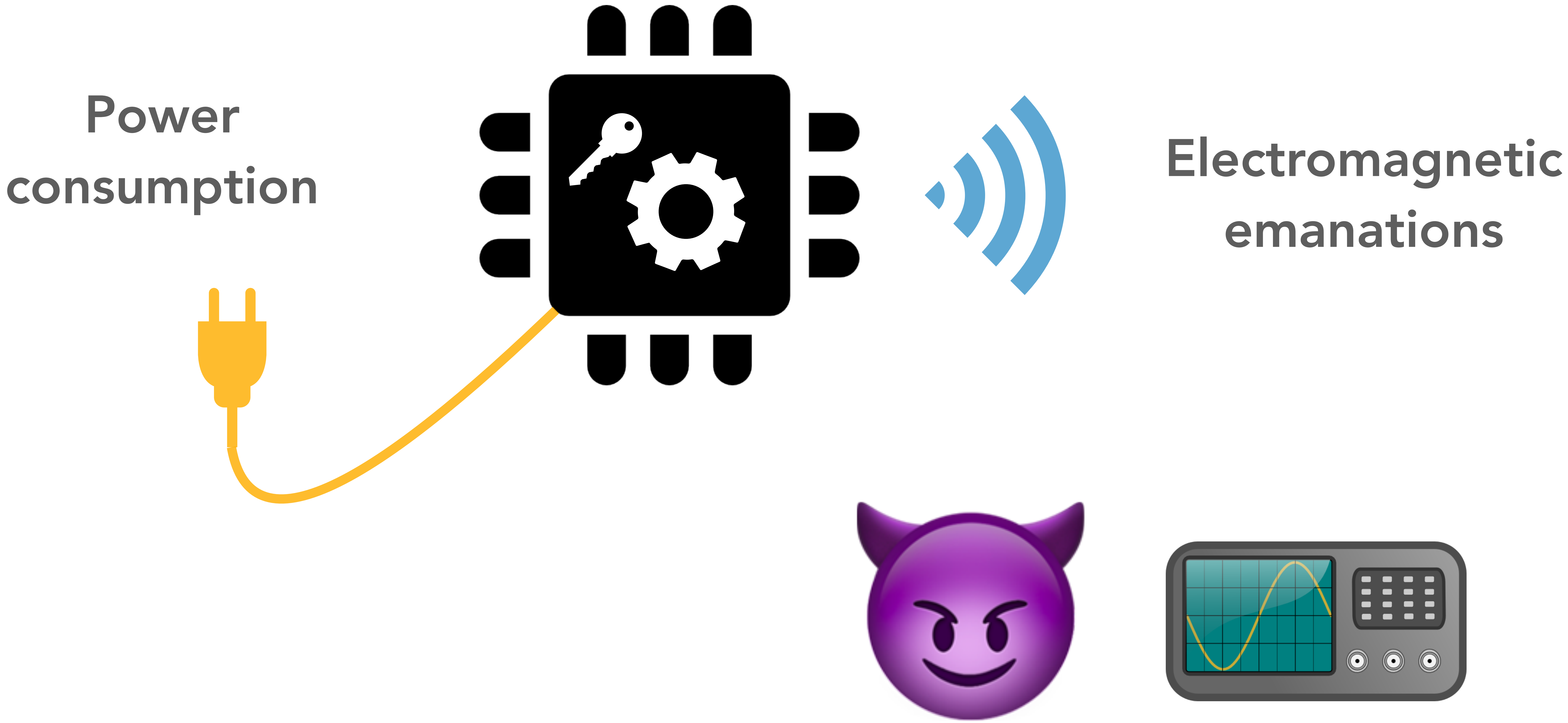
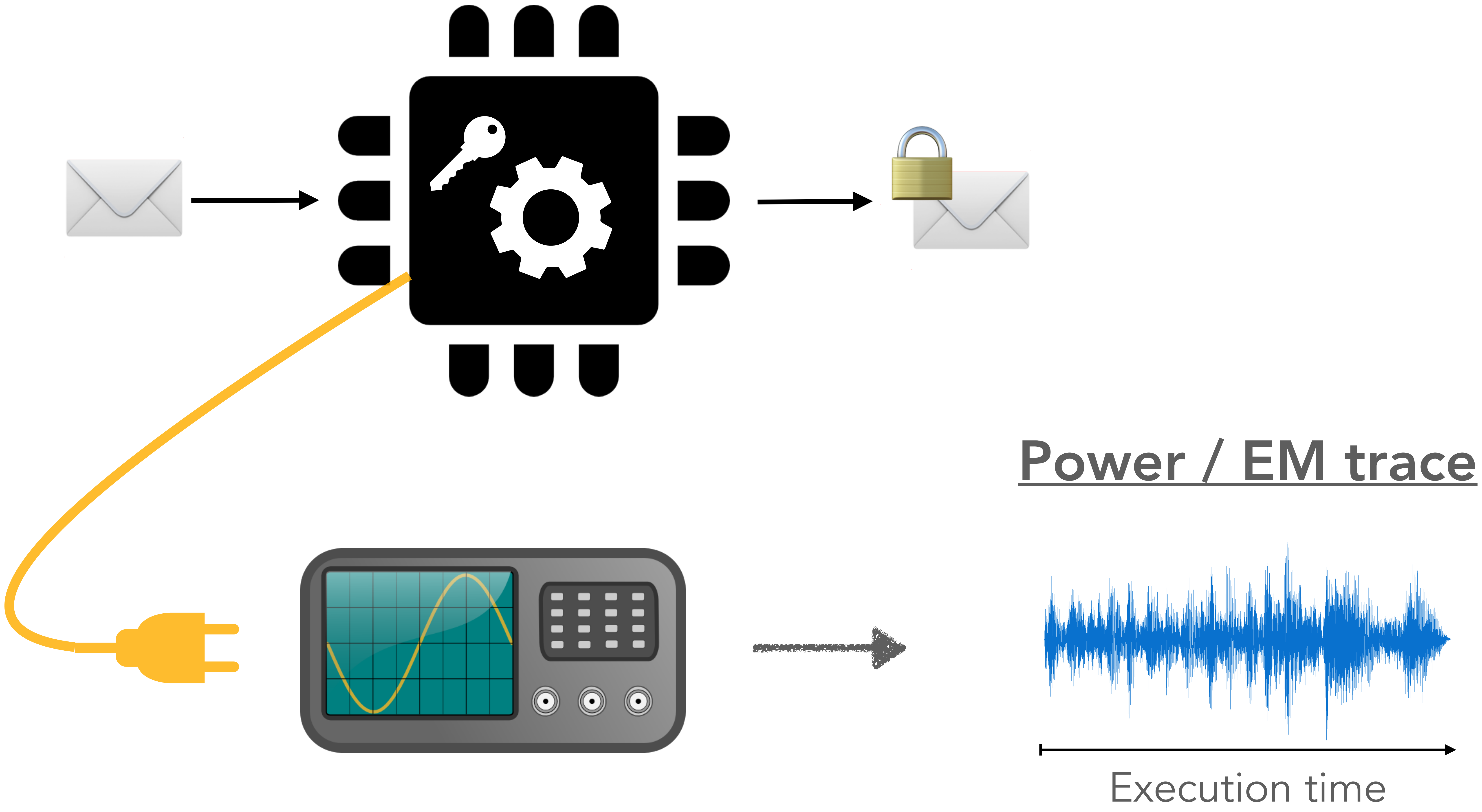


Differential power analysis



Masking

Apply secret sharing at the computation level

$$x = x_1 \oplus \cdots \oplus x_n$$

Masking

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randomly generated

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$$x = x_1 \oplus \cdots \oplus x_n$$

randomly generated ($x_2 \oplus \cdots \oplus x_n$)

constrained to keep the correctness (x_1)

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Apply secret sharing at the computation level

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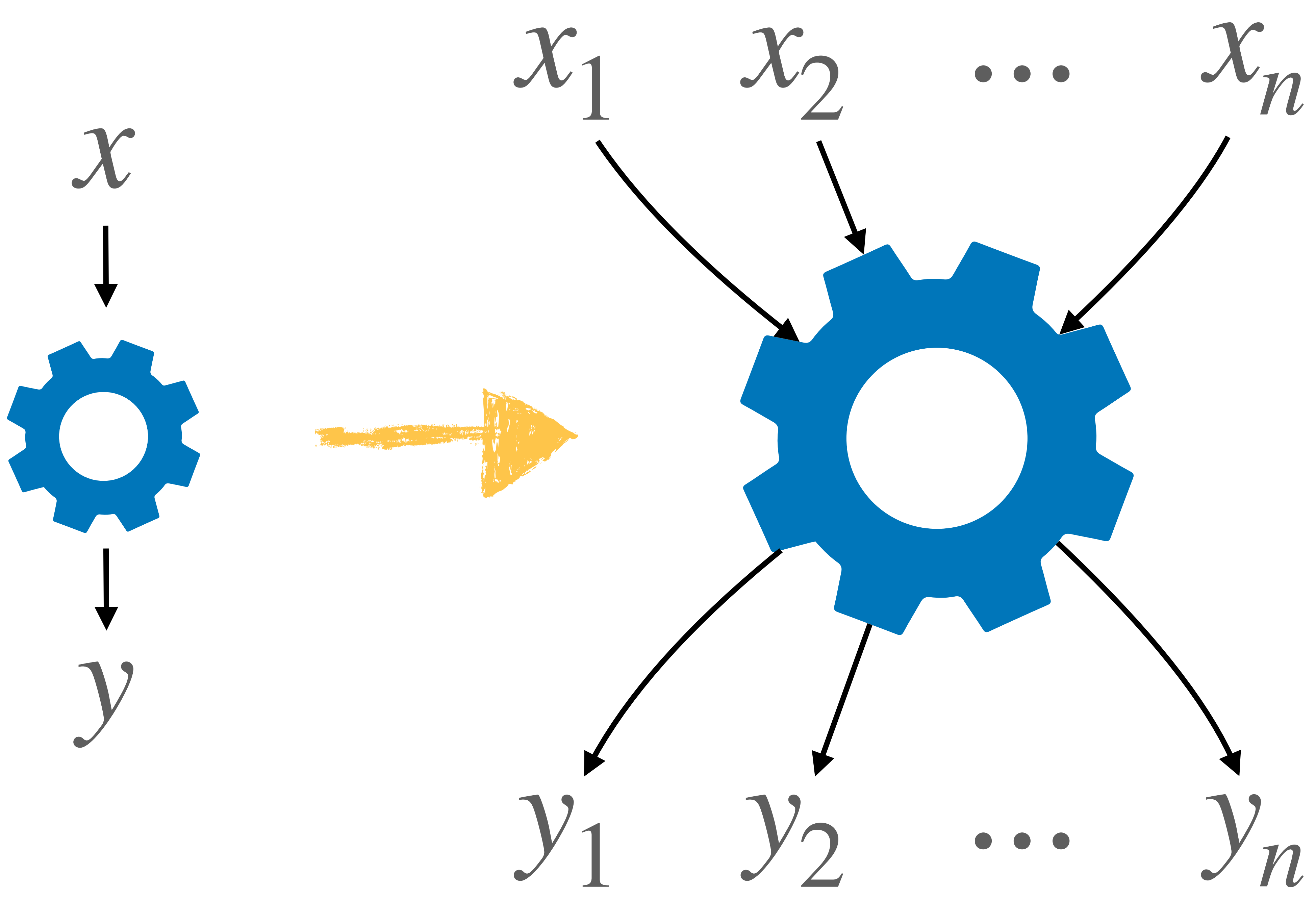
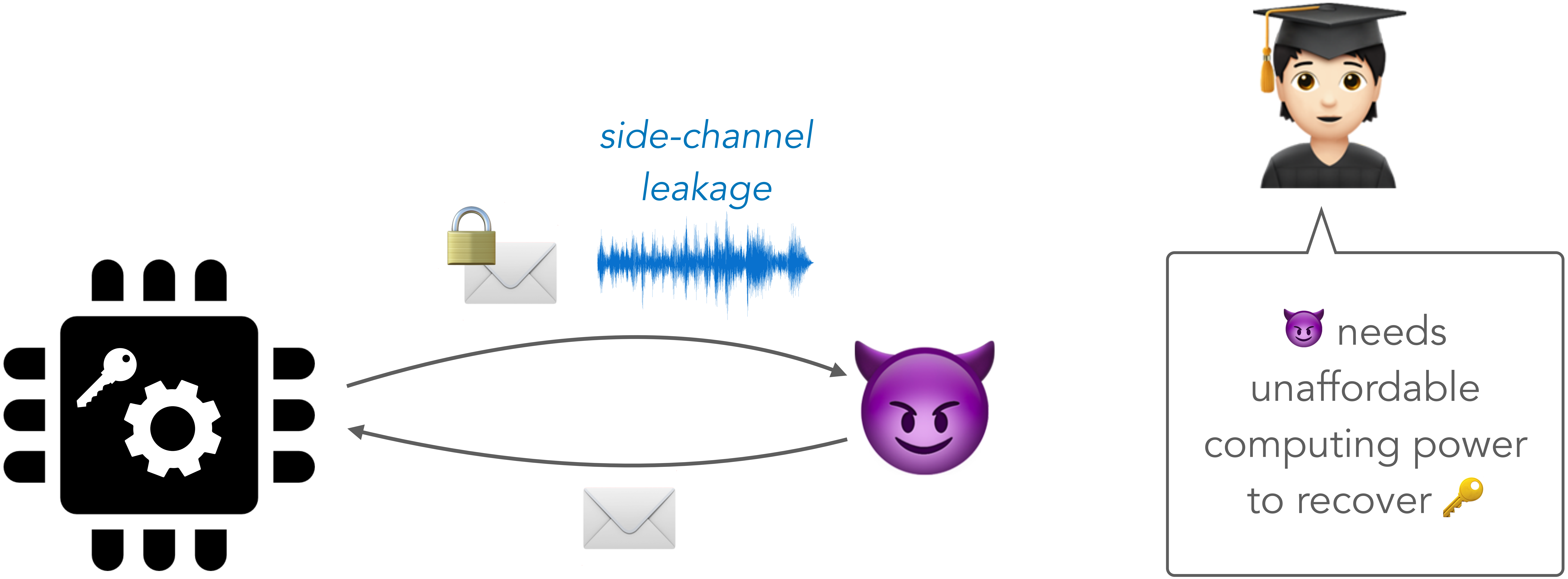
randomly generated

constrained to keep the correctness



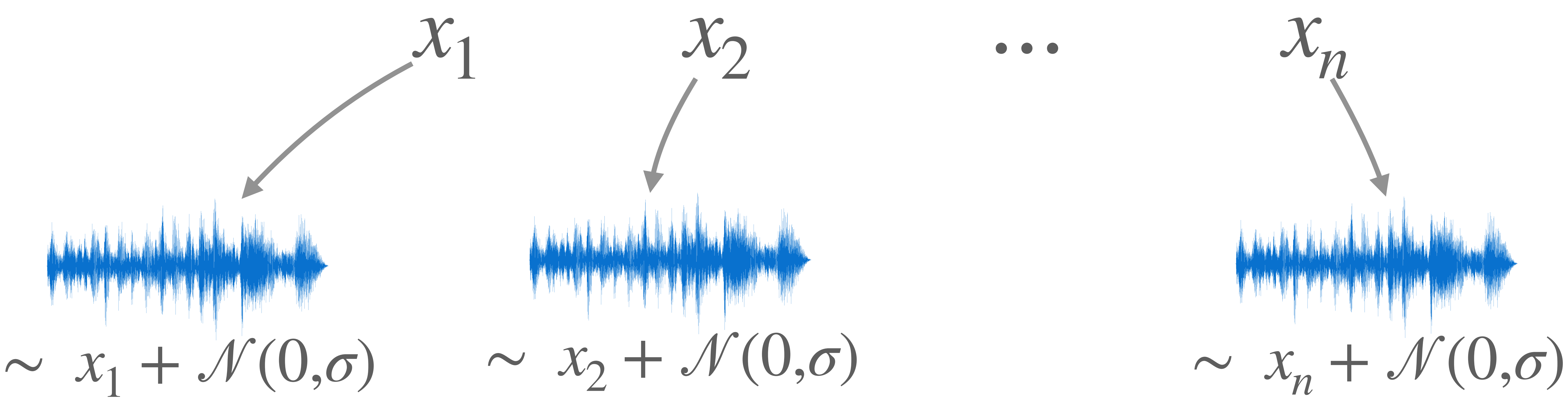
Provable security in the presence of leakage



security proof

Modelling noisy leakage

Motivation: soundness of masking with noise



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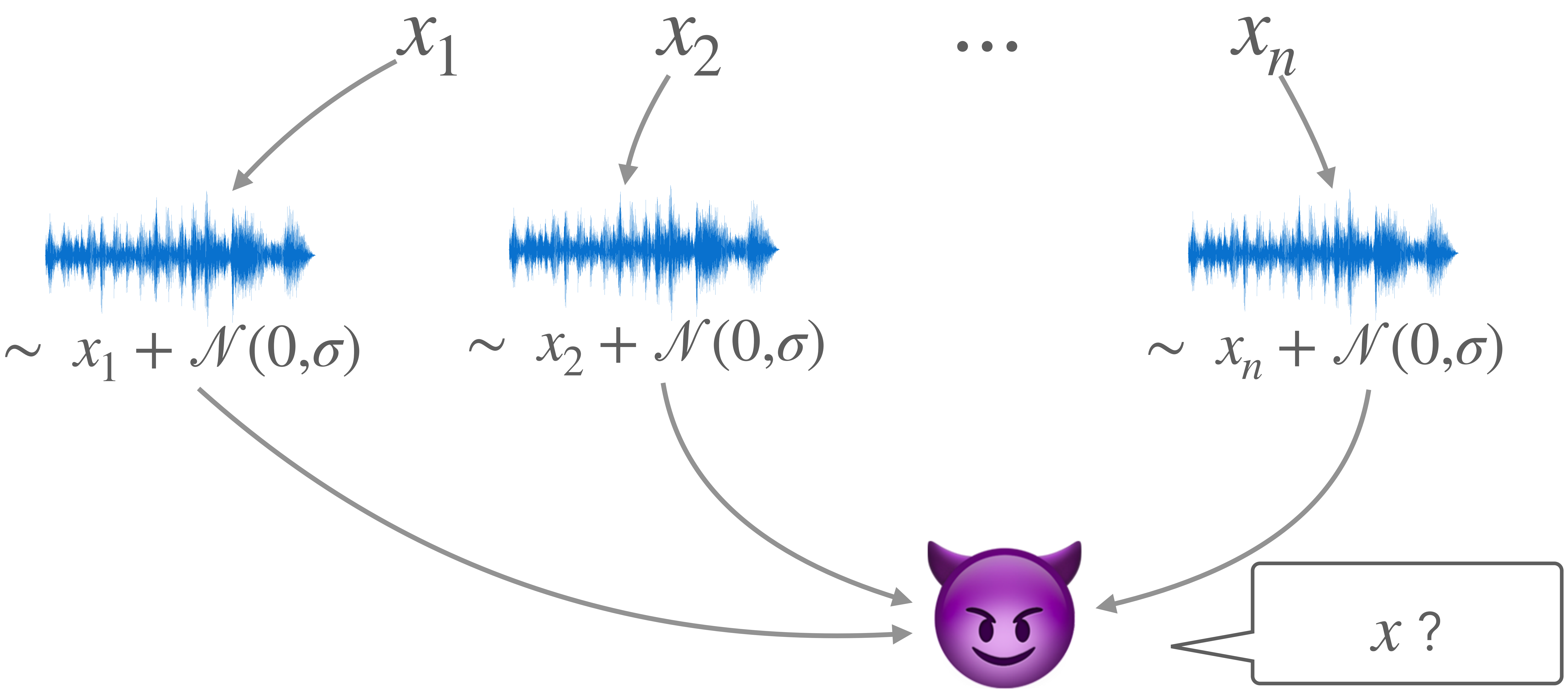

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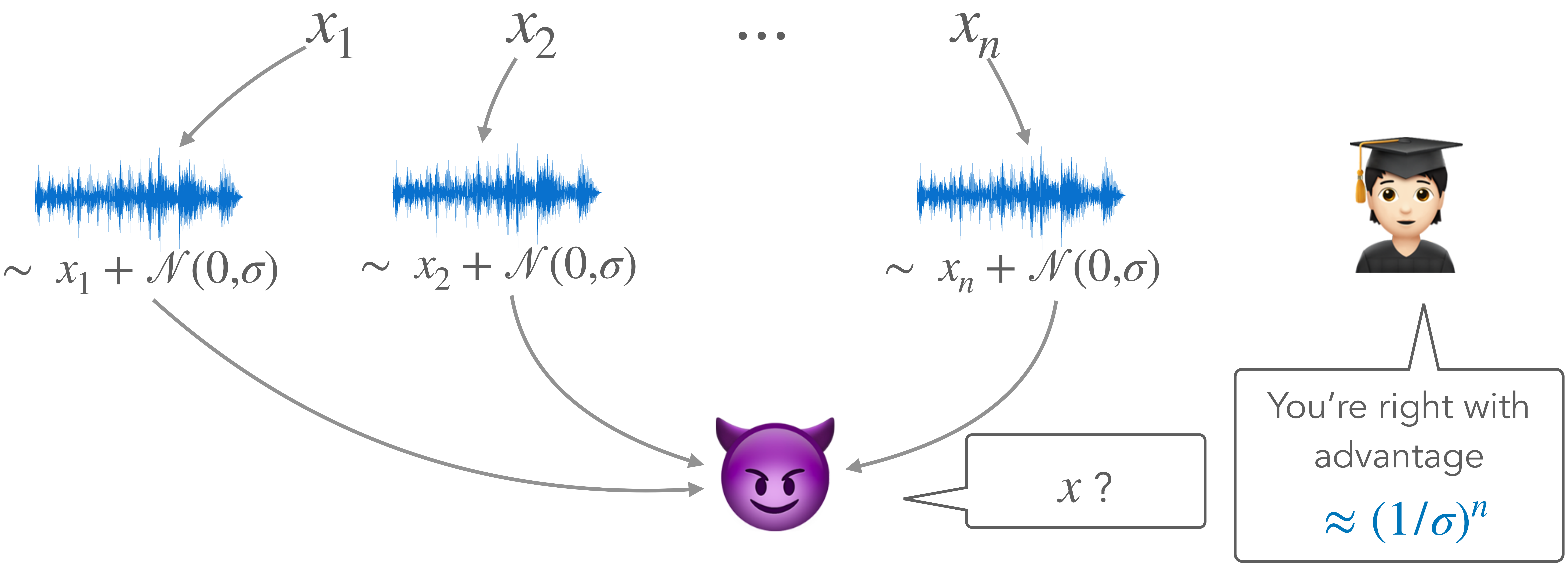

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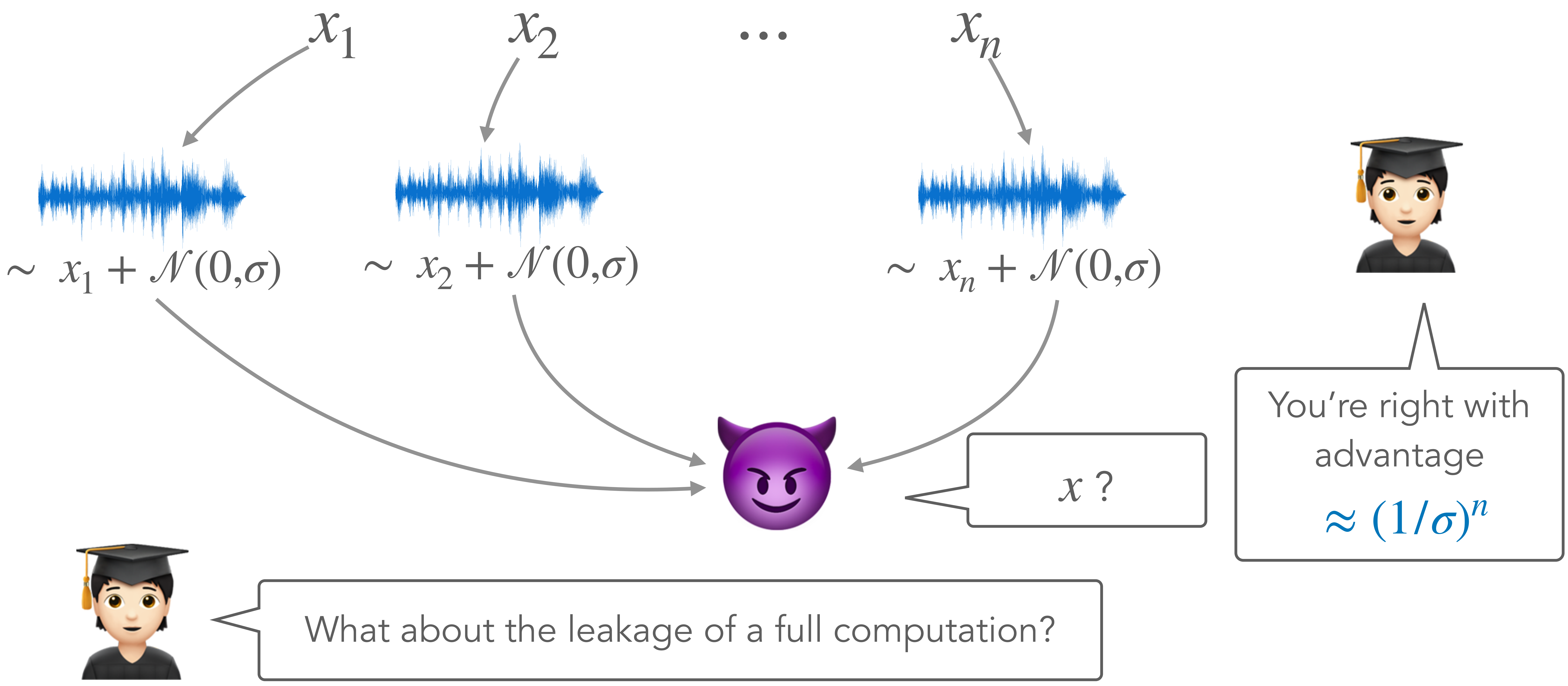
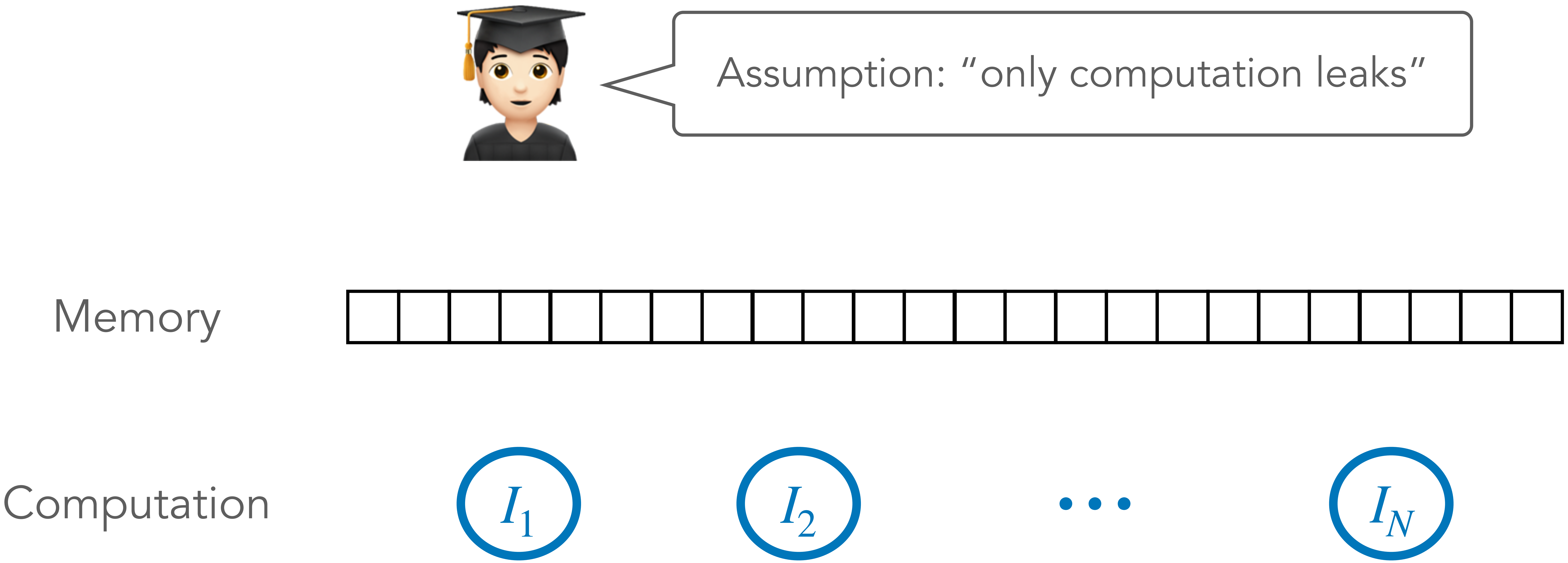
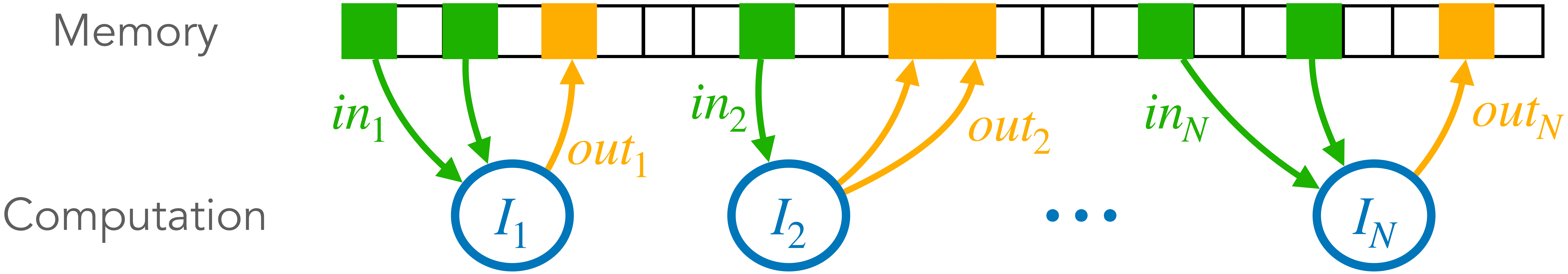


The noisy leakage model



The noisy leakage model

Assumption: "only computation leaks"



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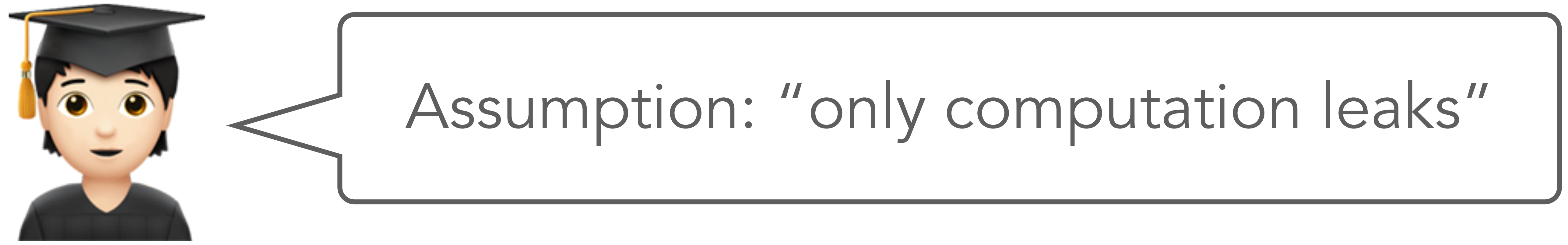

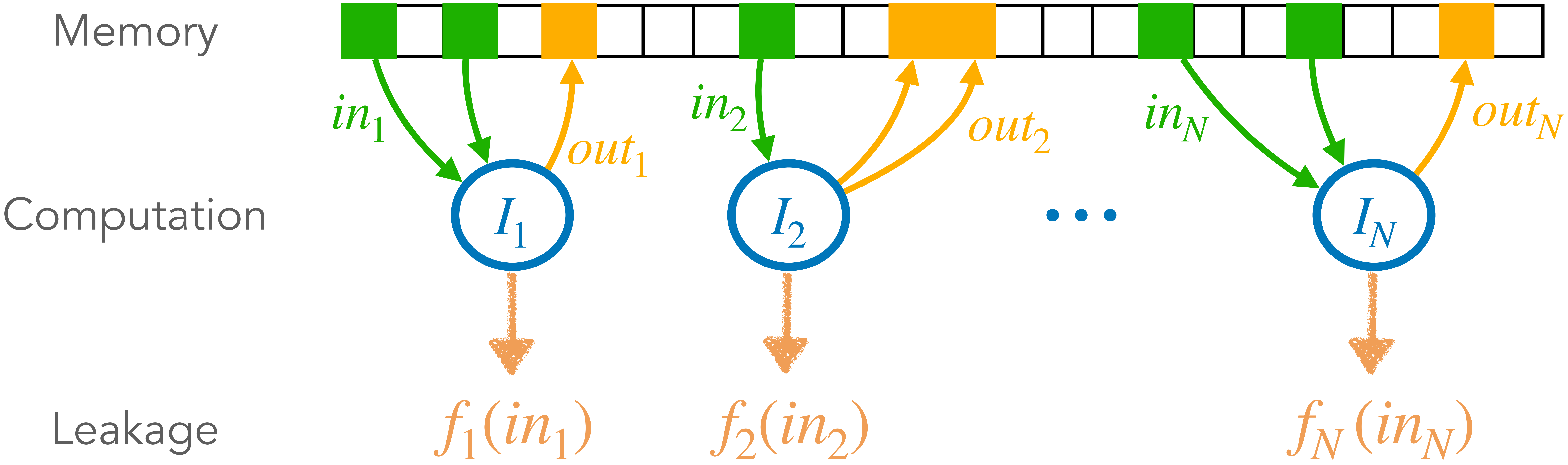

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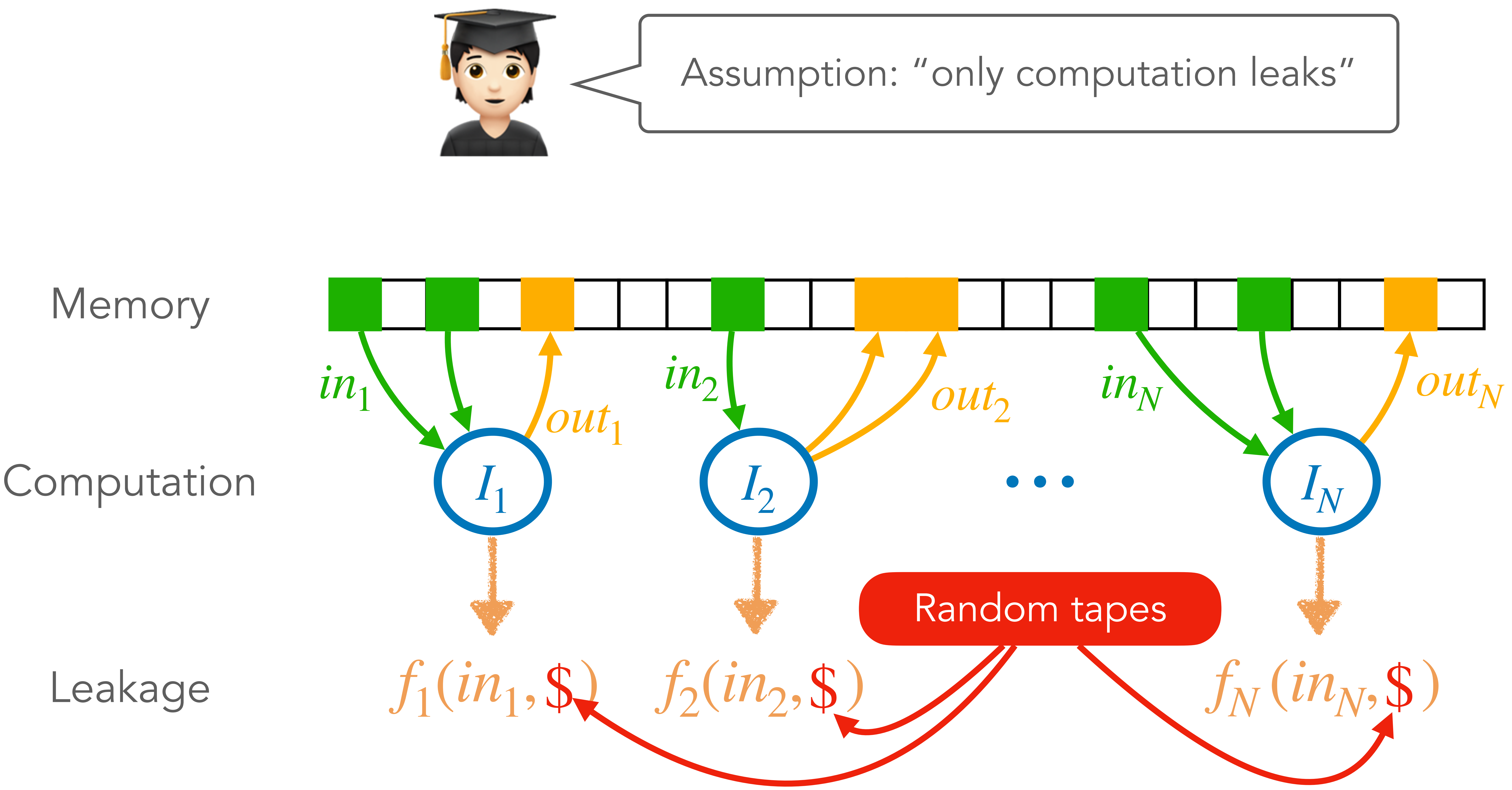

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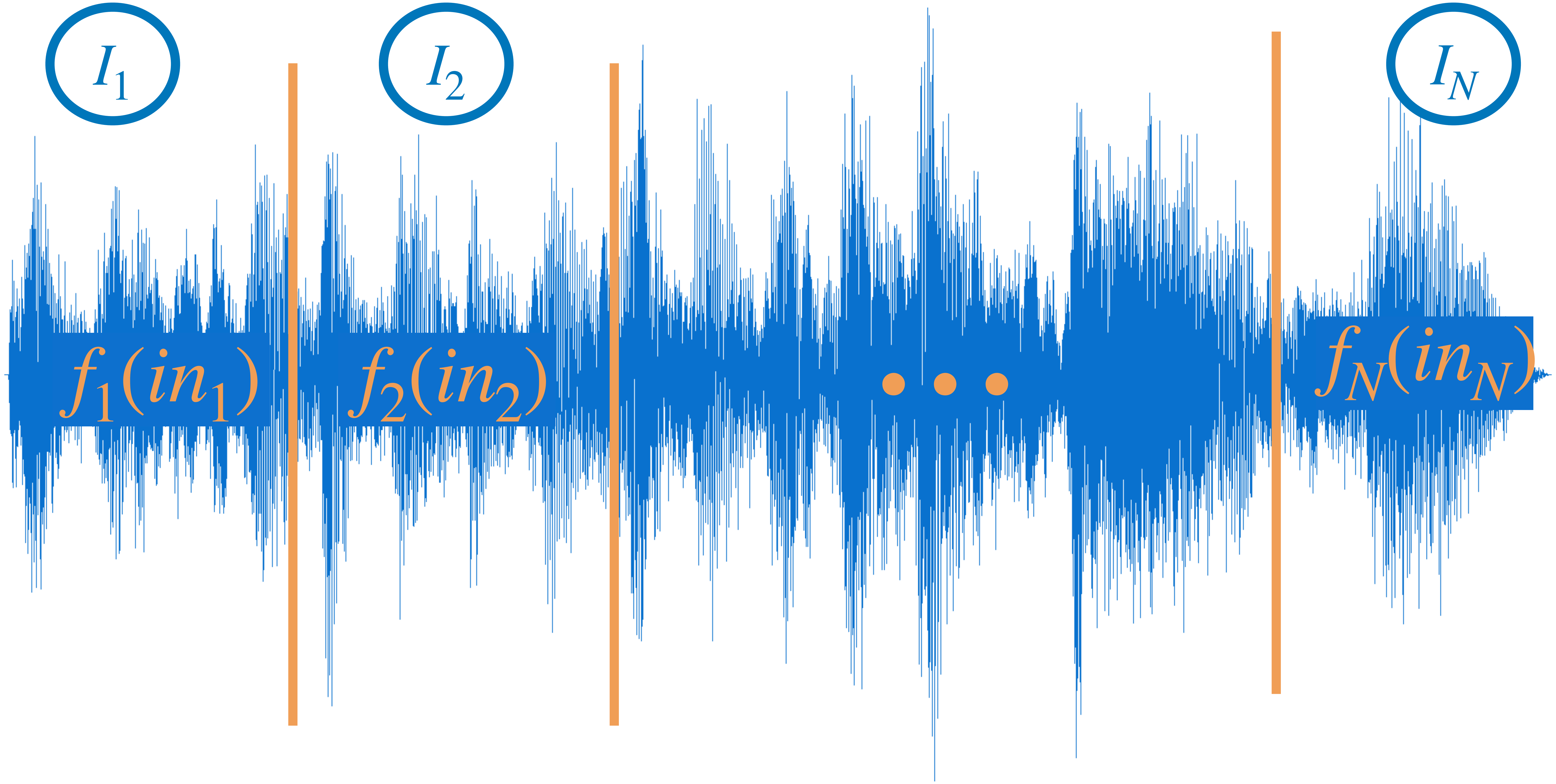

$f_i(in_i) \Rightarrow$ *multivariate noisy leakage*

Noisy leakage functions

A function is δ**-noisy** if (for $X \sim \mathcal{U}$):

$$\mathbb{E}_y\left[\Delta\left(X;\ \left(X \mid f(X) = y\right)\right)\right] \ \leq\ \delta$$

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statistical distance
between X and X
and given $f(X) = y$

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expectation on the possible leakage values

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Noisy leakage functions

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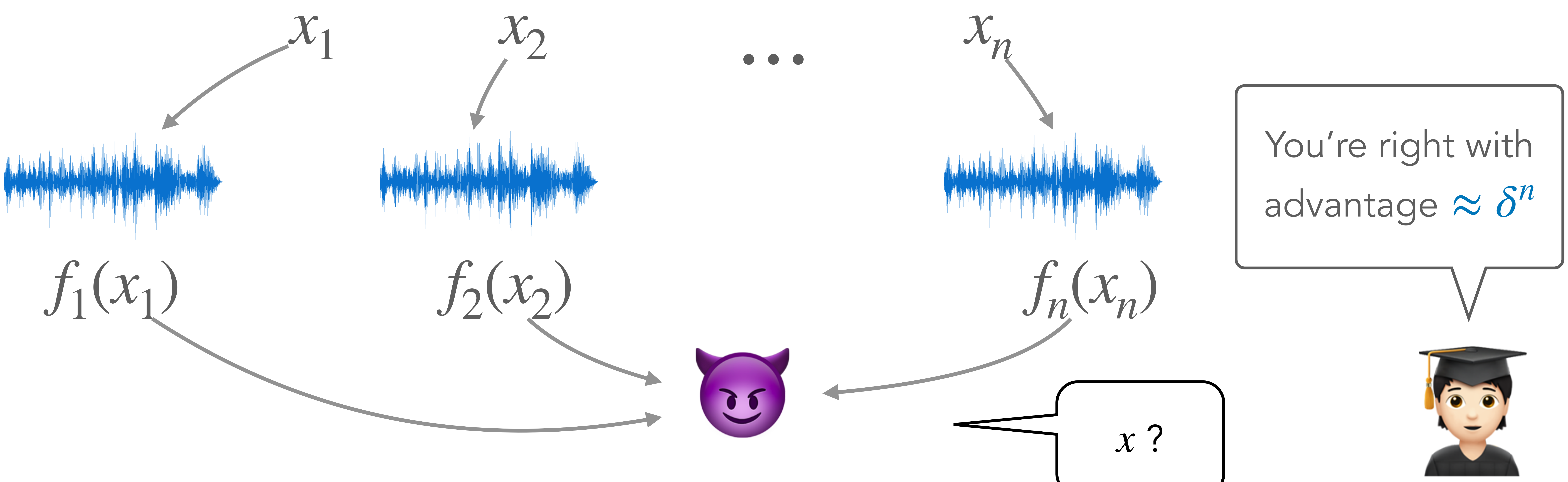
expectation on the possible leakage values

statistical distance between X and X and given $f(X) = y$

more noise $\Rightarrow$ smaller δ

$\begin{cases} 1 = \text{lot of leakage (low noise)} \\ 0 = \text{no leakage (infinite noise)} \end{cases}$

Soundness of masking with noise (generalised)



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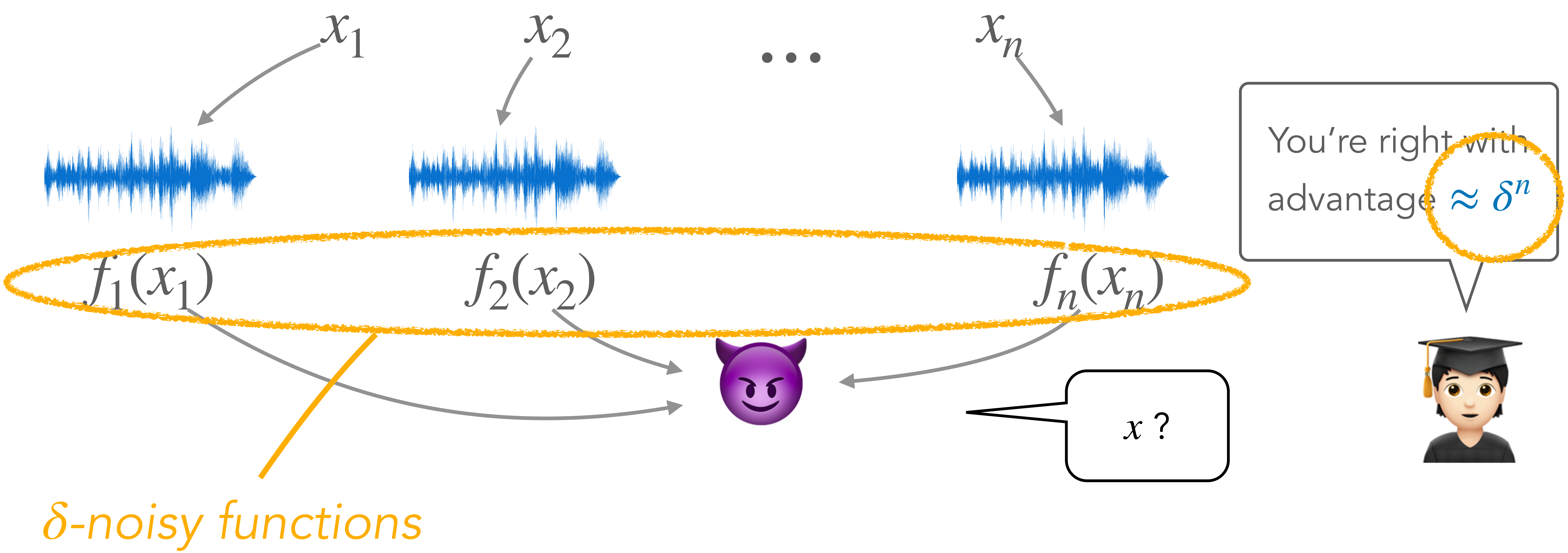
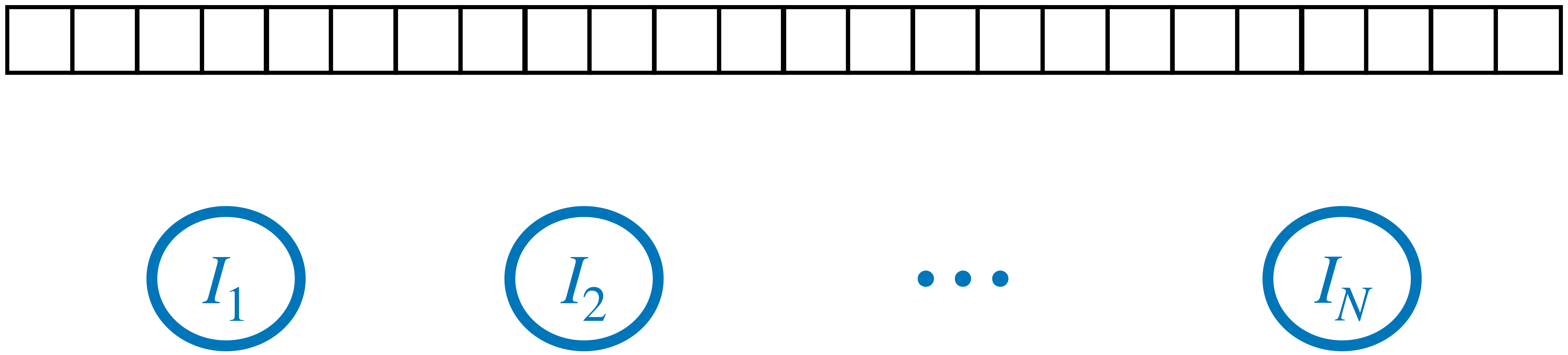


Simulation security

Secret input



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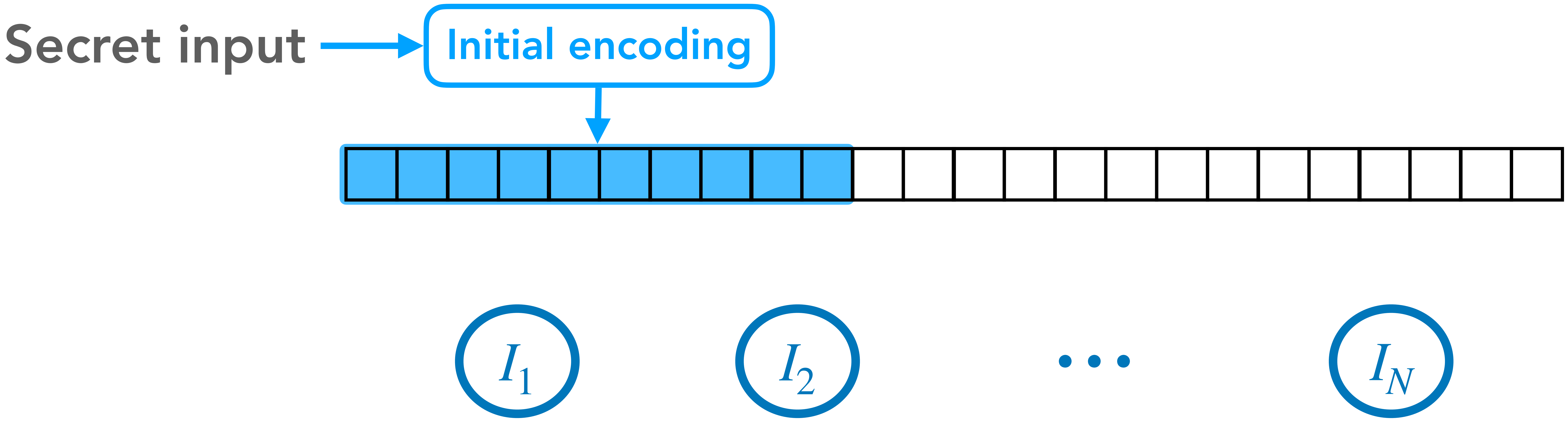

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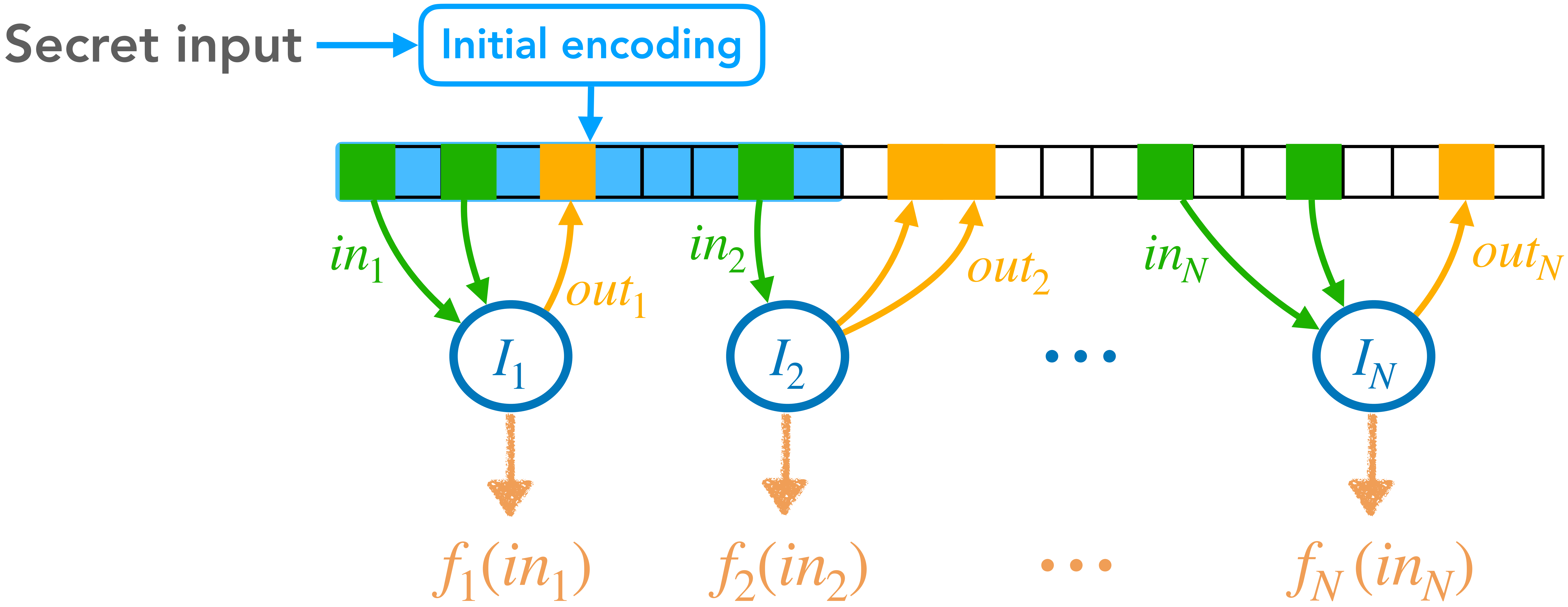

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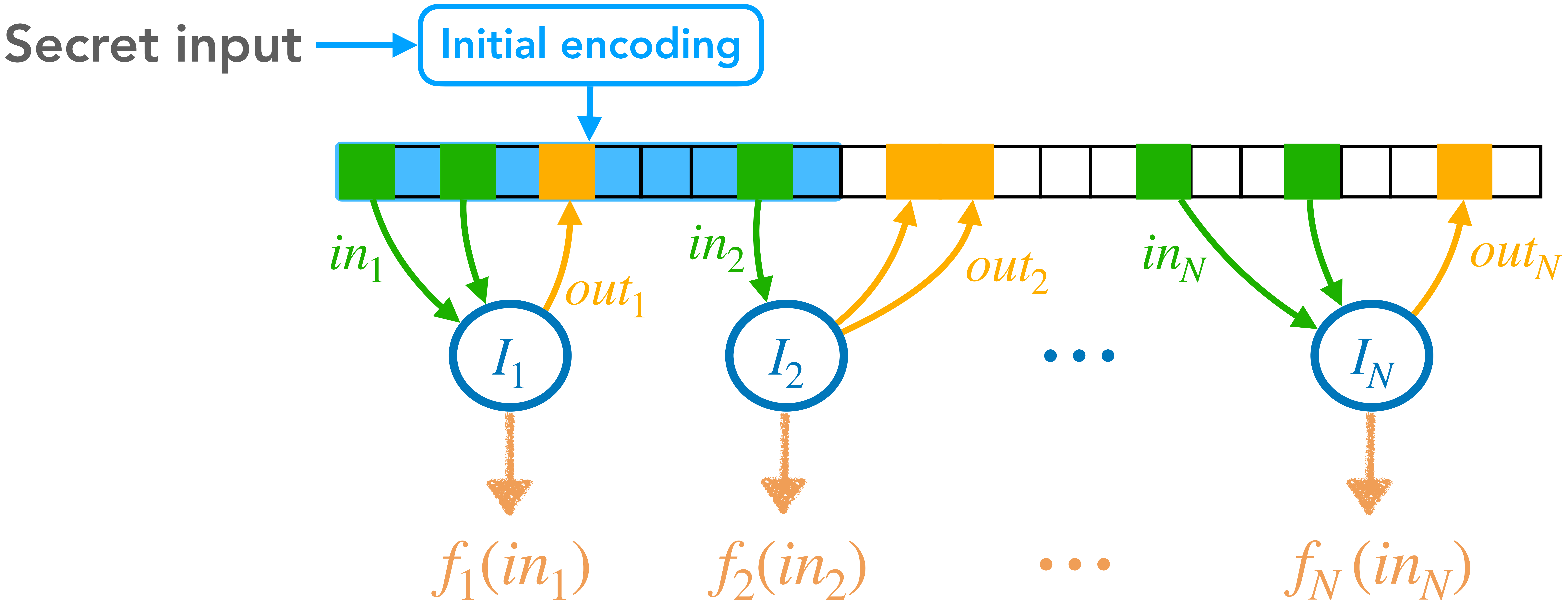

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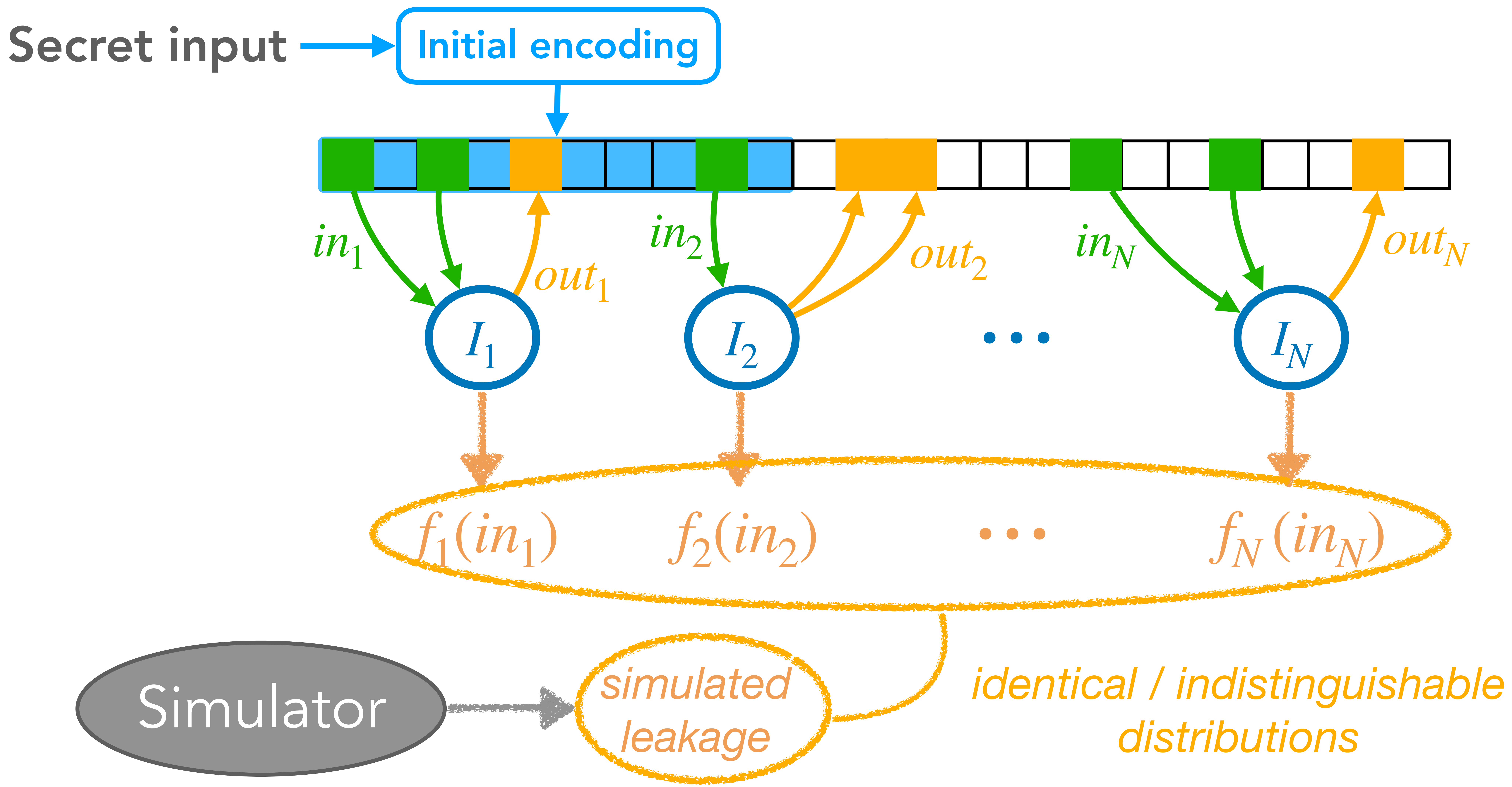

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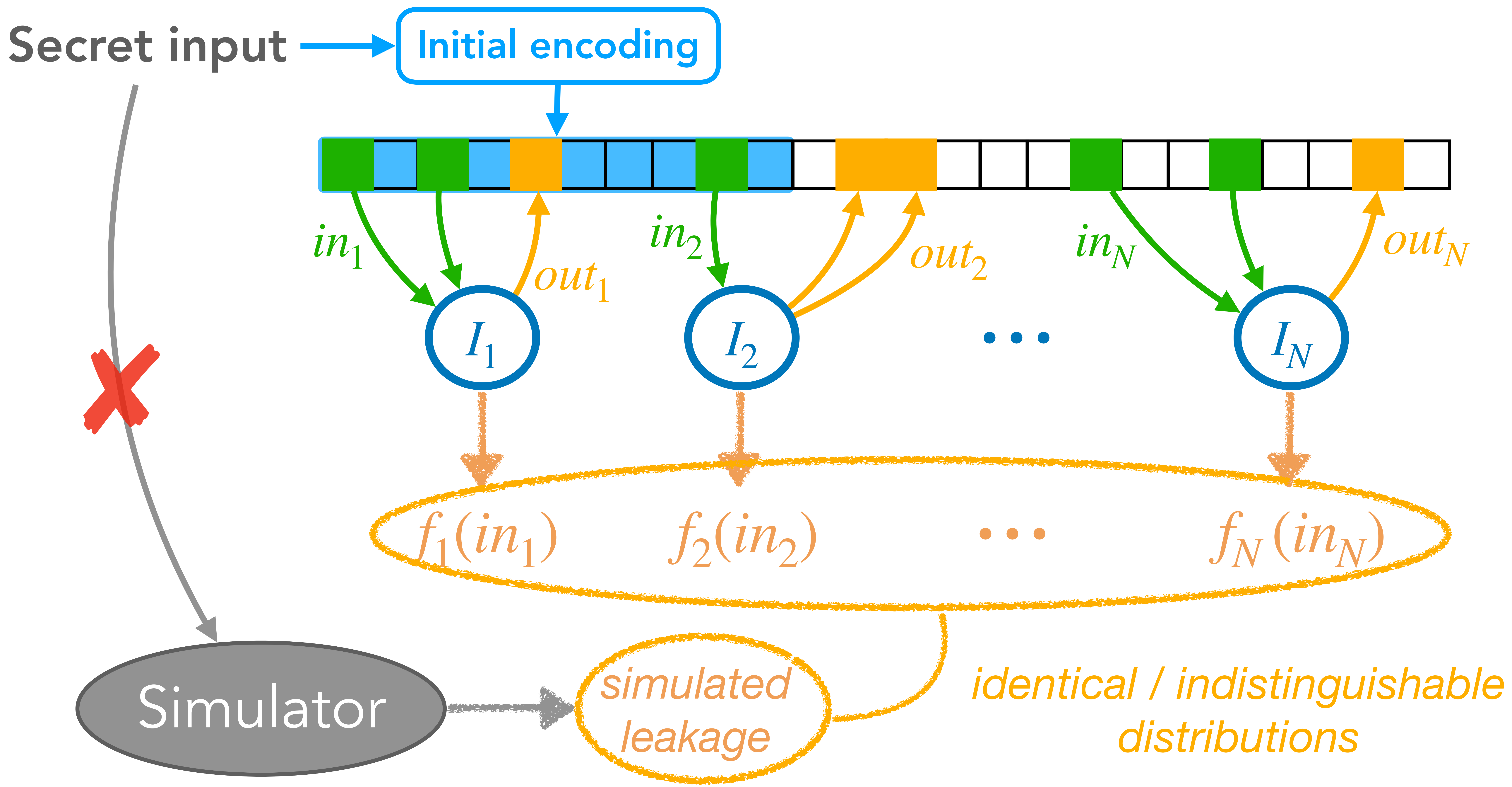


Masked computation

Masking

sharing

the shares

$$x \longrightarrow \vec{x} = (x_1, \ldots, x_n)$$

$$x = x_1 + \cdots + x_n$$

(on a field $\mathbb{K}$)

number of shares

Masking

$$x \longrightarrow \vec{x} = (x_1, \ldots, x_n)$$

sharing — the shares

$$x = x_1 + \cdots + x_n$$

(on a field $\mathbb{K}$)

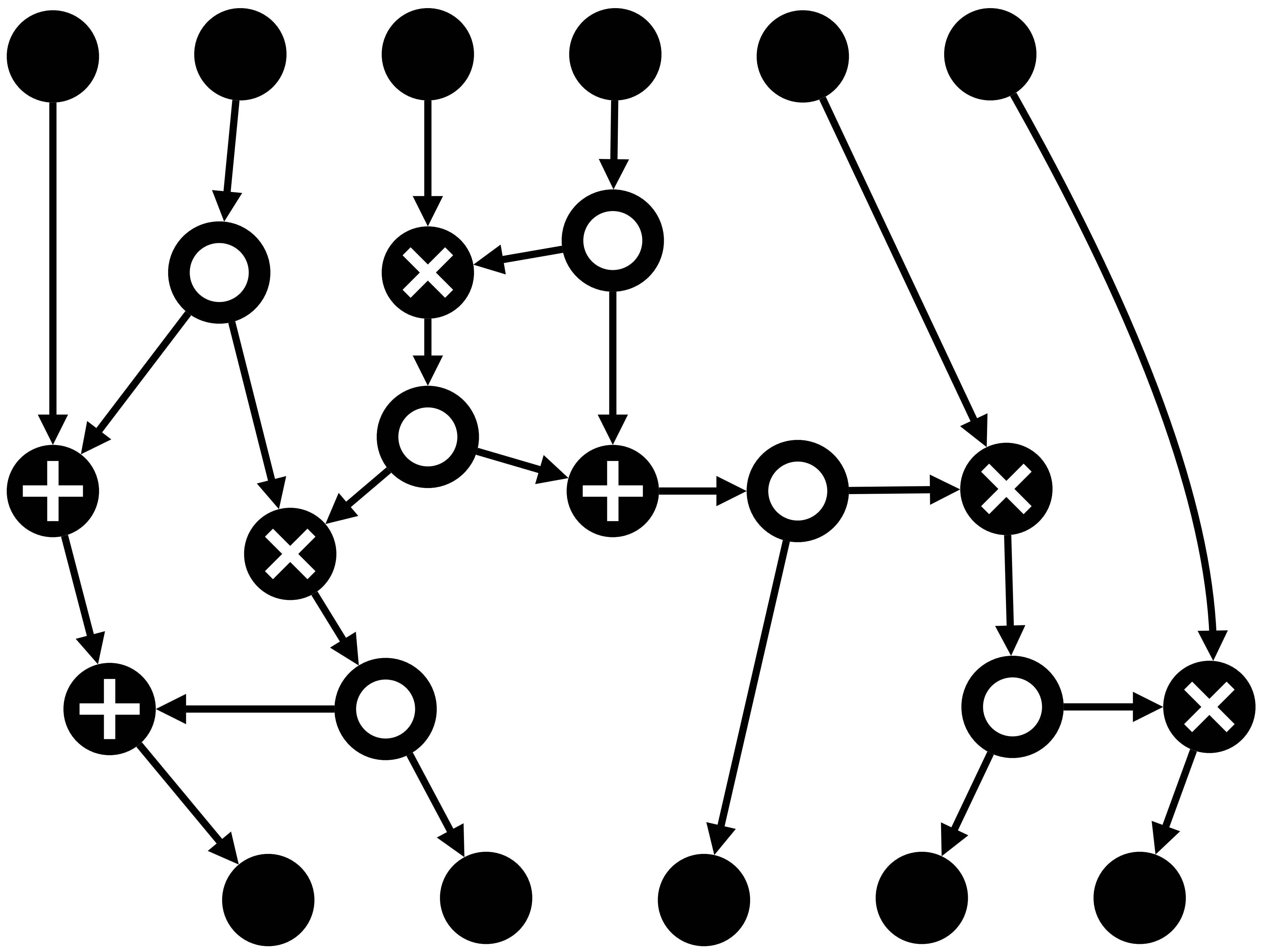
number of shares

⚠️ all the shares are necessary to recover x

🎲 any $n - 1$ shares are completely random

Circuit model

Crypto computation modelled as an arithmetic circuit on $\mathbb{K}$



Circuit model

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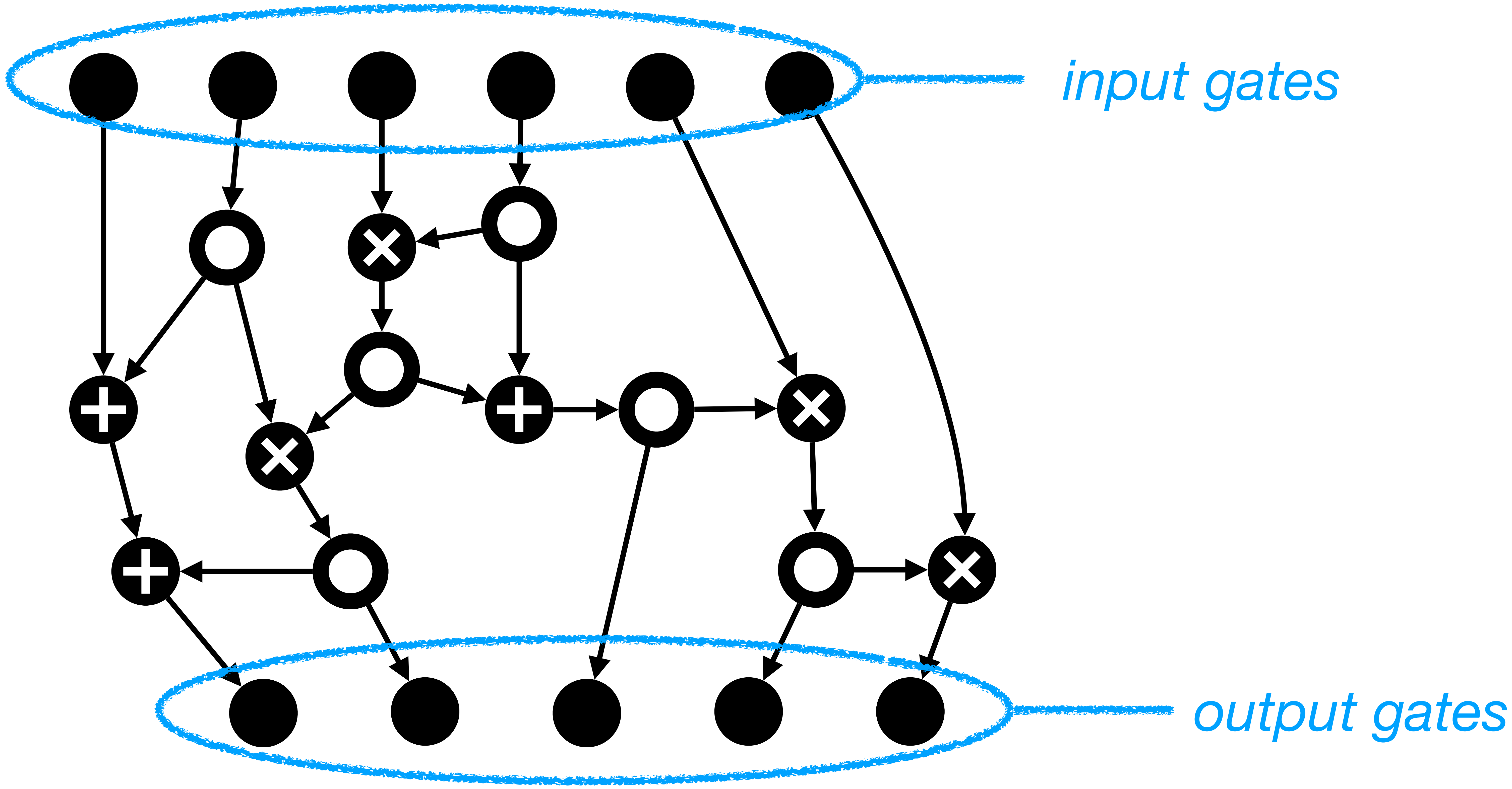

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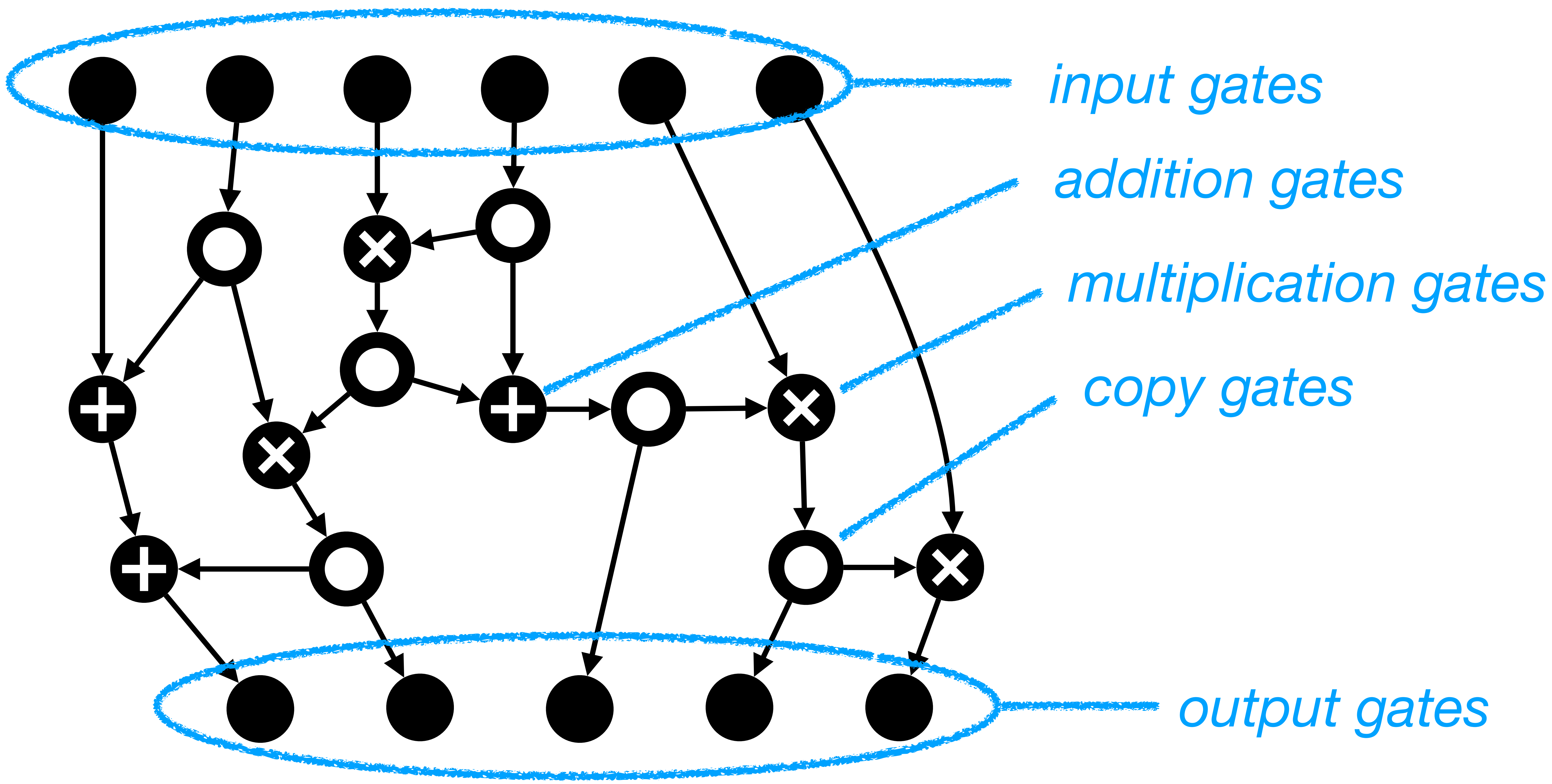

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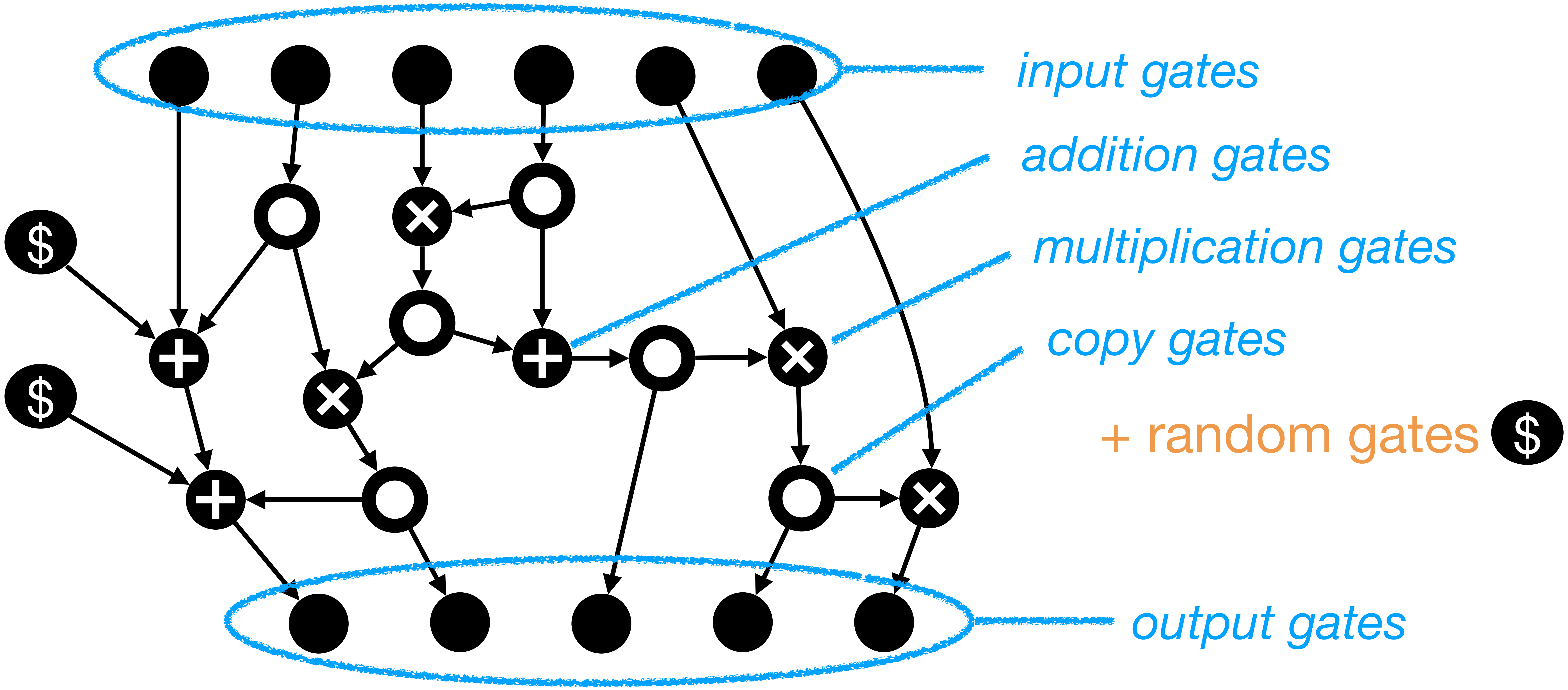

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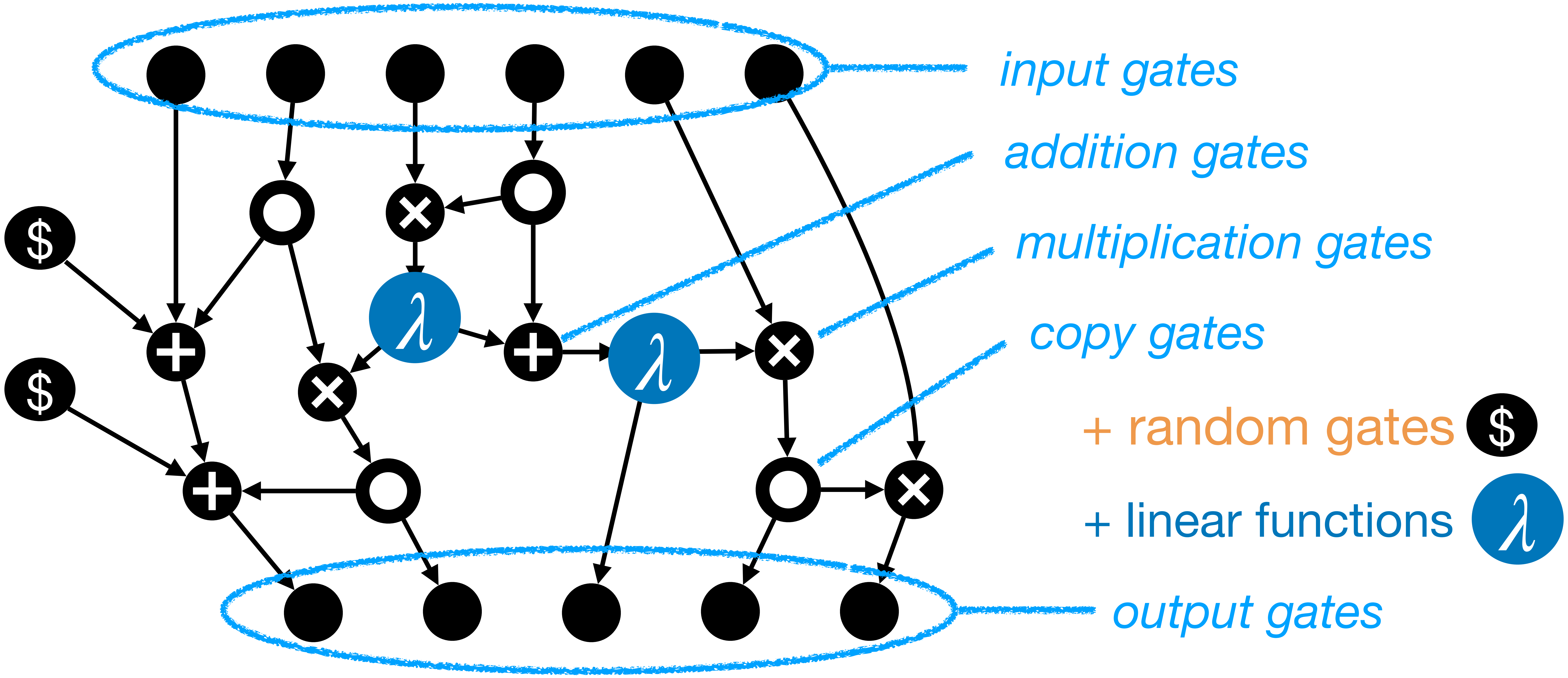
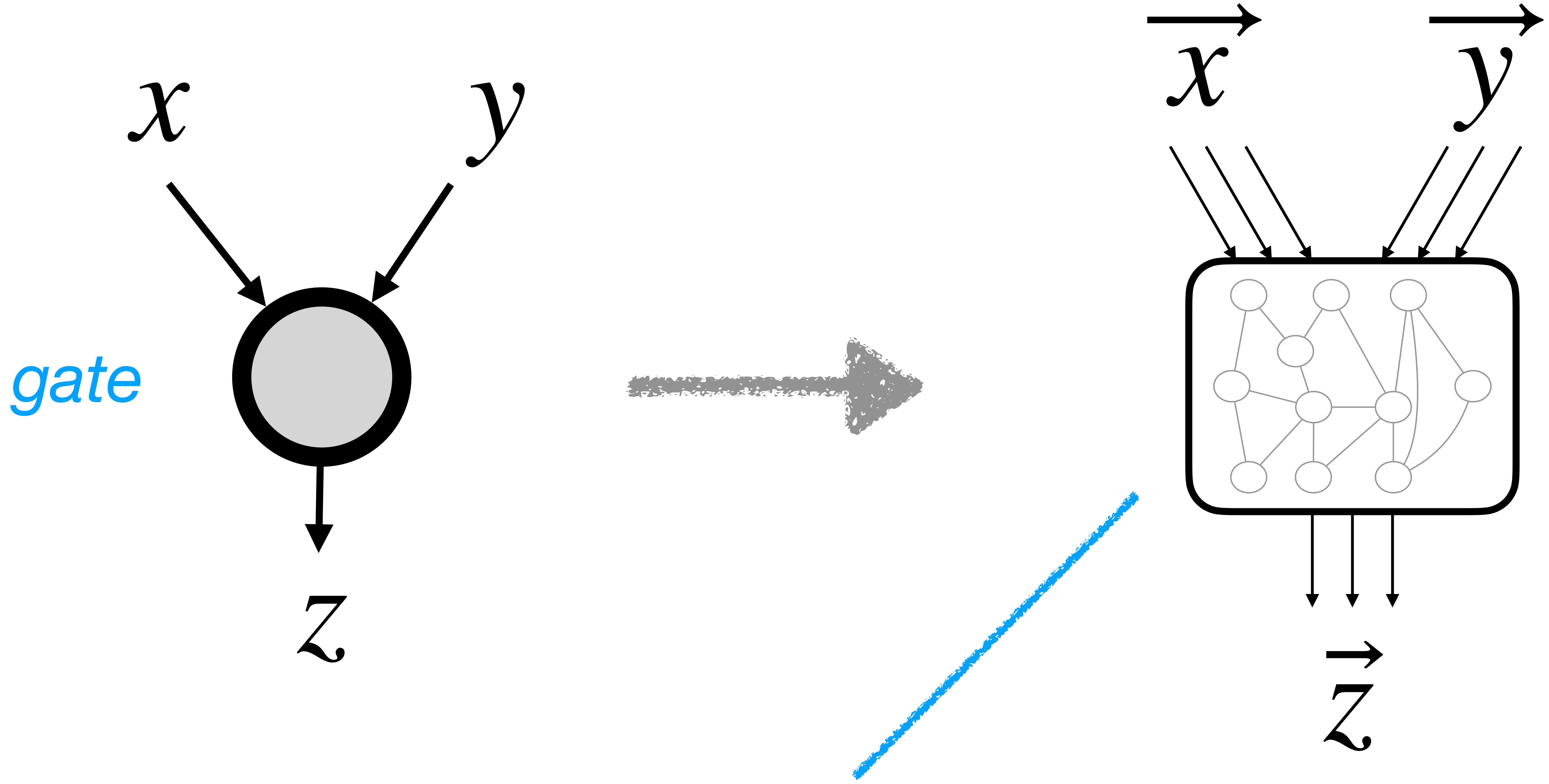
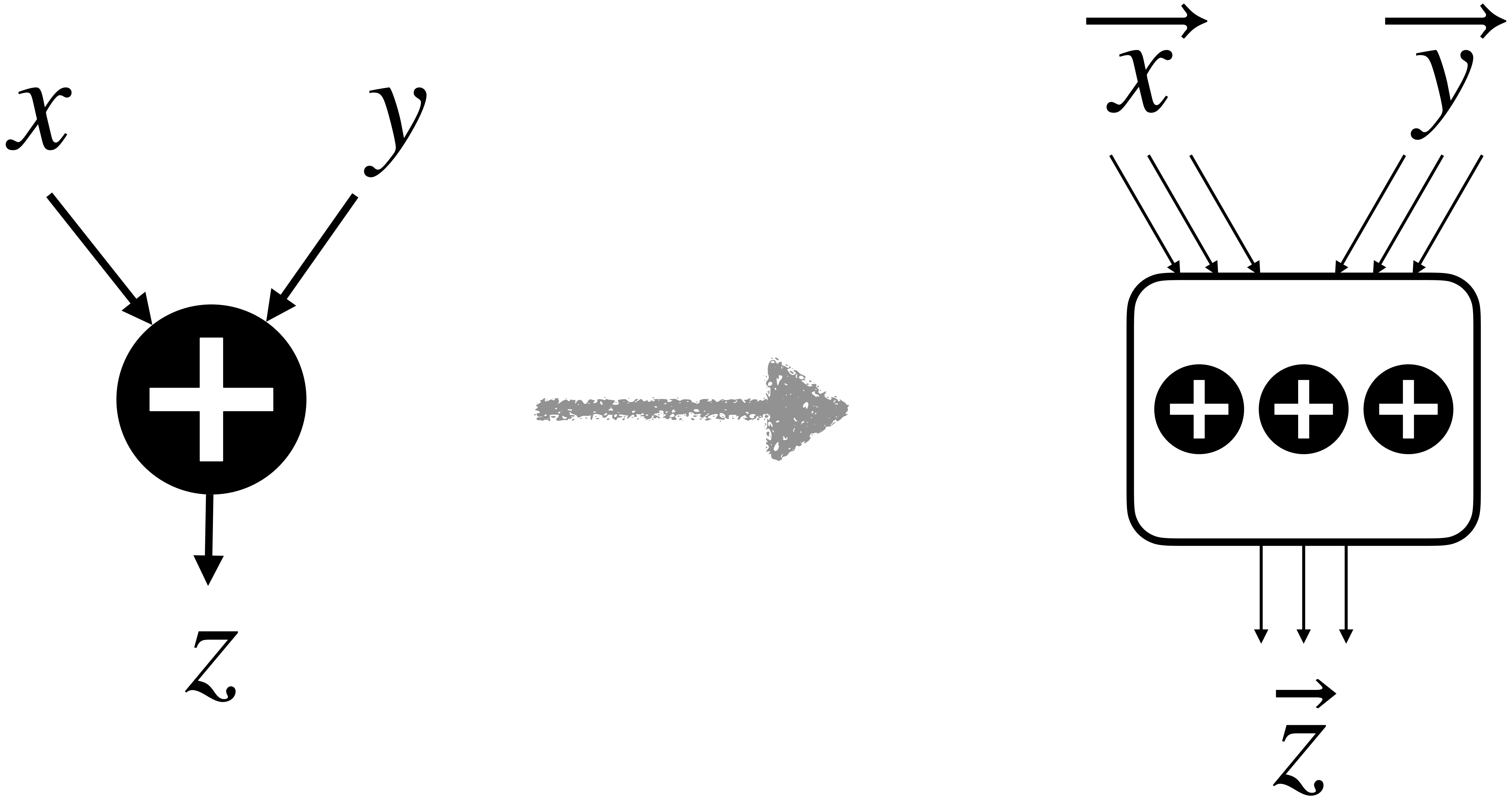


Gadgets



gadget : small circuit computing an operation on sharings

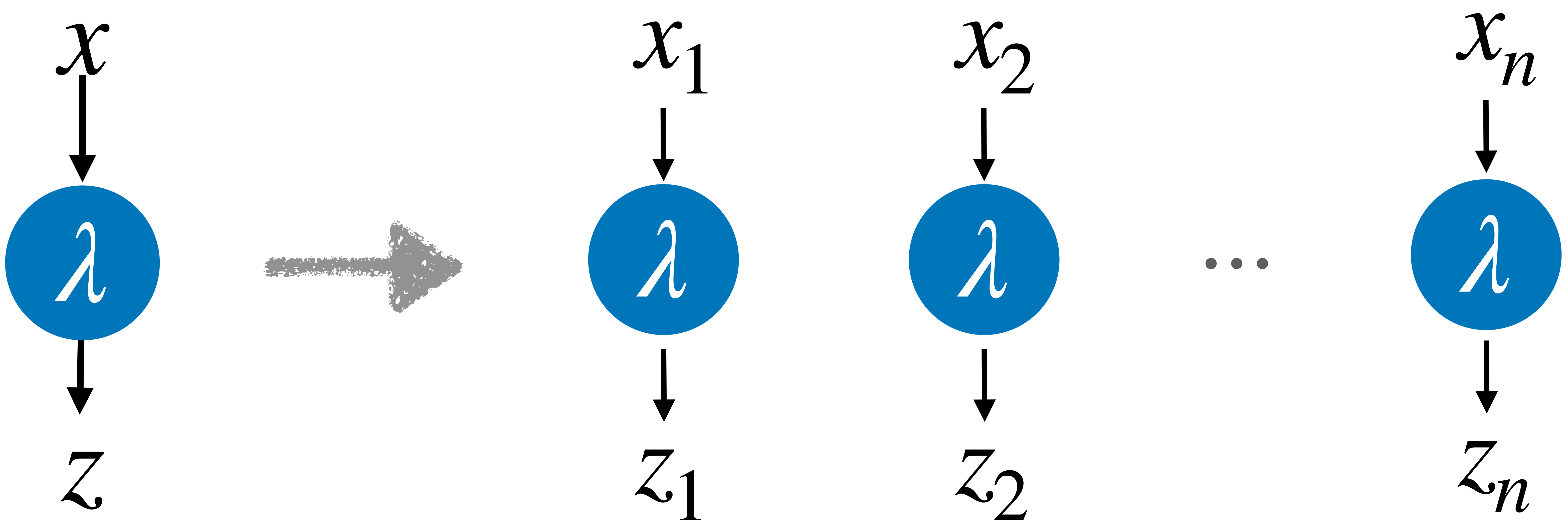
Adition gadget



sharewise computation

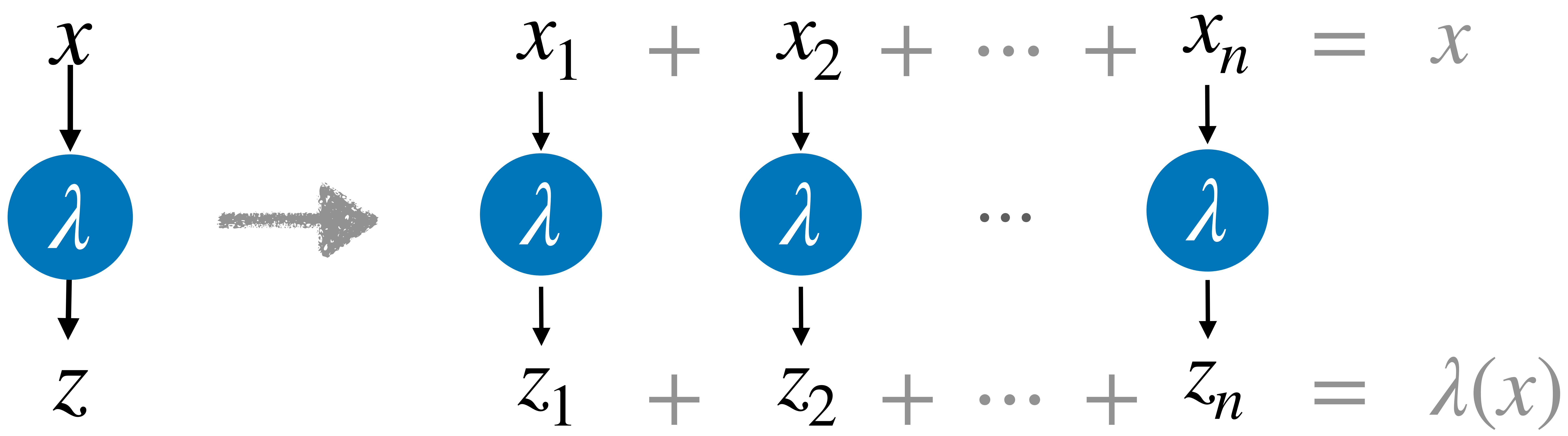
⇒ n addition gates

Linear gadget



sharewise computation $\Rightarrow$ n *evaluations of* λ

Linear gadget



sharewise computation $\Rightarrow$ *n evaluations of λ*

Multiplication gadget

$$z = x \cdot y = \left(\sum_i x_i\right)\left(\sum_i y_i\right) = \sum_{i,j} x_i y_j$$

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$$z = x \cdot y = \left(\sum_i x_i\right)\left(\sum_i y_i\right) = \sum_{i,j} x_i y_j$$

split into n shares

$$z_1$$

$$z_2$$

$$\vdots$$

$$z_n$$

Multiplication gadget

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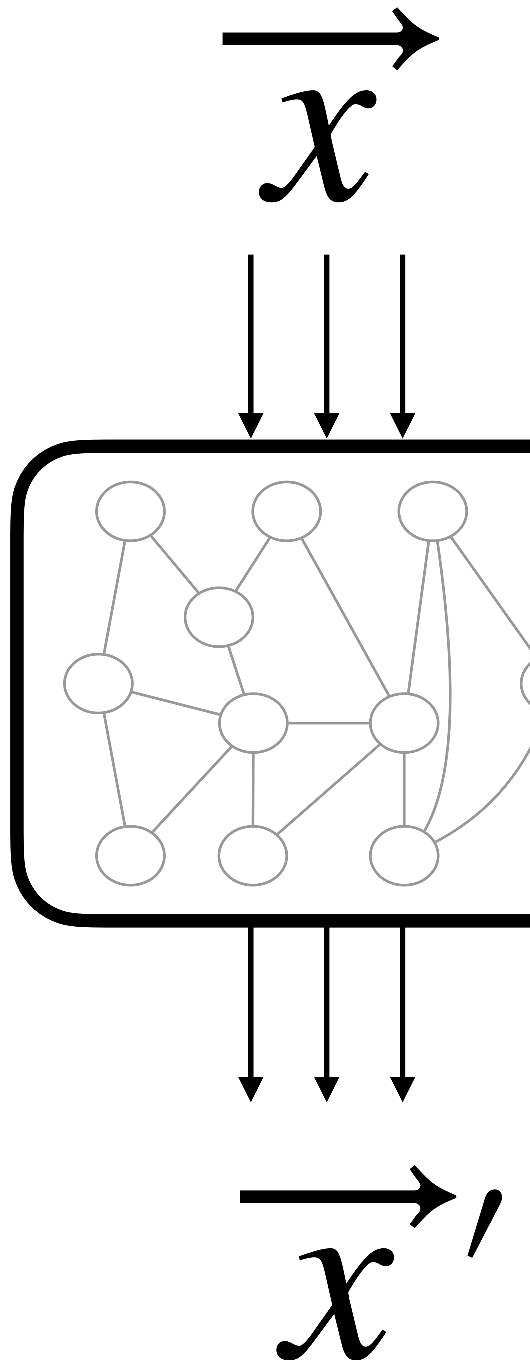
$\vdots$

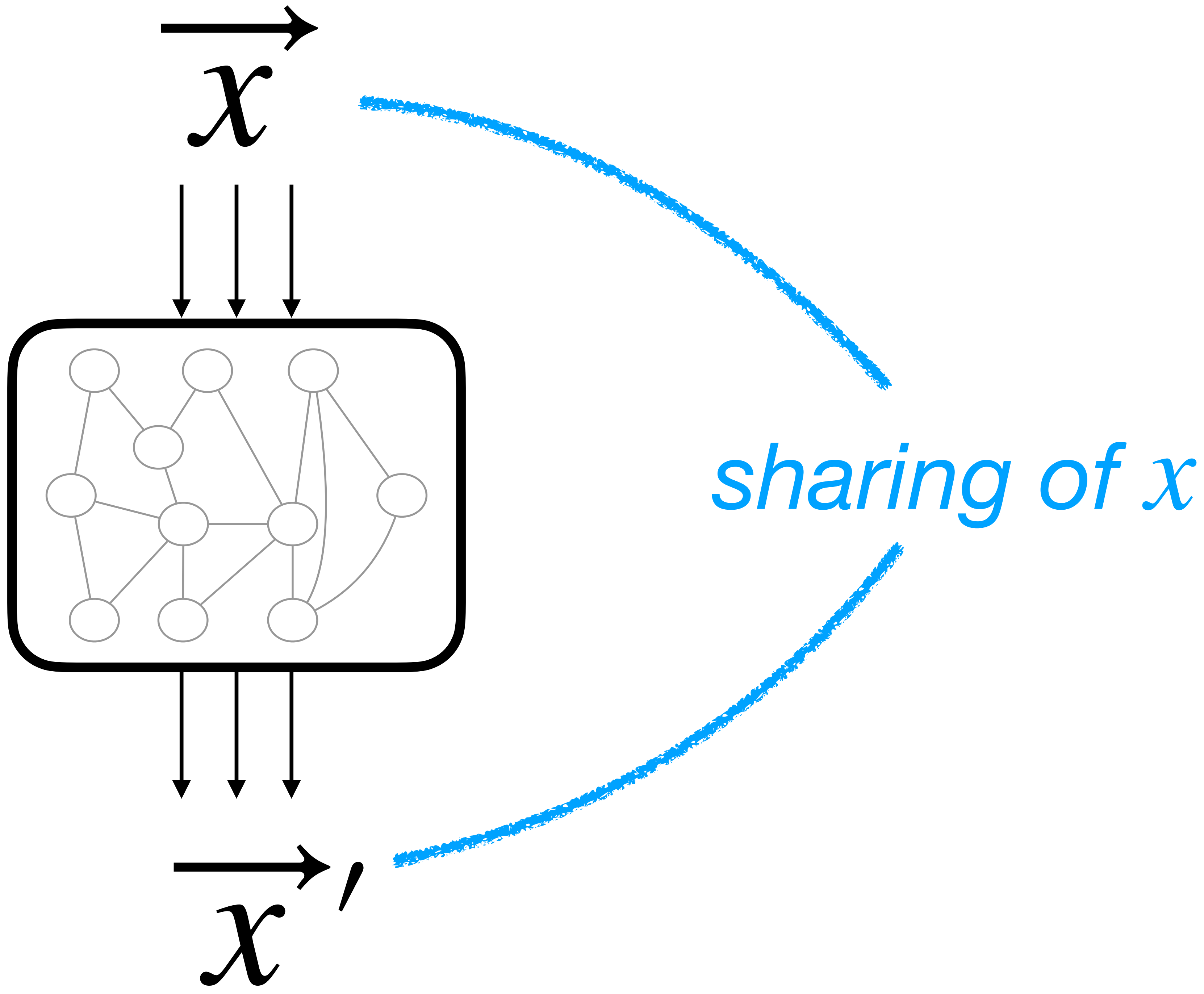
z_n

+ fresh randomness

Refresh gadget



Refresh gadget

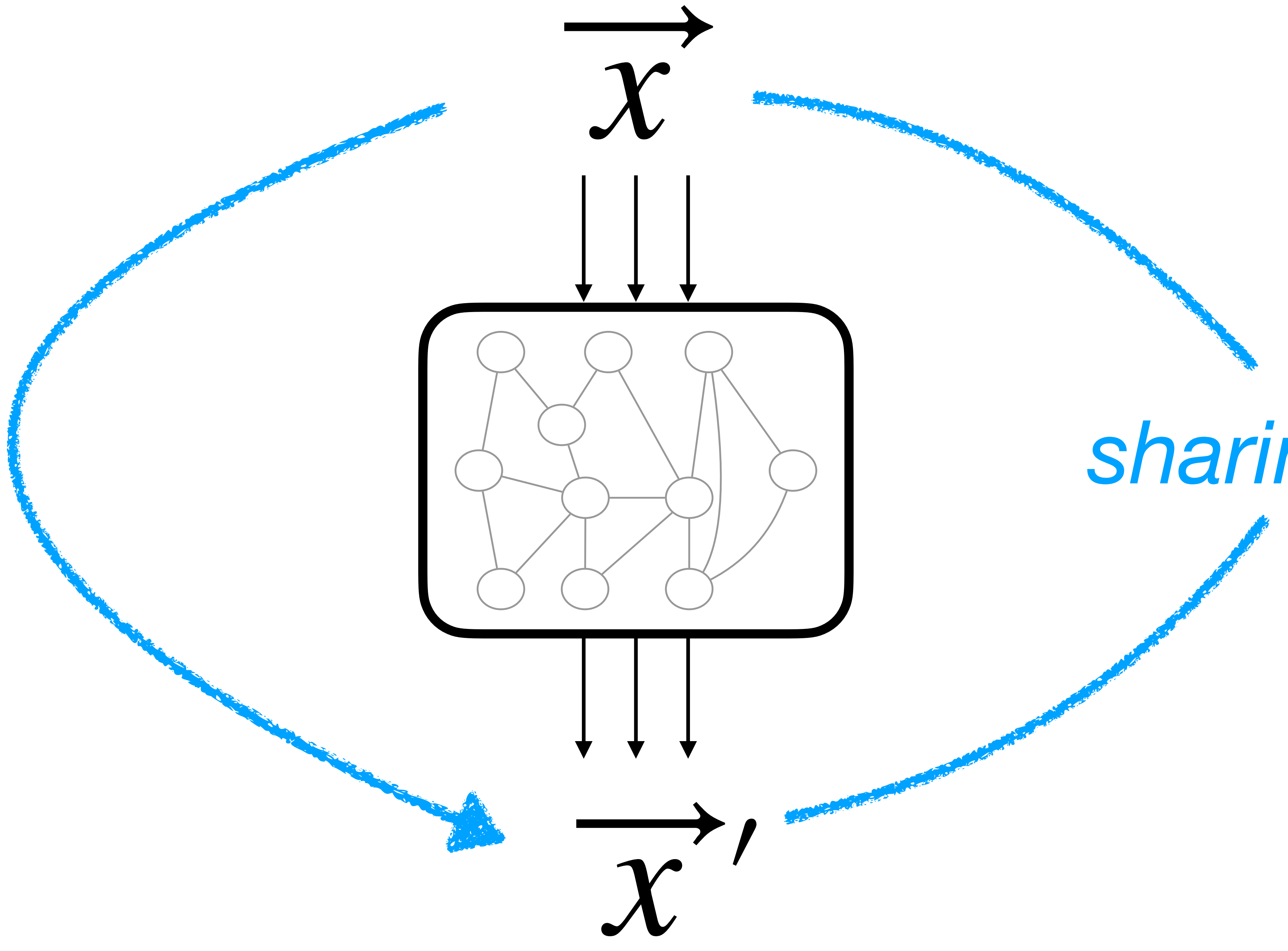


Refresh gadget

fresh randomness

$\Rightarrow$

statistical independence

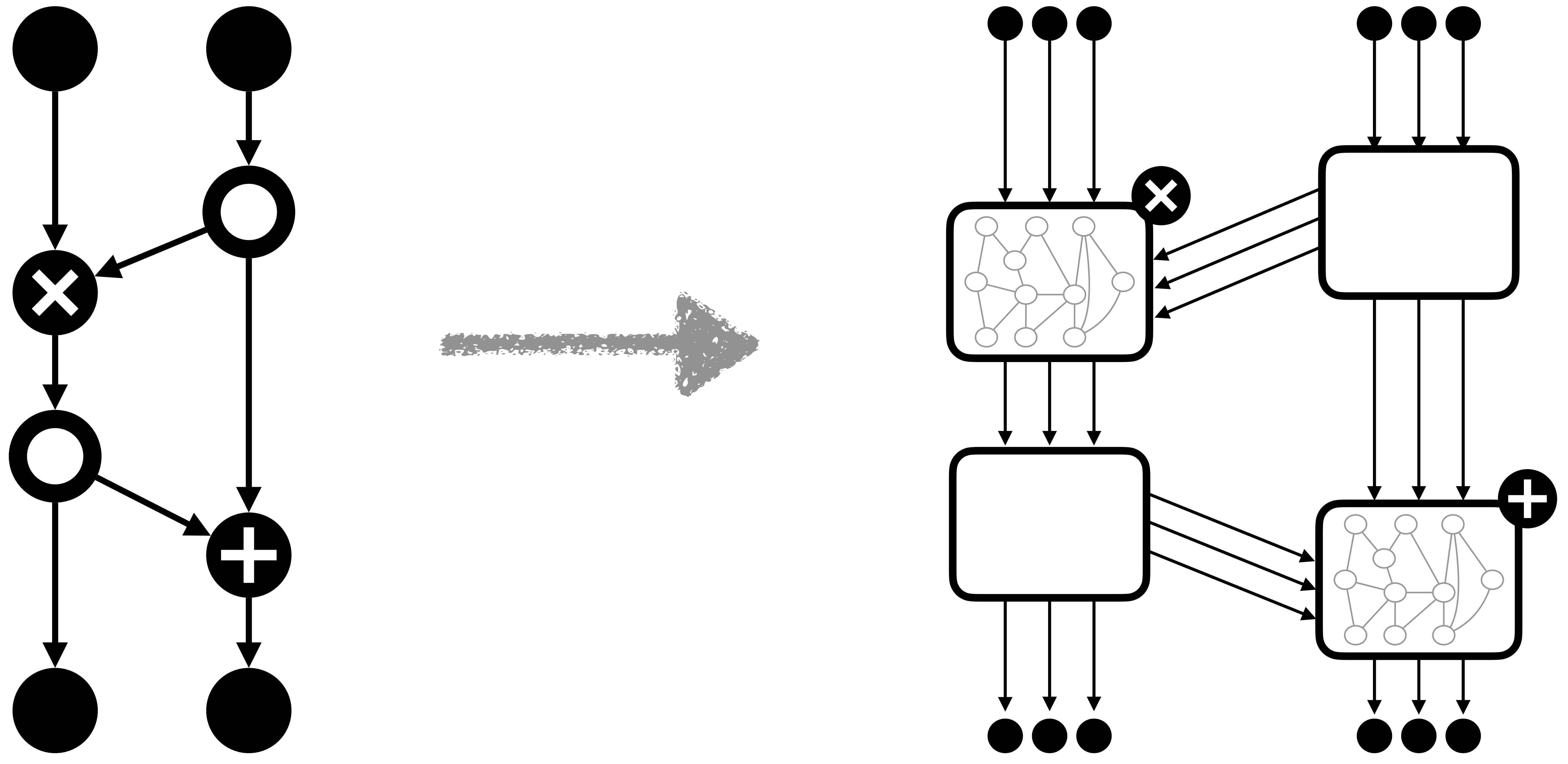


sharing of x

Standard circuit compiler

wire $\to$ n wires (sharing)

gate $\to$ gadget



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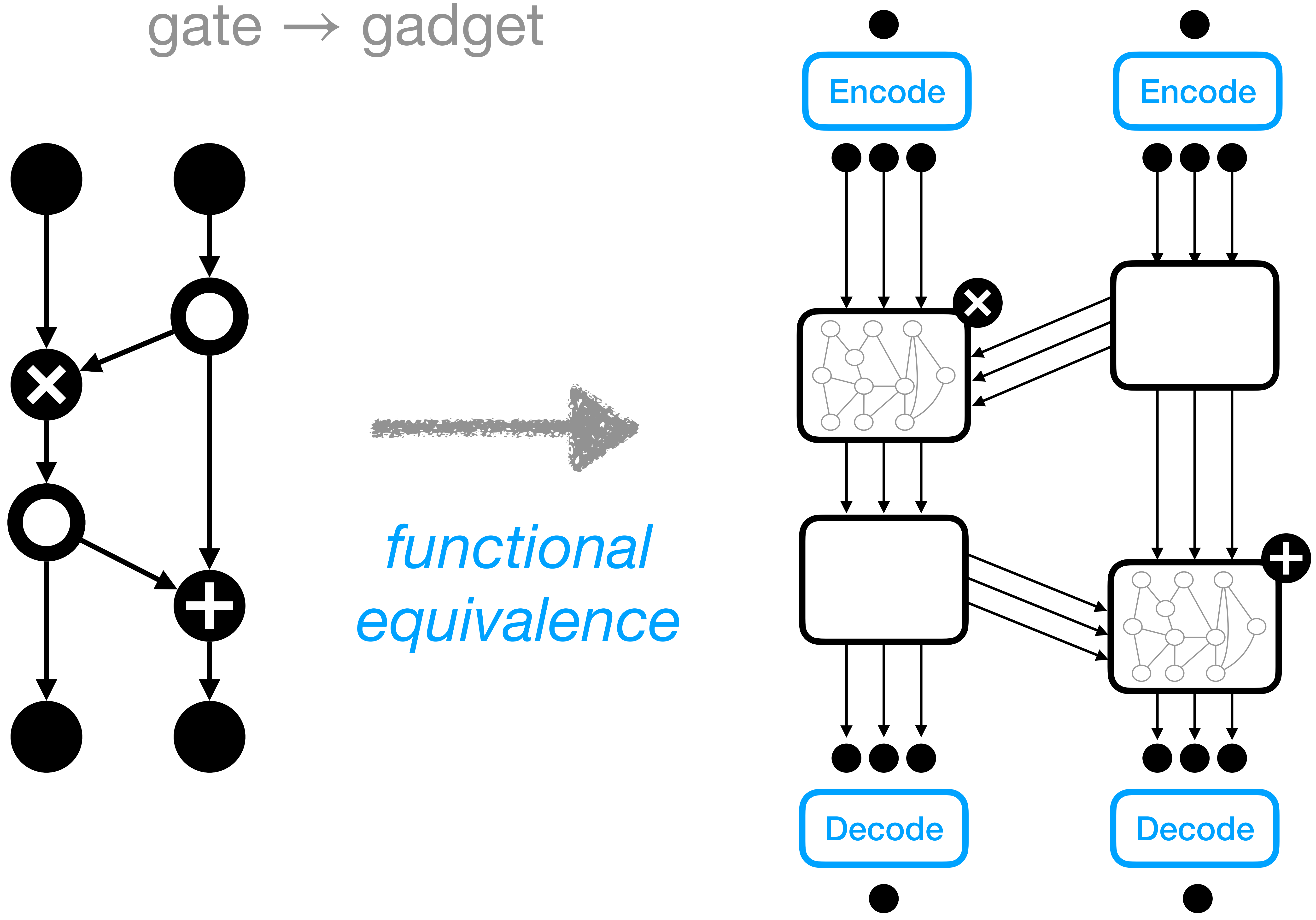

Standard circuit compiler

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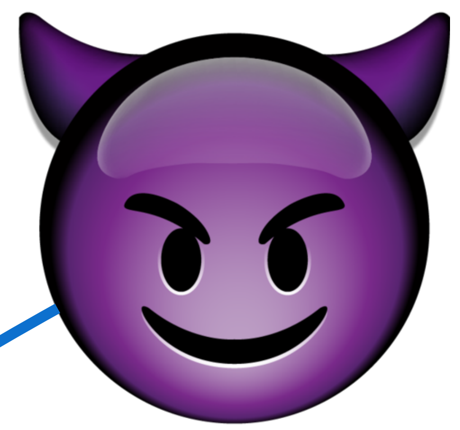
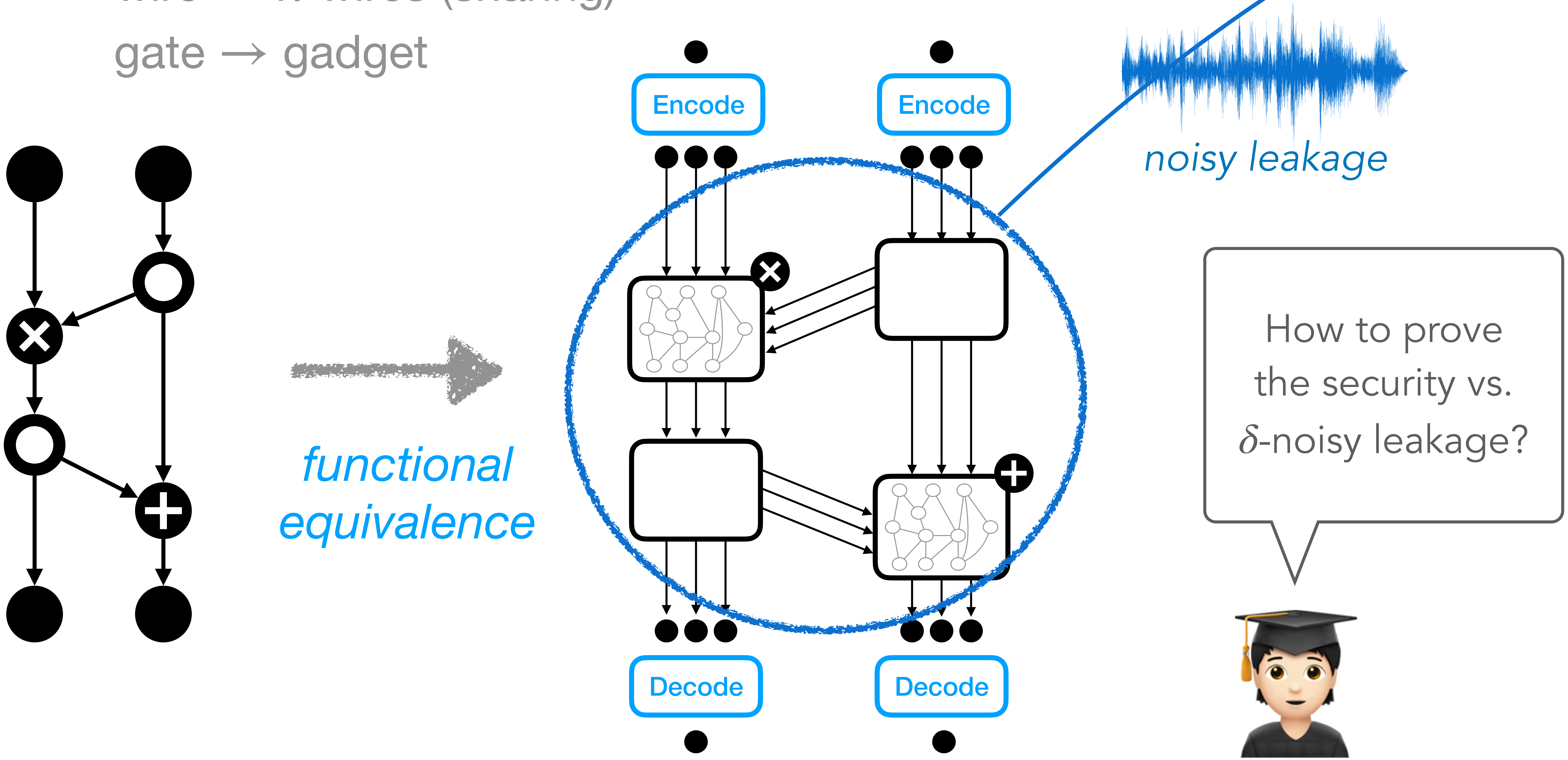
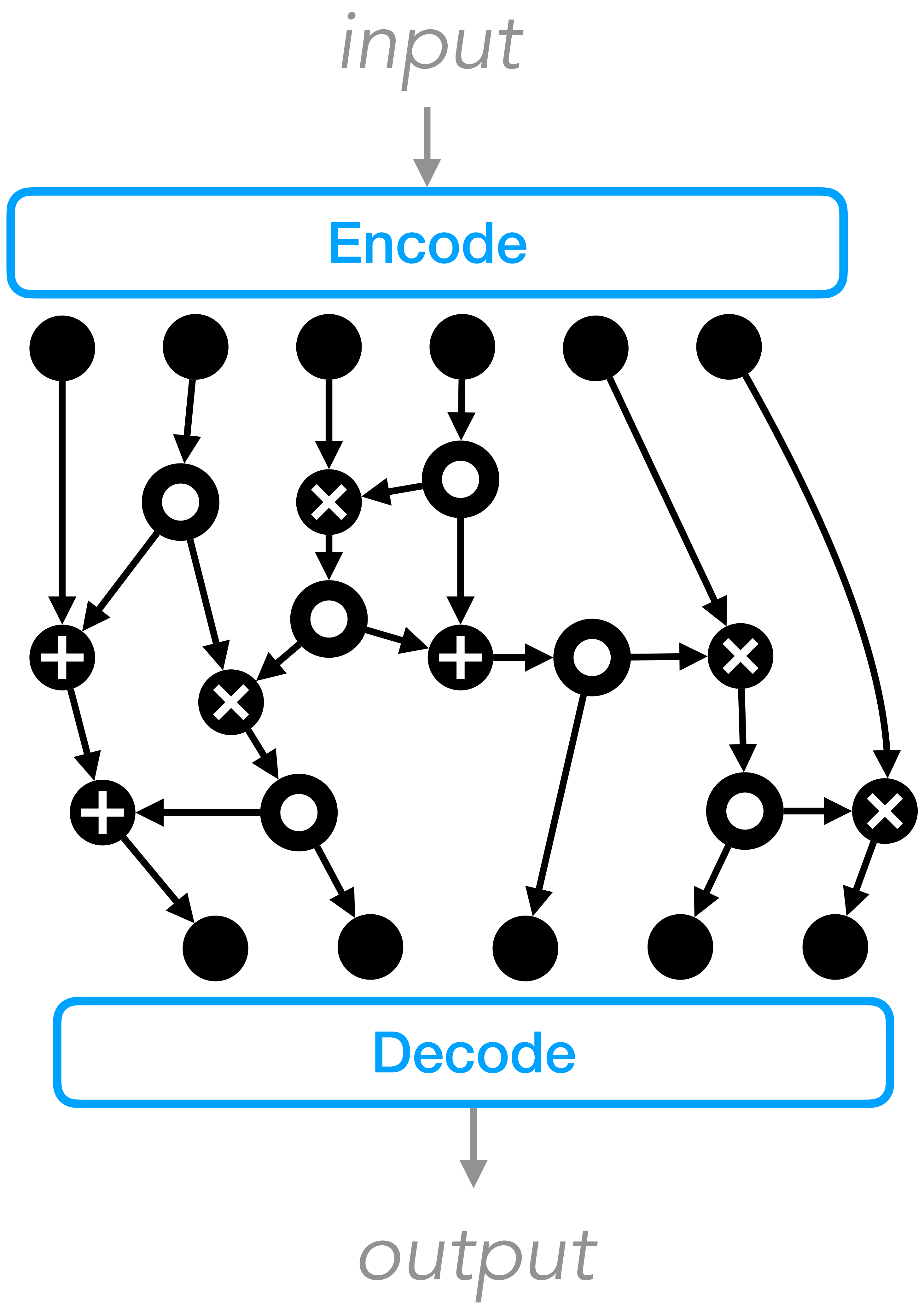




Probing models

Probing model



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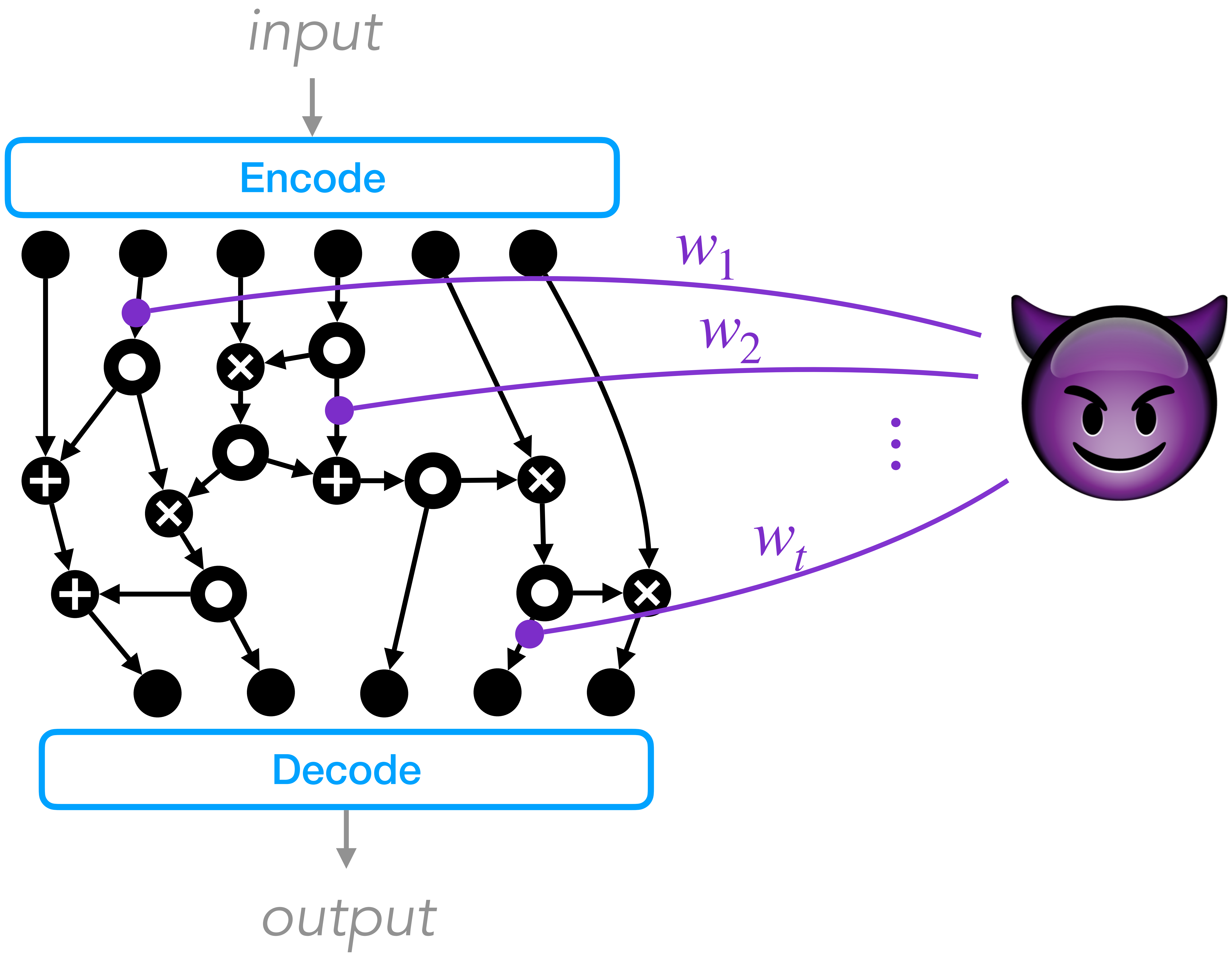
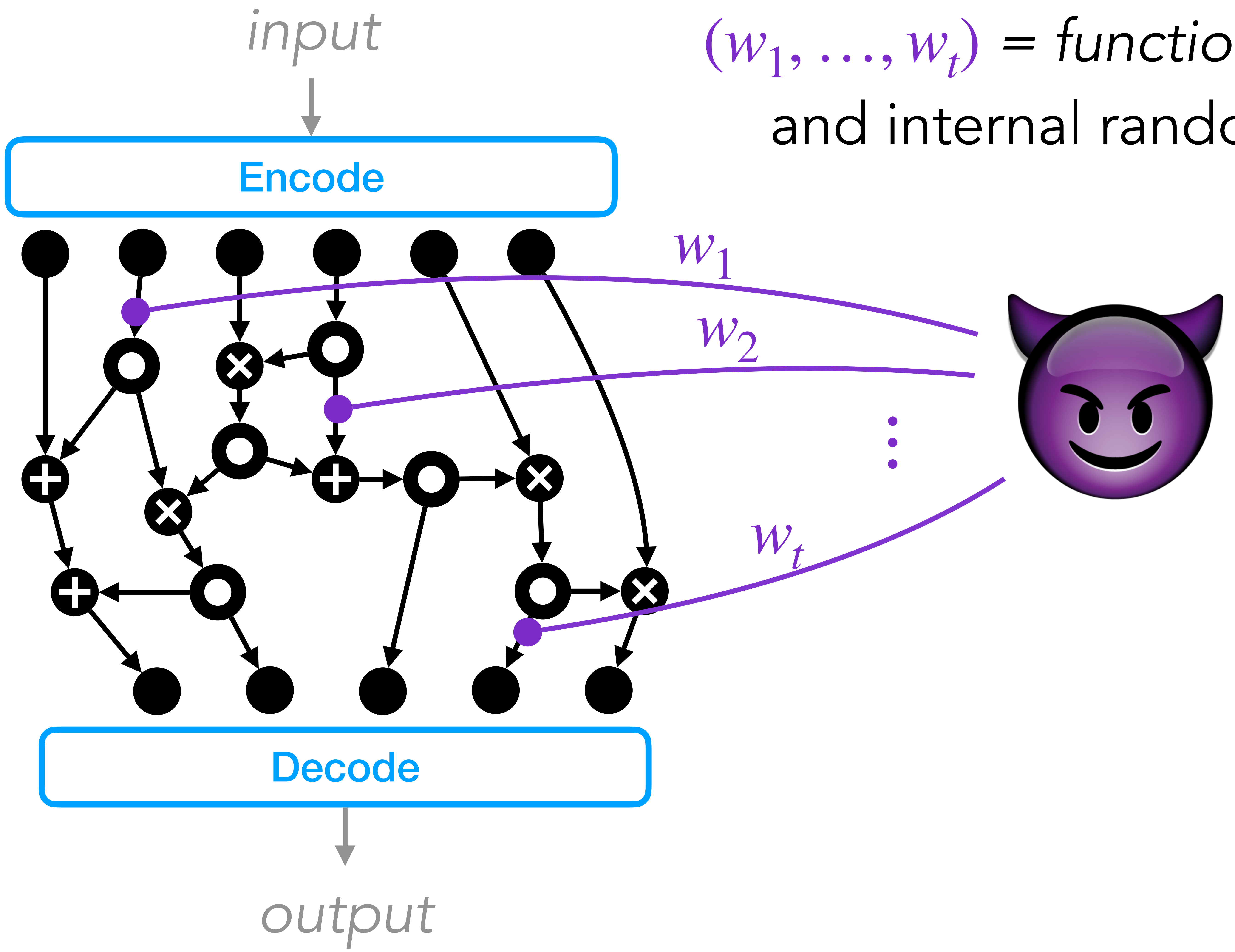


Probing model

$(w_1, \ldots, w_t)$ = *function of input*
and internal randomness



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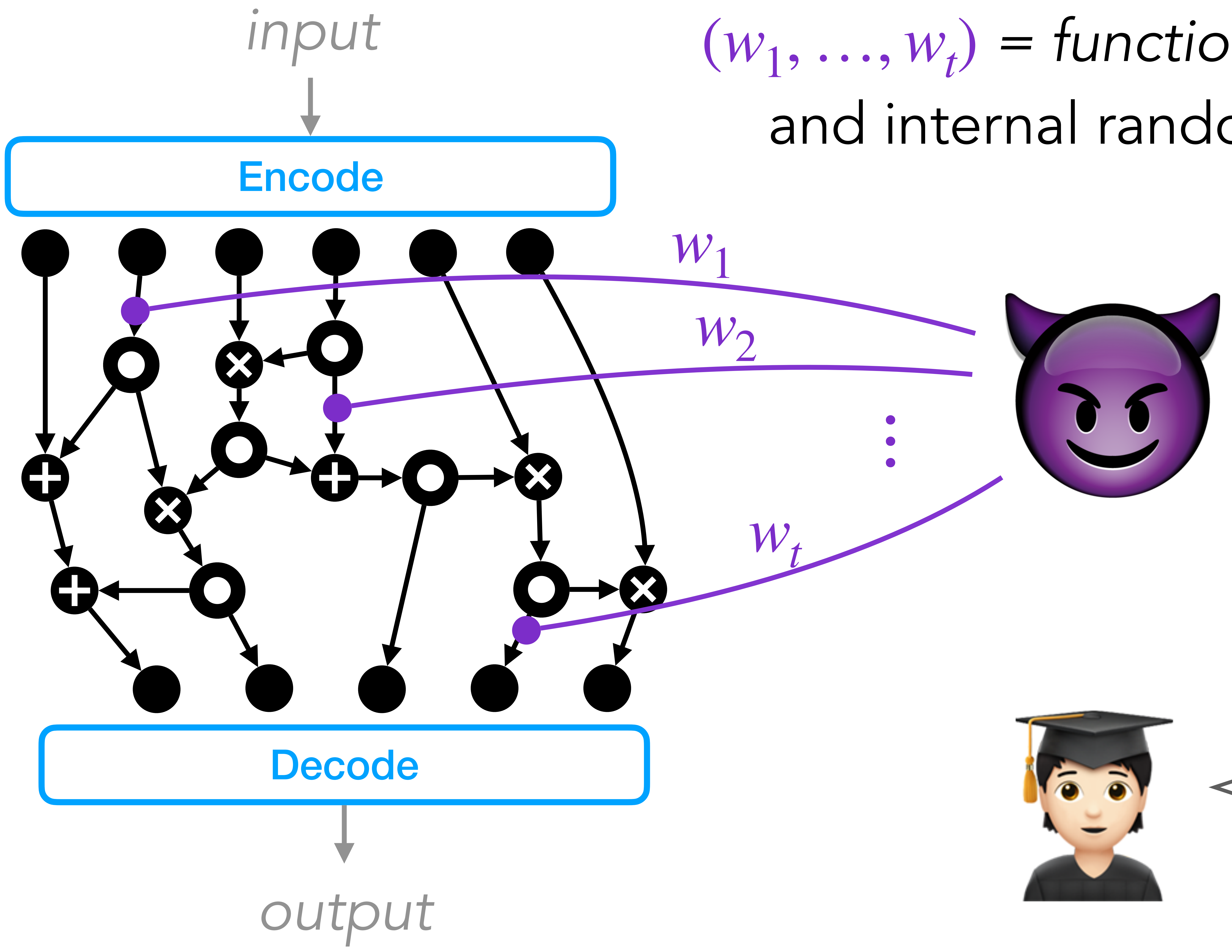


$(w_1, \ldots, w_t)$ = *function of* *input* and internal randomness

t-probing security:

$(w_1, \ldots, w_t)$ *can be perfectly simulated w/o any knowledge about the input*

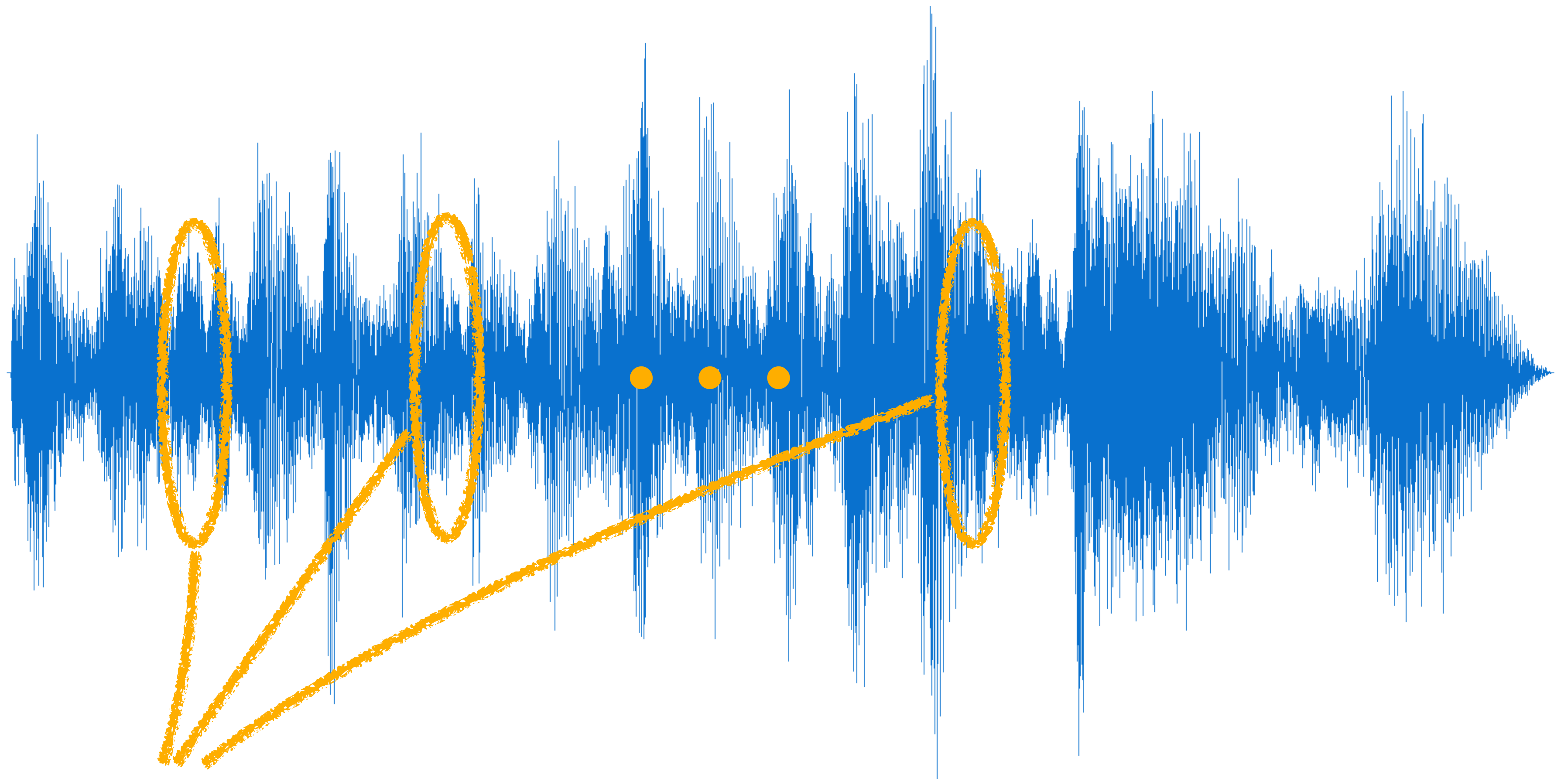
Probing model

t-probing security

$\Leftrightarrow$

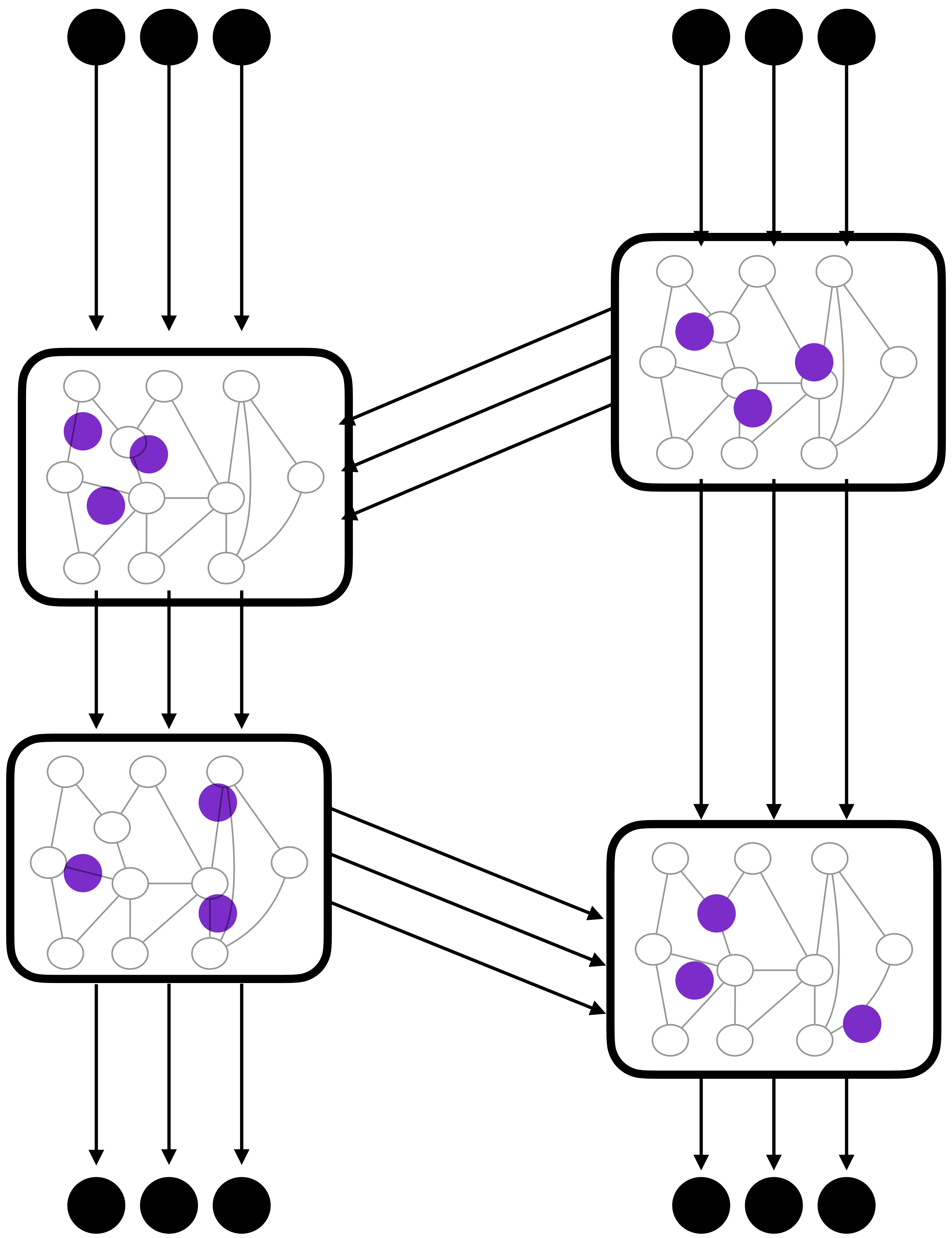
security against

t-order DPA



any t leakage points
independent of the secrets

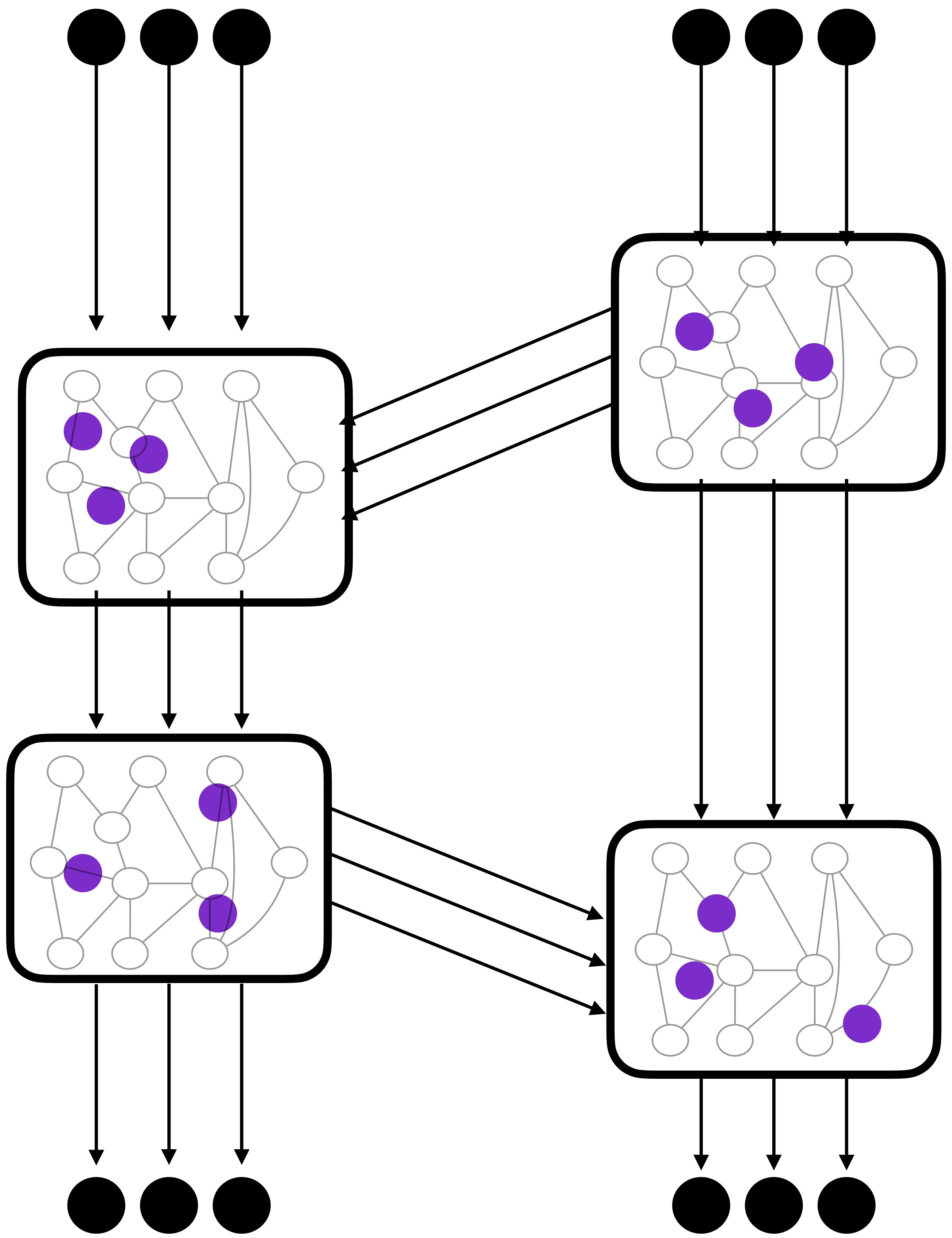
Region probing model



t probes per gadget (or region)

with $t = r \times |G|$

Region probing model



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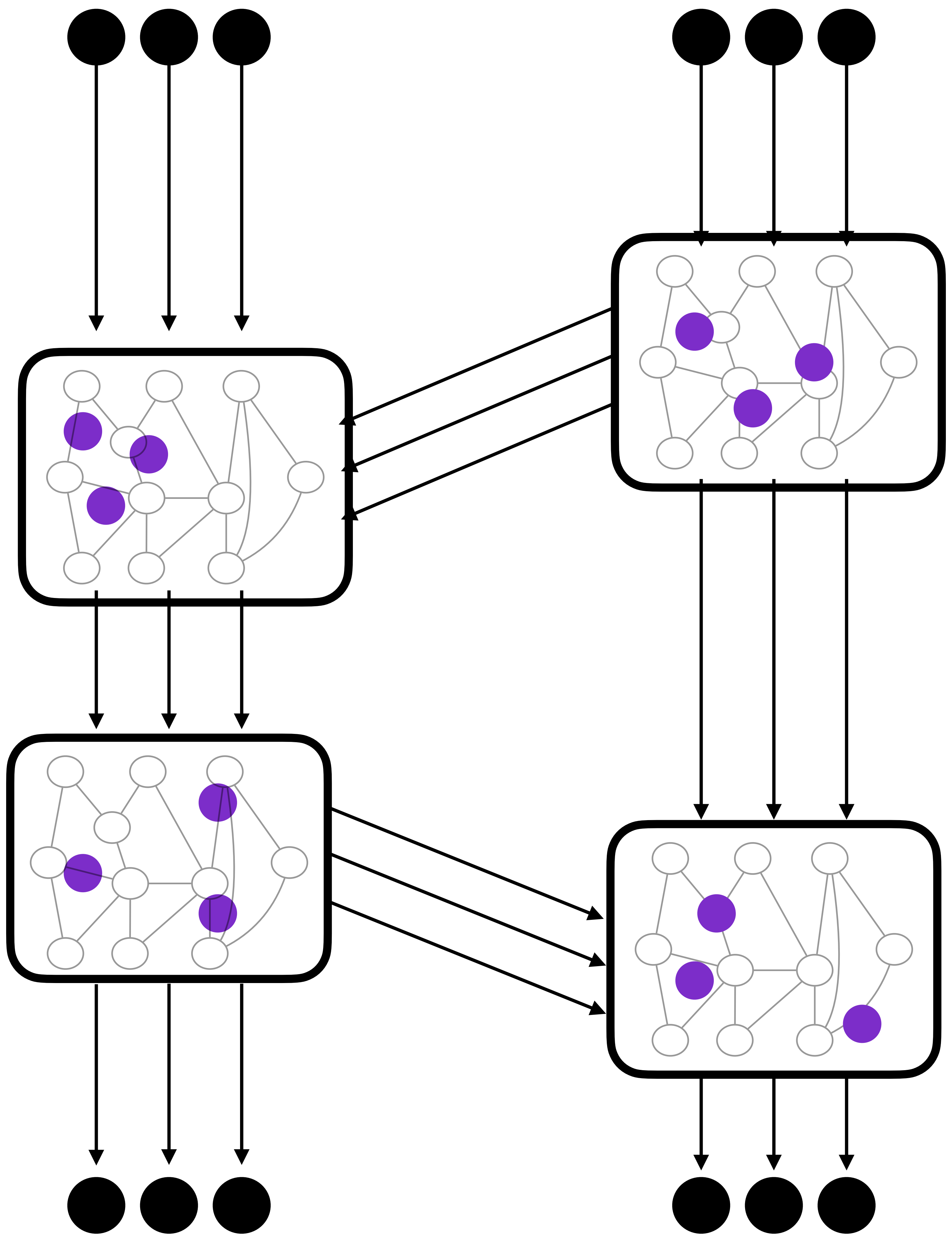
probing rate

number of wires in G

Region probing model



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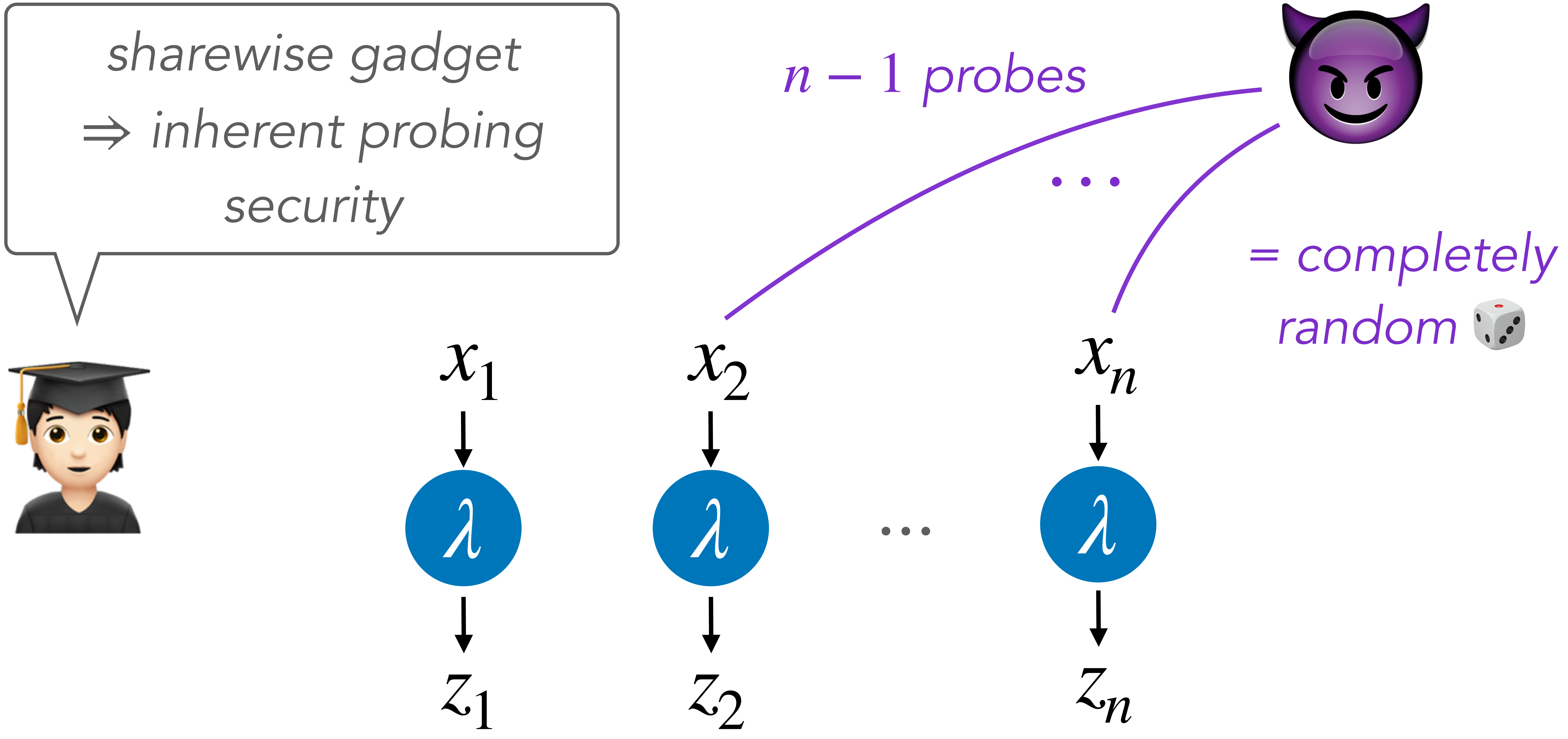
probing rate

number of wires in G

$\Rightarrow$ r-region probing security

Security of sharewise gadgets



ISW multiplication gadget

$$\begin{pmatrix} x_1y_1 & x_1y_2 & x_1y_3 \\ & x_2y_2 & x_2y_3 \\ & & x_3y_3 \end{pmatrix} + \begin{pmatrix} & & \\ x_2y_1 & & \\ x_3y_1 & x_3y_2 & \end{pmatrix}^{\mathsf{T}}$$

ISW multiplication gadget

cross-products $\sum_{i,j} x_i y_j$

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ISW multiplication gadget

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$$\left(\left(\begin{matrix} x_1y_1 & x_1y_2 & x_1y_3 \\ & x_2y_2 & x_2y_3 \\ & & x_3y_3 \end{matrix}\right) + \left(\begin{matrix} & & \\ x_2y_1 & & \\ x_3y_1 & x_3y_2 & \end{matrix}\right)^{\top}\right) + \left(\begin{matrix} & r_{1,2} & r_{1,3} \\ -r_{1,2} & & r_{2,3} \\ -r_{1,3} & -r_{2,3} & \end{matrix}\right)$$

ISW multiplication gadget

cross-products $\sum_{i,j} x_i y_j$

fresh randomness
(cancelling out)

$$\left(\begin{pmatrix} x_1y_1 & x_1y_2 & x_1y_3 \\ & x_2y_2 & x_2y_3 \\ & & x_3y_3 \end{pmatrix} + \begin{pmatrix} & & \\ x_2y_1 & & \\ x_3y_1 & x_3y_2 & \end{pmatrix}\right)^{\mathsf{T}} + \begin{pmatrix} & r_{1,2} & r_{1,3} \\ -r_{1,2} & & r_{2,3} \\ -r_{1,3} & -r_{2,3} & \end{pmatrix}$$

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row sum →

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

ISW multiplication gadget

cross-products $\sum_{i,j} x_i y_j$

fresh randomness (cancelling out)

$$\begin{pmatrix} x_1y_1 & x_1y_2 & x_1y_3 \\ & x_2y_2 & x_2y_3 \\ & & x_3y_3 \end{pmatrix} + \begin{pmatrix} & & \\ x_2y_1 & & \\ x_3y_1 & x_3y_2 & \end{pmatrix}^{\top} + \begin{pmatrix} & r_{1,2} & r_{1,3} \\ -r_{1,2} & & r_{2,3} \\ -r_{1,3} & -r_{2,3} & \end{pmatrix}$$

row sum → $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$

t probes ⇒ info on at most t shares

Composition security

⚠️ probing security for gadgets $\not\Longrightarrow$ global (region) probing security

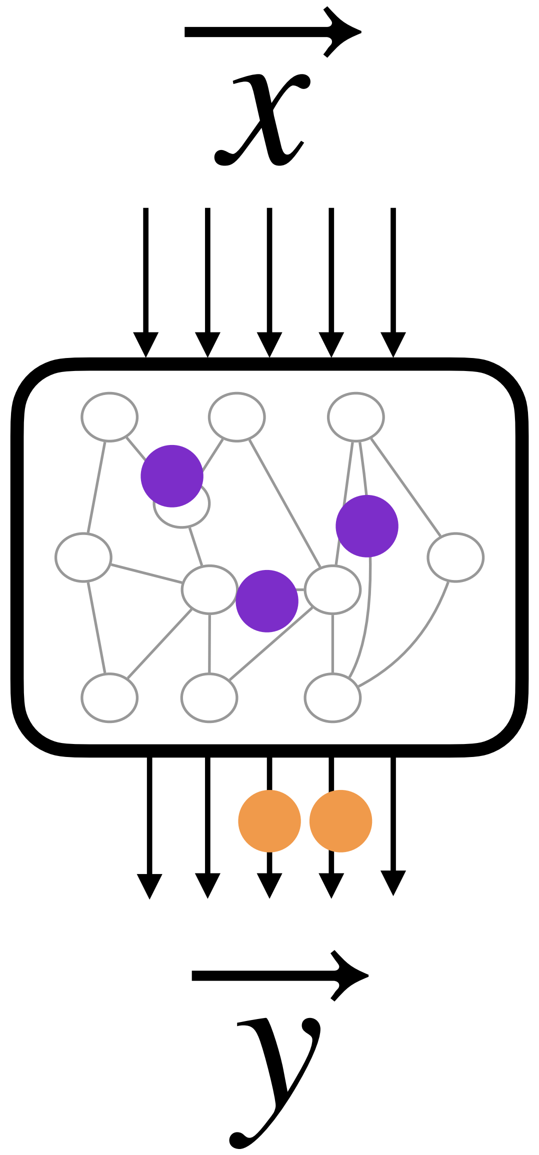
💡 composition security notions

Composition security

⚠️ probing security for gadgets $\not\Rightarrow$ global (region) probing security

💡 composition security notions

Example: strong non-interference (SNI)



t_1 *internal probes*

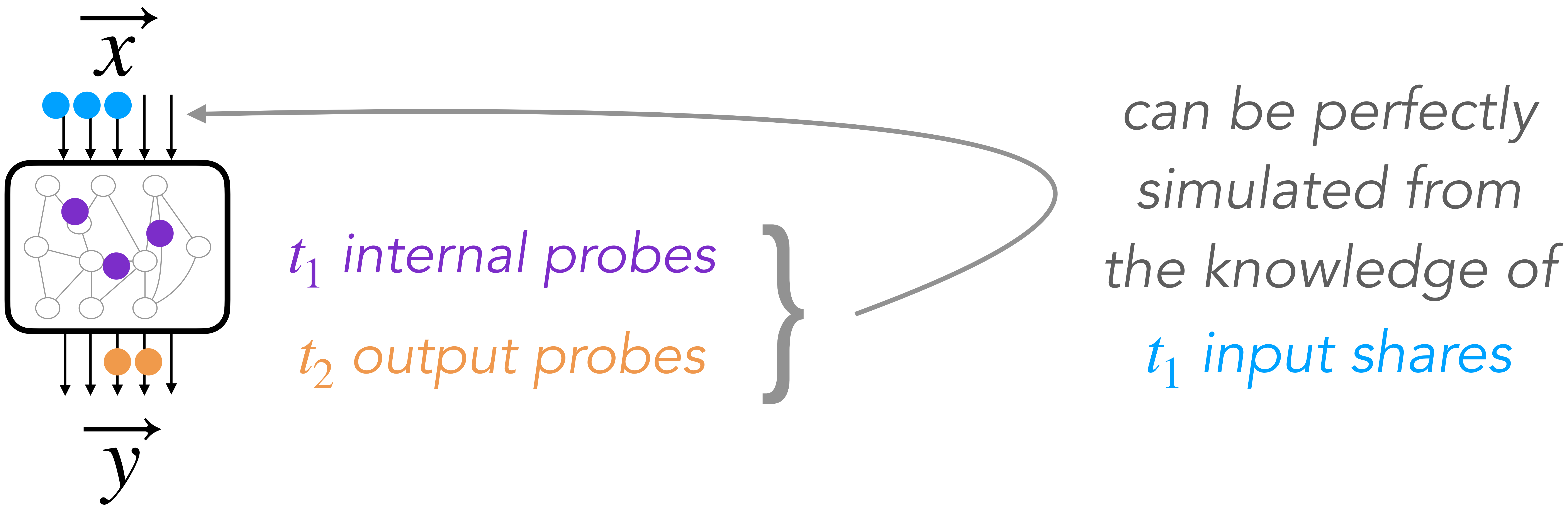
t_2 *output probes*

Composition security

⚠️ probing security for gadgets ⇏ global (region) probing security

💡 composition security notions

Example: strong non-interference (SNI)



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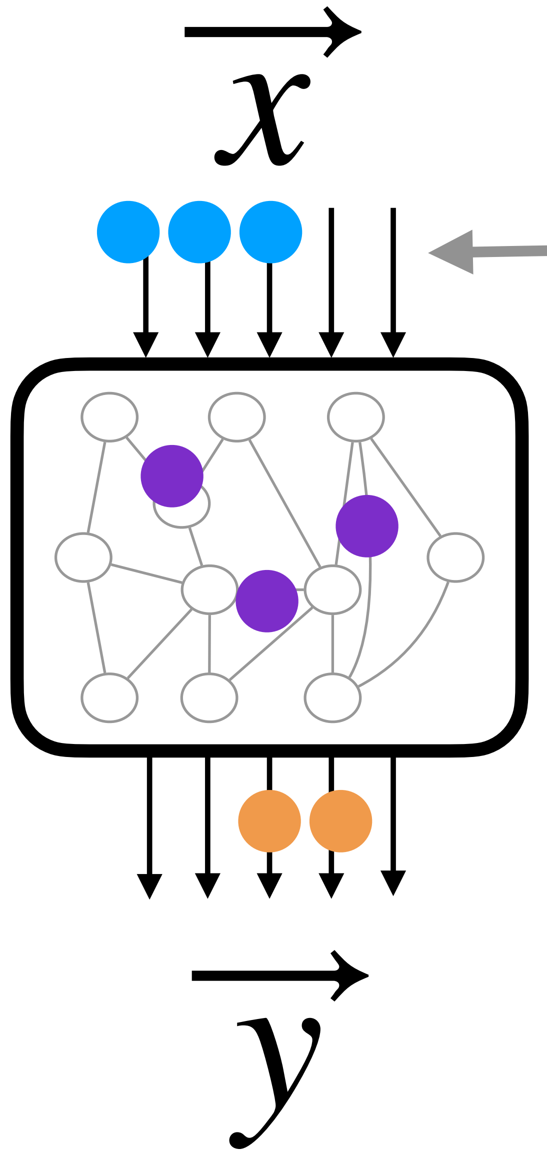
Example: strong non-interference (SNI)

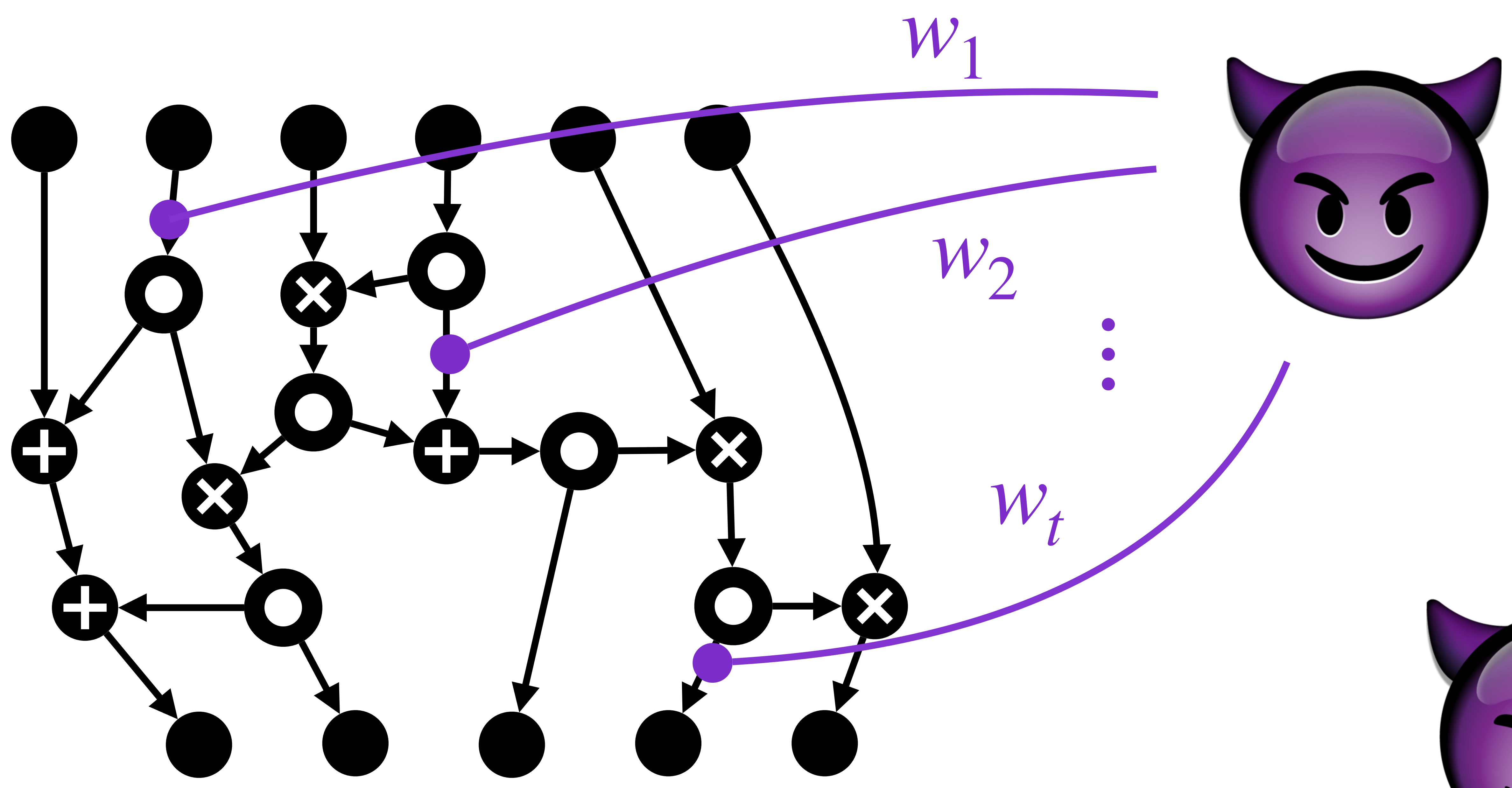


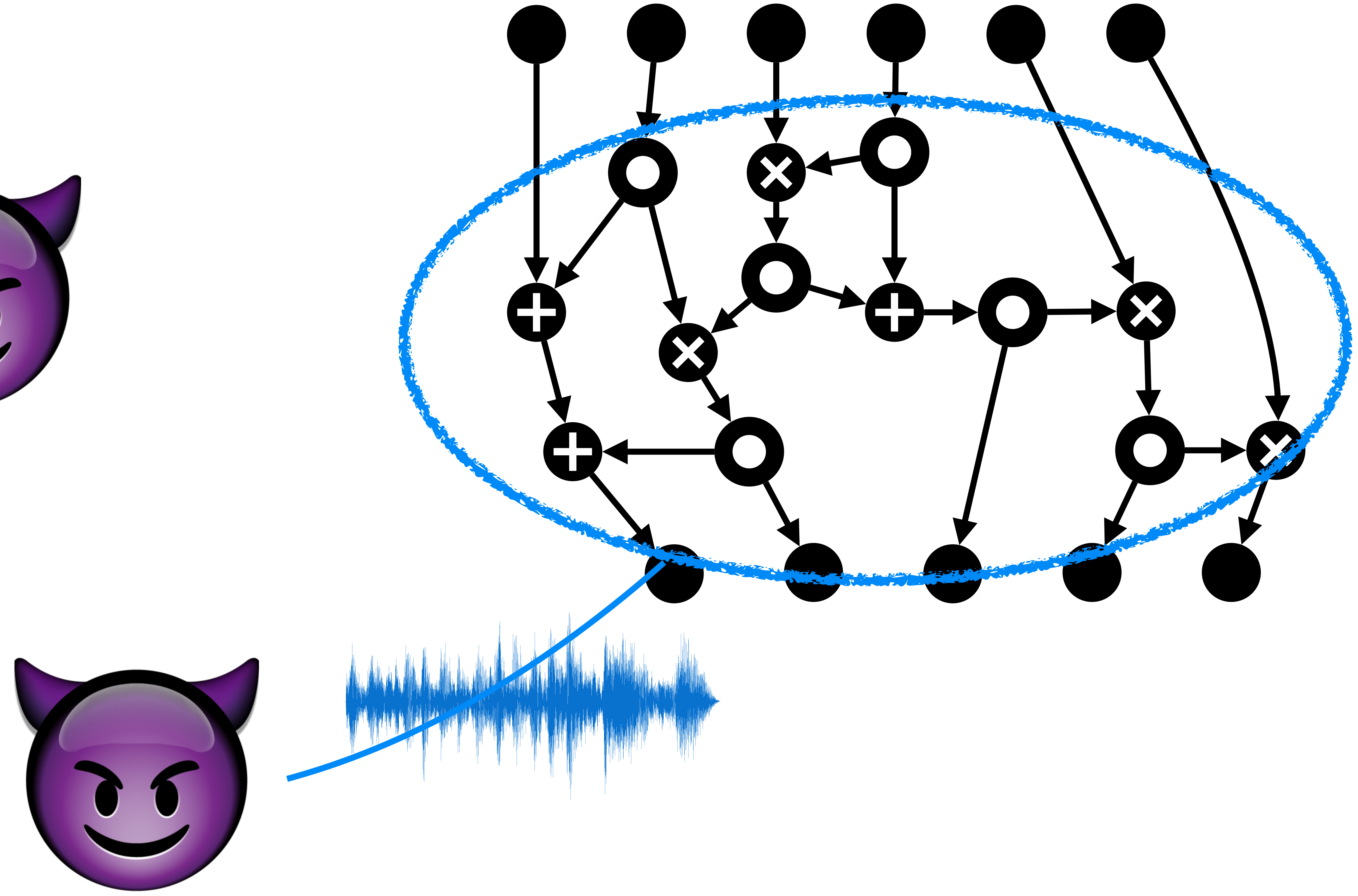
t_1 internal probes
t_2 output probes

can be perfectly simulated from the knowledge of t_1 input shares

SNI gadgets $\Rightarrow$ global region probing security

But… wait a minute!

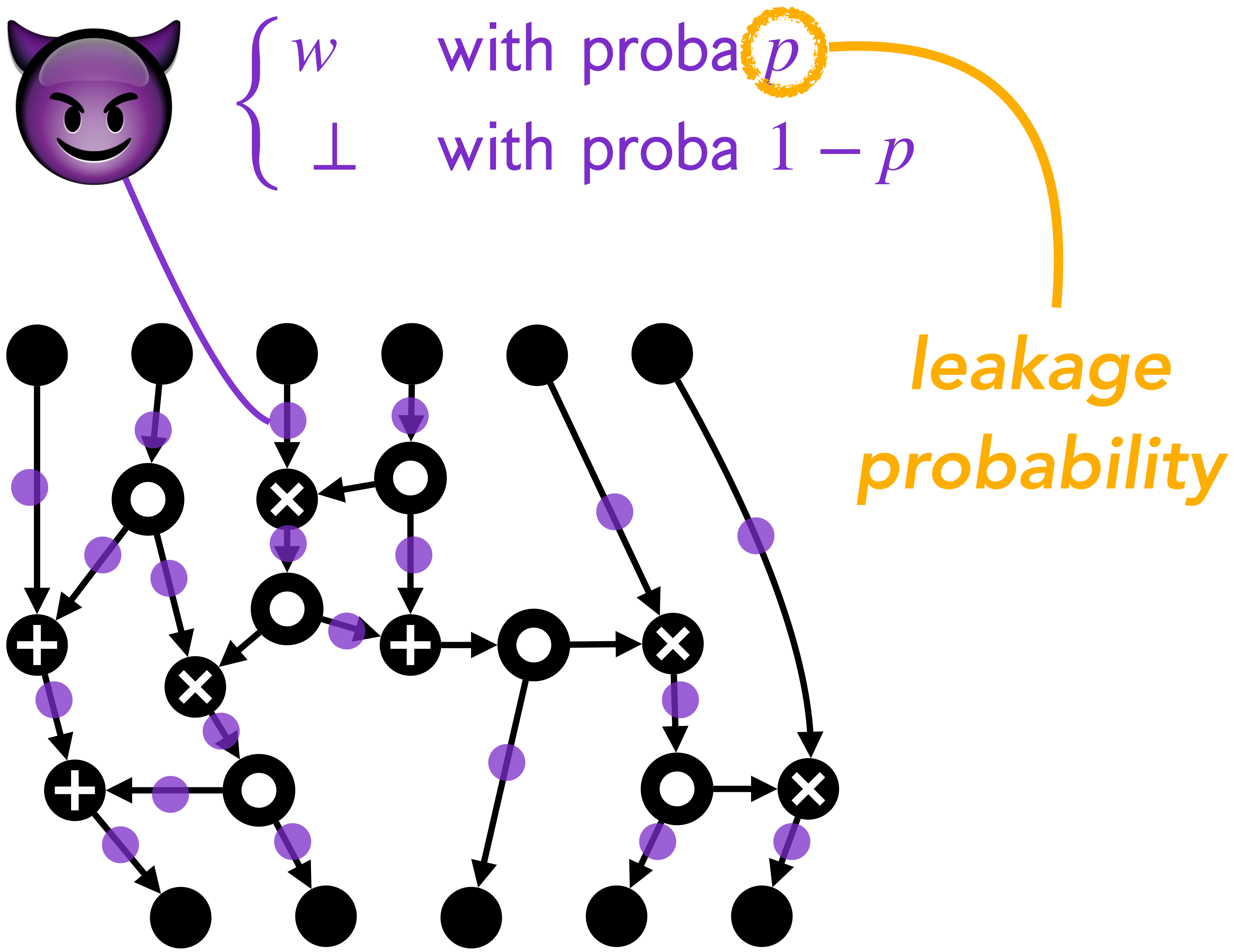




How do I get from probing security …

… to noisy leakage security?!

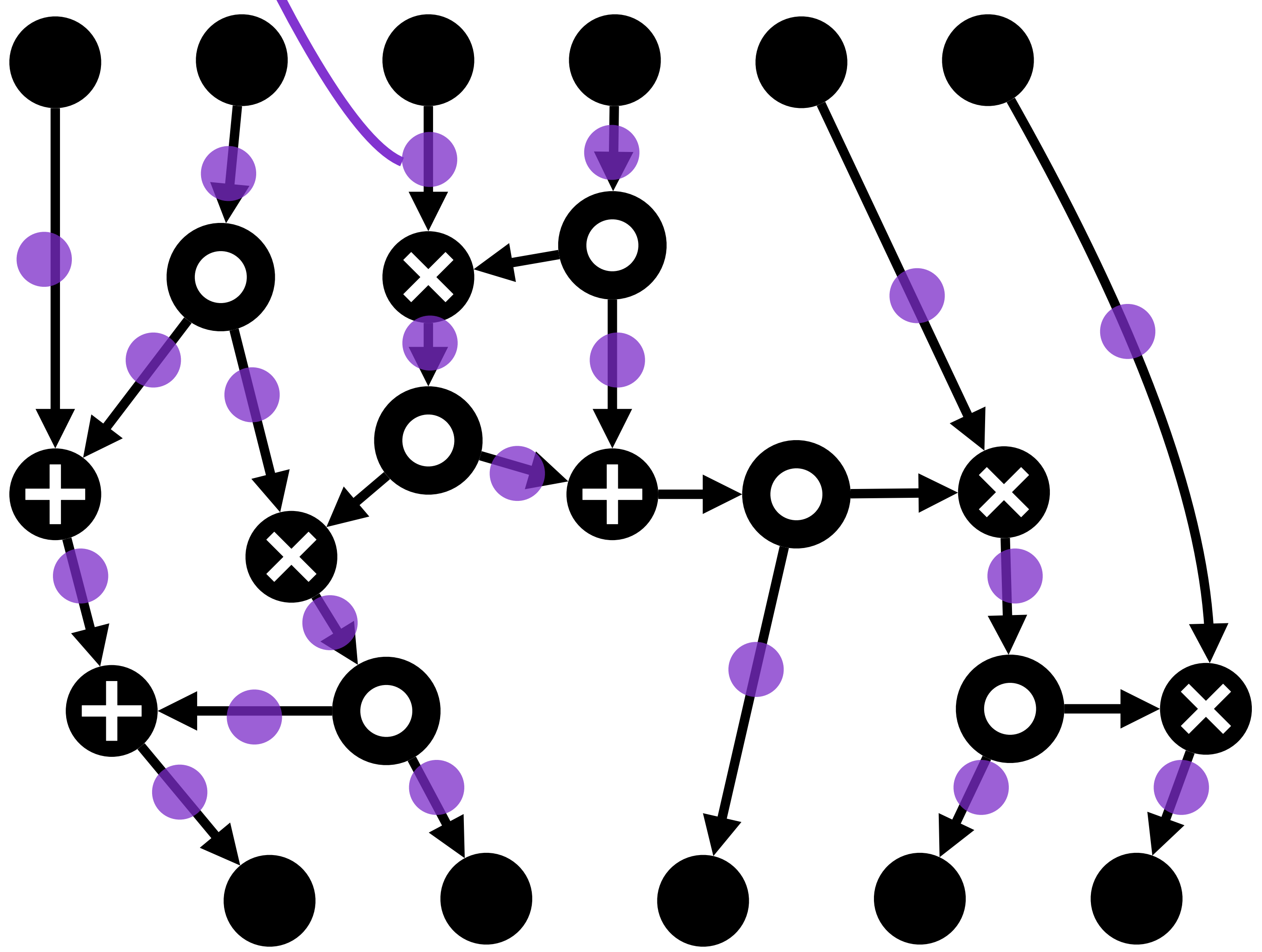
Random probing model



Random probing model

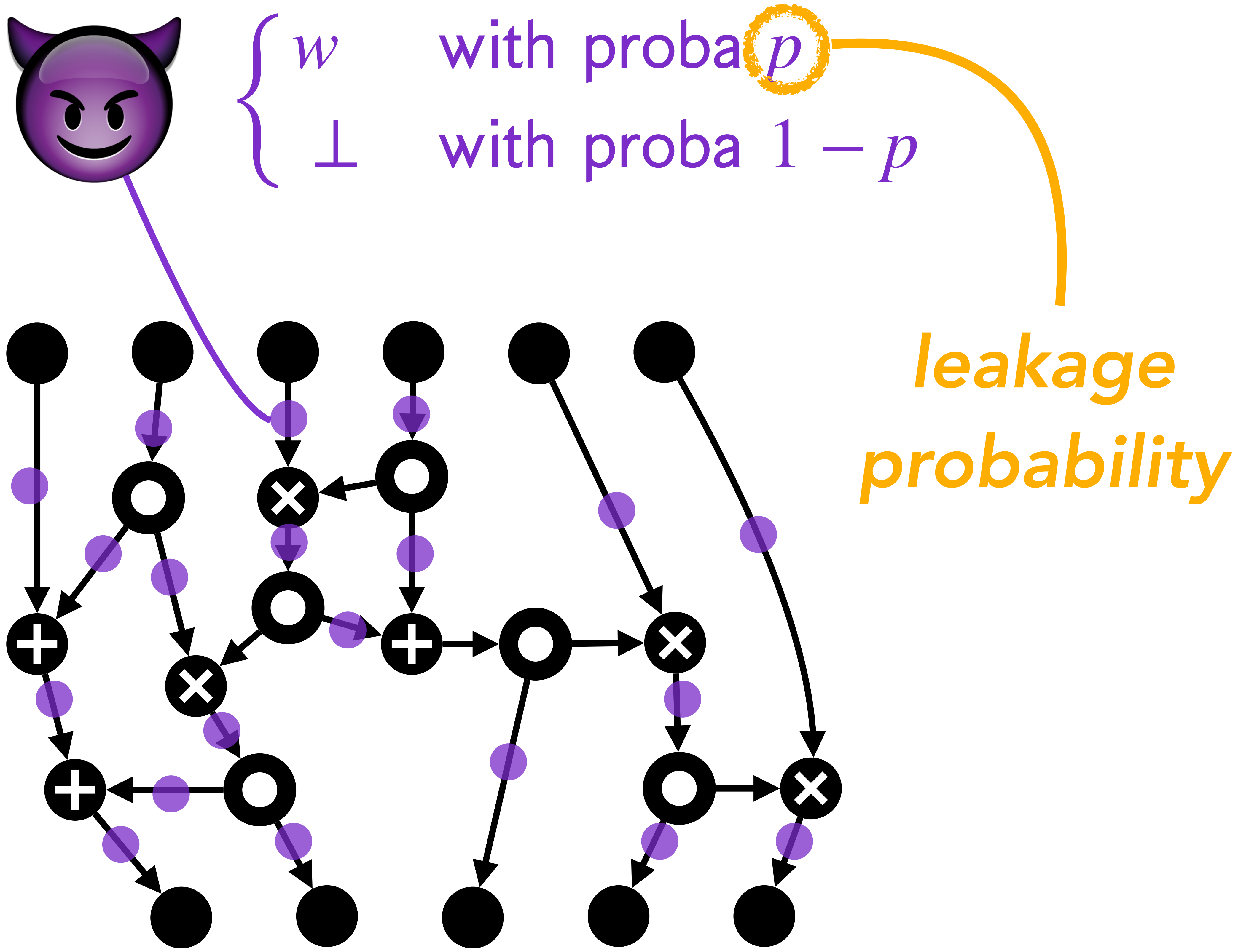
$$\begin{cases} w & \text{with proba } p \\ \perp & \text{with proba } 1-p \end{cases}$$

leakage probability



In average $p \cdot |G|$ leaking wires per gadget

Random probing model

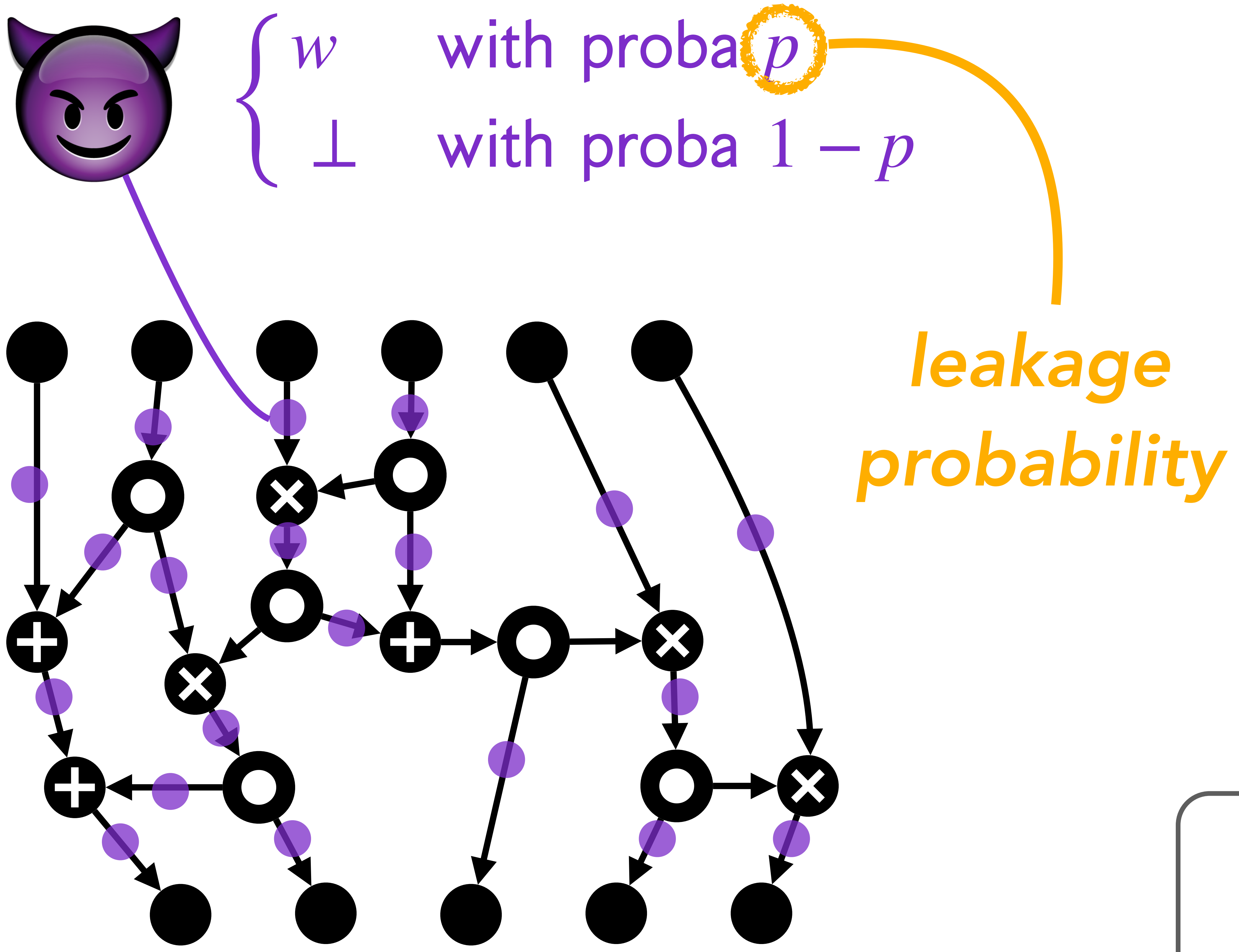


In average $p \cdot |G|$ leaking wires per gadget

Chernoff bound:

$< 2p\,|G|$ leaking wires with overwhelming probability

Random probing model



In average $p \cdot |G|$ leaking wires per gadget

Chernoff bound:

$< 2p|G|$ leaking wires with overwhelming probability

r-region probing security

$\Rightarrow$ p-random probing security

with $p = \Theta(r)$

Unifying probing and noisy models

Key lemma:

If f is δ-noisy, then $\exists f'$ s.t.

$$f(x) = f'(\phi(x))$$

where

$$\phi(x) := \begin{cases} x & \text{with proba } p \leq \delta \cdot |\mathcal{X}| \\ \perp & \text{with proba } 1 - p \end{cases}$$

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⇒ δ-noisy leakage can be simulated from p-random probing leakage

Random probing leakage

$$\phi(w_1),\ \phi(w_2),\ \ldots\ ,\ \phi(w_N)$$

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$\Rightarrow$ ***δ-noisy leakage can be simulated from p-random probing leakage***

Random probing leakage

$\phi(w_1), \phi(w_2), \ \dots \ , \phi(w_N)$

Apply $f_1', \dots, f_N'$

Noisy leakage

$f_1(w_1), f_2(w_2), \ \dots \ , f_N(w_N)$

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p-random probing security

$\Rightarrow$ δ-noisy security with $\delta = \Theta(p)$

Unifying probing and noisy models

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$\Rightarrow$

p-random probing security

with $p = \Theta(r)$

$\Rightarrow$

δ-noisy leakage security

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Unifying probing and noisy models

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leakage rate

$$\begin{cases} 1 = \text{lot of leakage (low noise)} \\ 0 = \text{no leakage (infinite noise)} \end{cases}$$

Unifying probing and noisy models

r-region probing security

$\Rightarrow$

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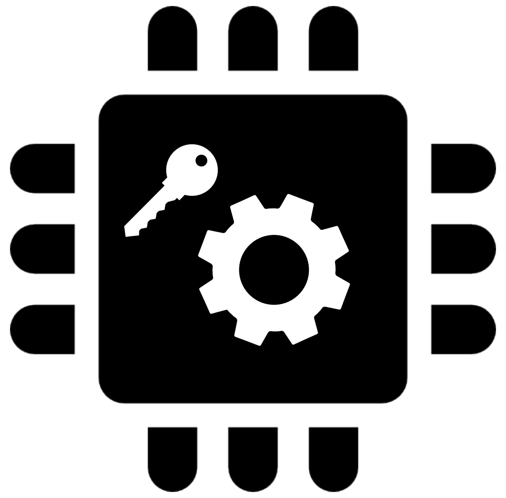

the noise / leakage rate depends on the hardware

Unifying probing and noisy models

r-region probing security

$\Rightarrow$

p-random probing security
with $p = \Theta(r)$

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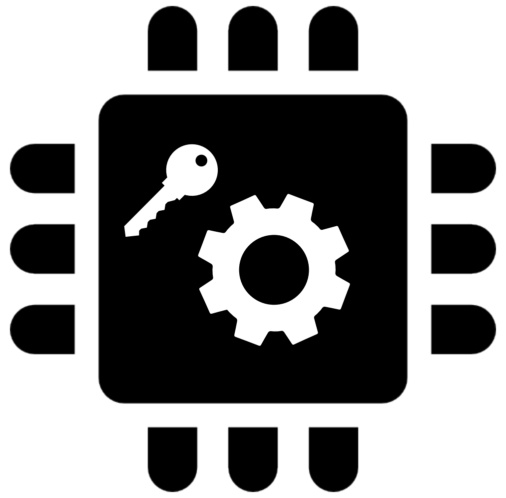

the noise / leakage rate depends on the hardware

efficient masking schemes secure vs. constant (high) leakage rate

Secure schemes

State of the art

- State-of-the-art noisy-leakage-secure schemes
  - most schemes with **at least $\mathcal{O}(n^2)$ complexity**
  - a few schemes with $\mathcal{O}(1)$ leakage rate, but **constant not explicit**
- In what follows
  - region probing security in **quasilinear complexity**
  - random probing security with **explicit constant leakage rate**

Security in quasilinear complexity

Quasilinear masking

A $\overrightarrow{v}$-sharing of x

$$\overrightarrow{x} = (x_0, x_1, \ldots, x_{n-1}) \quad \text{s.t.} \quad \langle \overrightarrow{v}, \overrightarrow{x} \rangle = x$$

Quasilinear masking

A $\overrightarrow{v}$-sharing of x

$$\overrightarrow{x} = (x_0, x_1, \ldots, x_{n-1}) \quad \text{s.t.} \quad \langle \overrightarrow{v}, \overrightarrow{x} \rangle = x = \sum_{i=0}^{n-1} x_i \cdot \omega^i$$

$$\overrightarrow{v} = (1, \omega, \omega^2, \ldots, \omega^{n-1}) \quad \text{for} \quad \omega \xleftarrow{\$} \mathbb{F}$$

Quasilinear masking

Polynomial $P_{\vec{x}}(\omega)$

(shares = coefficients)

A $\vec{v}$-sharing of x

$$\vec{x} = (x_0, x_1, \ldots, x_{n-1}) \quad \text{s.t.} \quad \langle \vec{v}, \vec{x} \rangle = x = \sum_{i=0}^{n-1} x_i \cdot \omega^i$$

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Efficient multiplication

- Let $\vec{t}$ such that

$$P_{\vec{t}} = P_{\vec{x}} \cdot P_{\vec{y}}$$

- We get

$$P_{\vec{t}}(\omega) = \sum_{i=0}^{2n-1} t_i \omega^i = x \cdot y$$

Efficient multiplication

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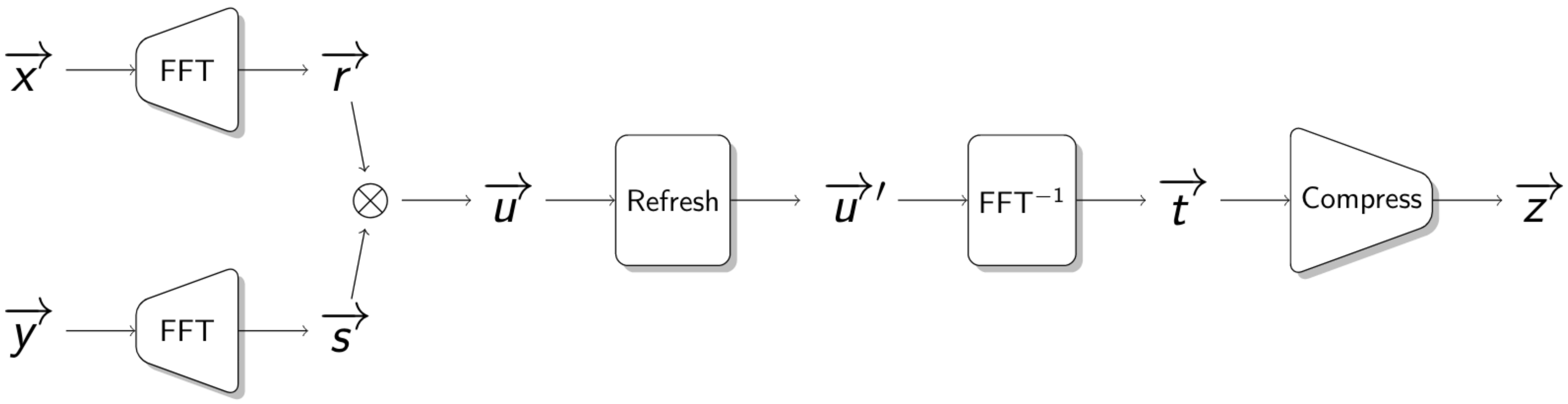
- We get

$$P_{\vec{t}}(\omega) = \sum_{i=0}^{2n-1} t_i \omega^i = x \cdot y$$

- Compression:

$$\vec{z} = (t_0, \ldots, t_{n-1}) + \omega^n \cdot (t_n, \ldots, t_{2n-1})$$

Multiplication gadget



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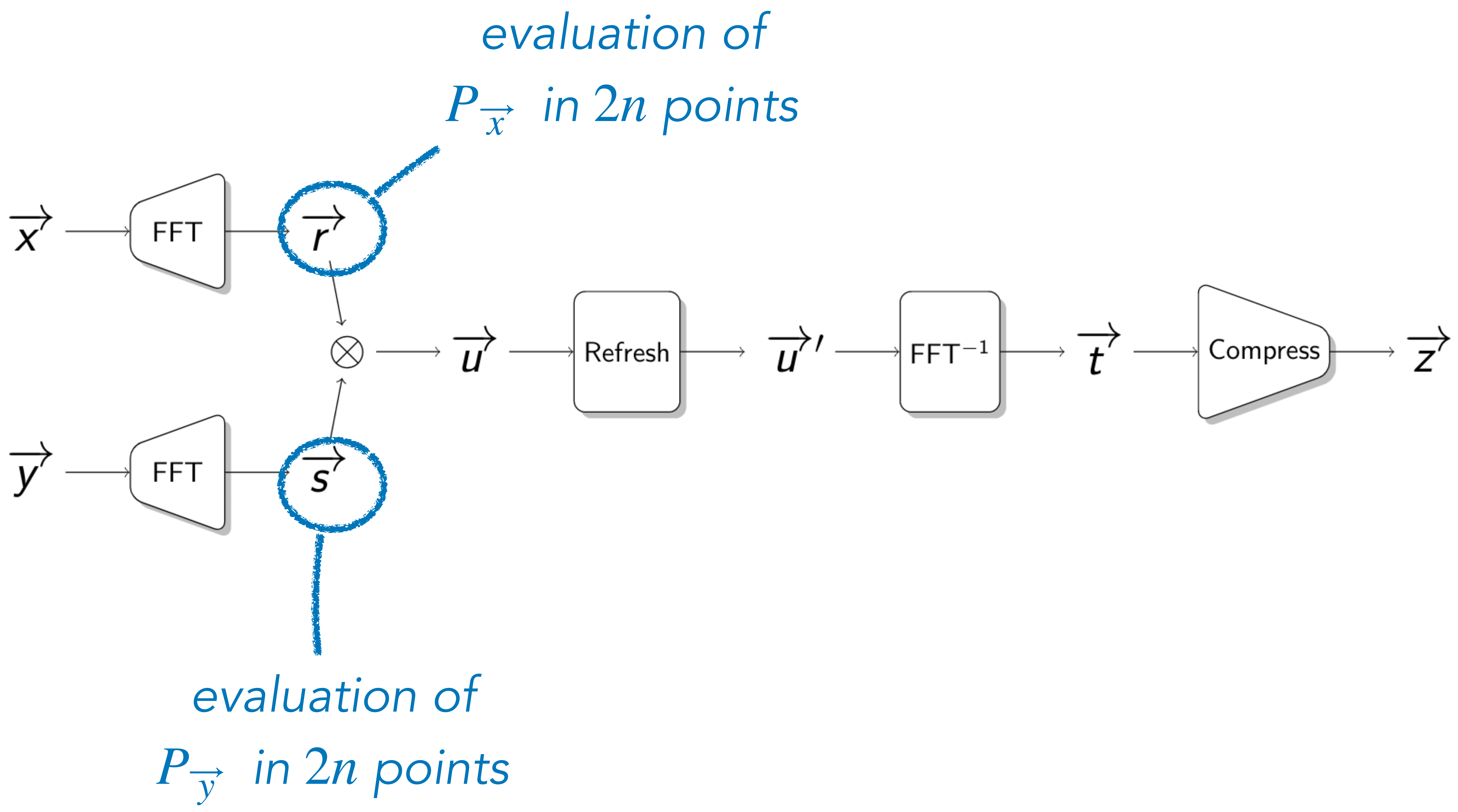

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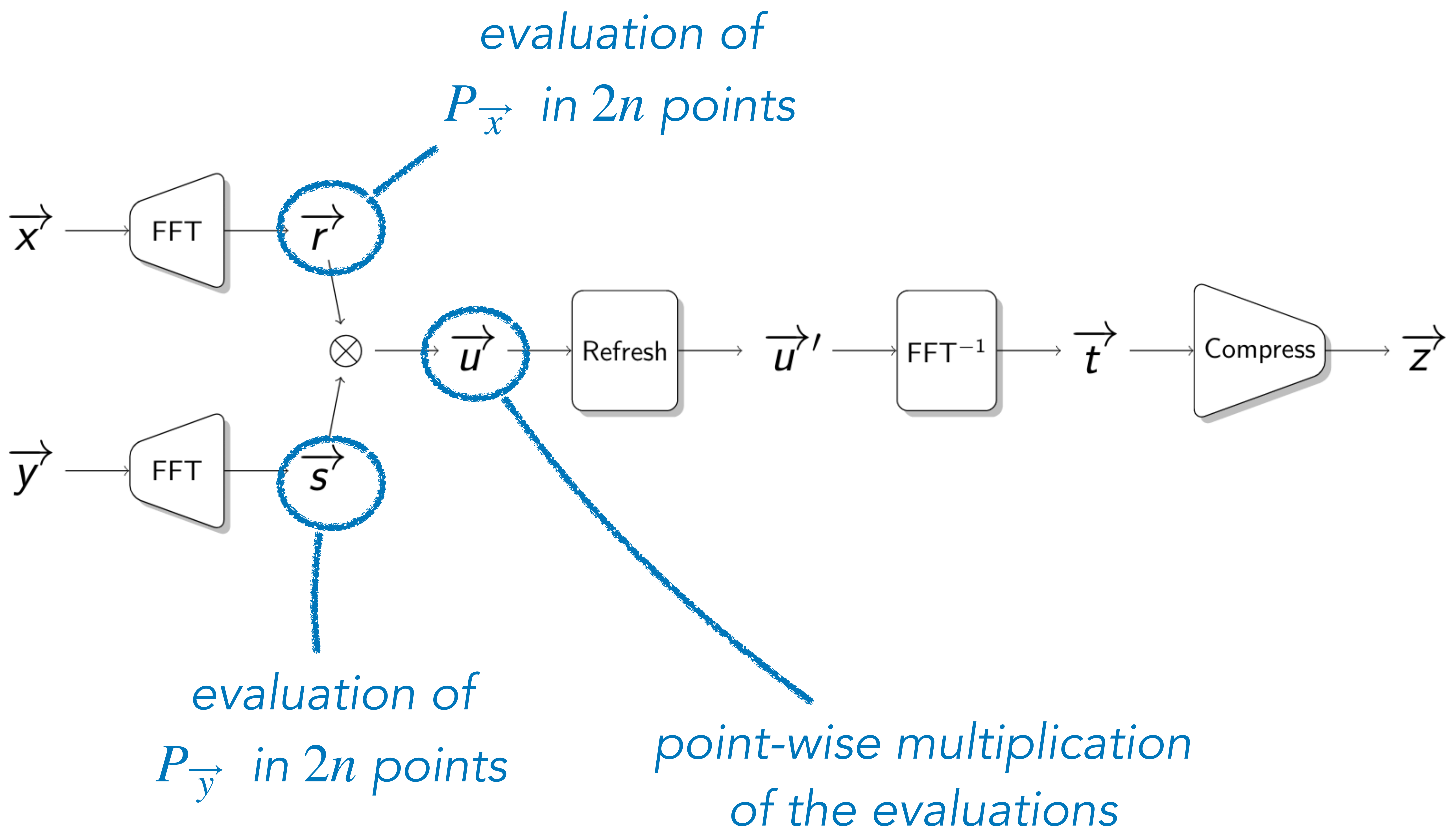

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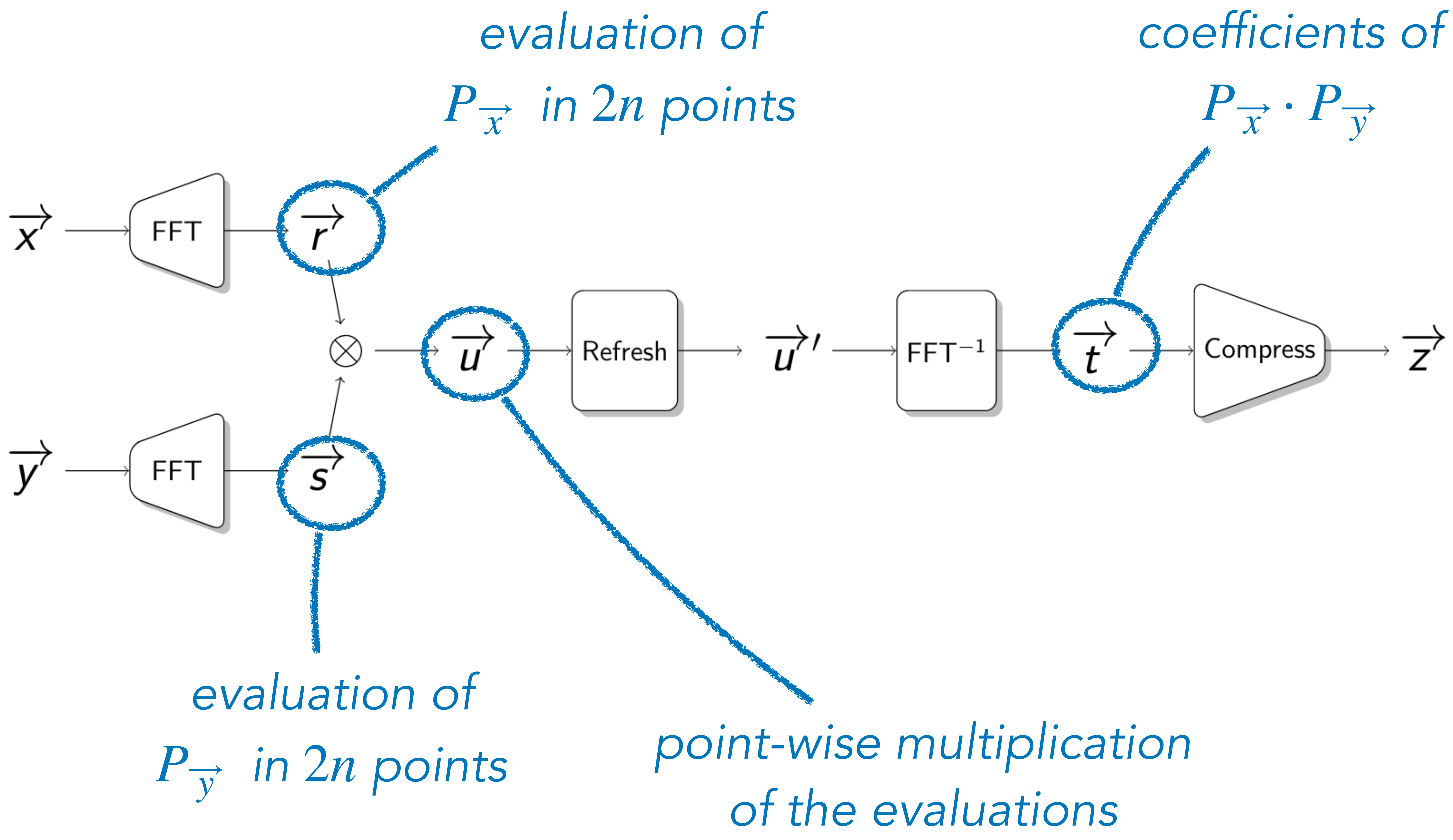

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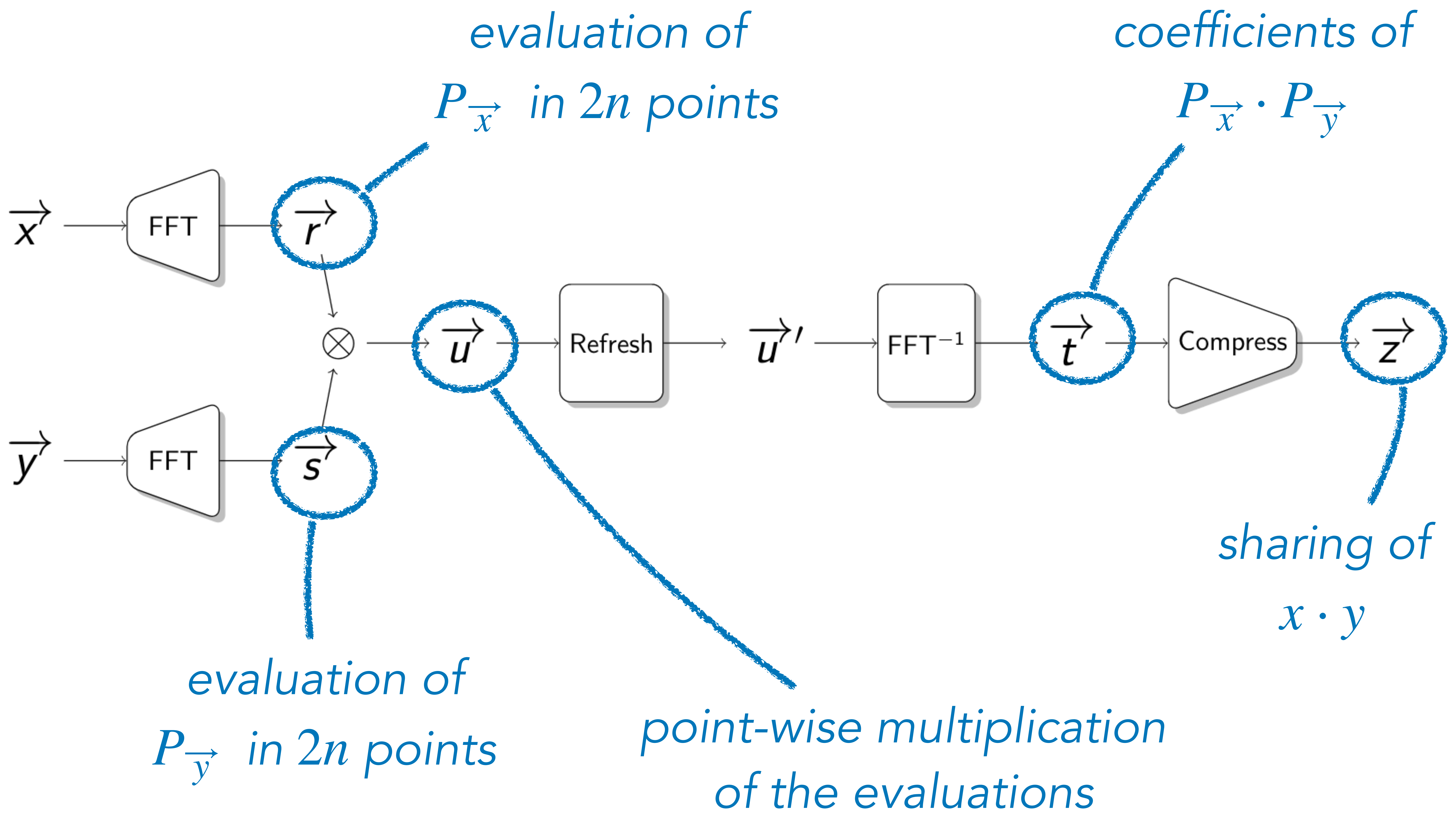

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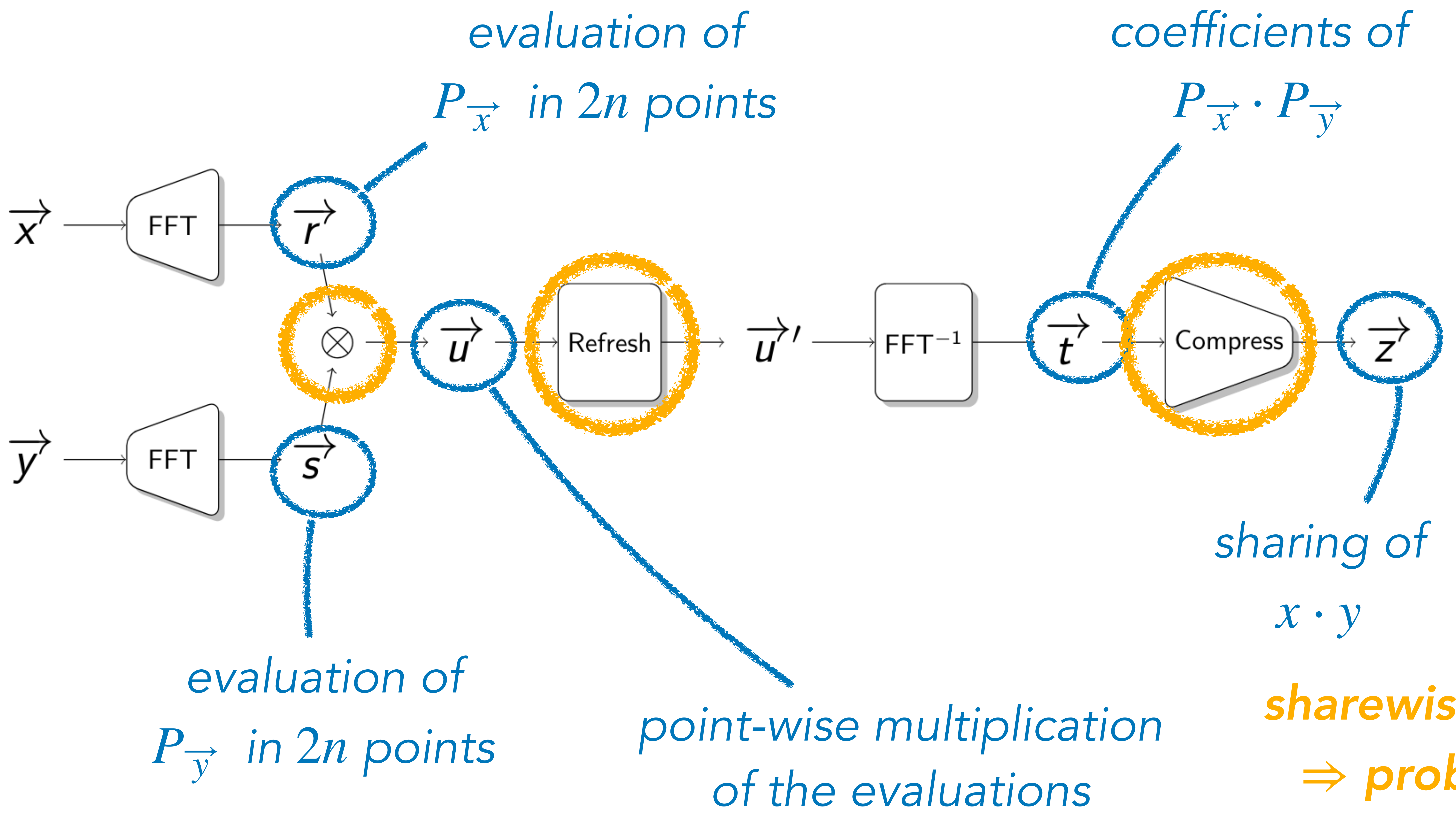


sharewise operations
$\Rightarrow$ probing secure

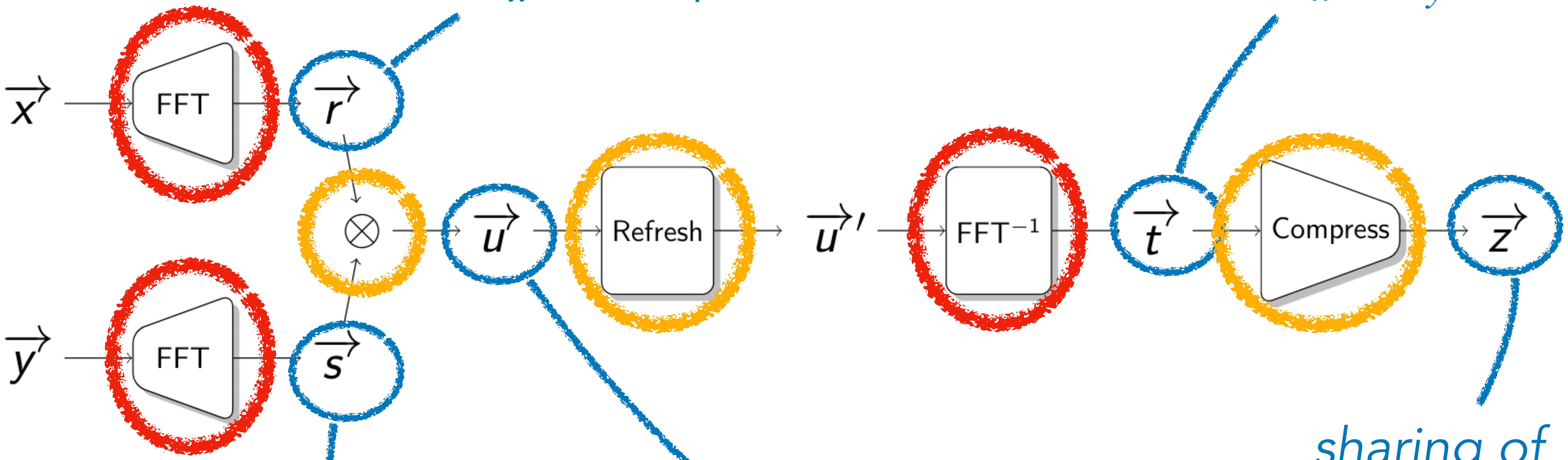
Multiplication gadget

evaluation of

$P_{\vec{x}}$ in $2n$ points

coefficients of

$P_{\vec{x}} \cdot P_{\vec{y}}$



sharing of

$x \cdot y$

what about the FFT?

evaluation of

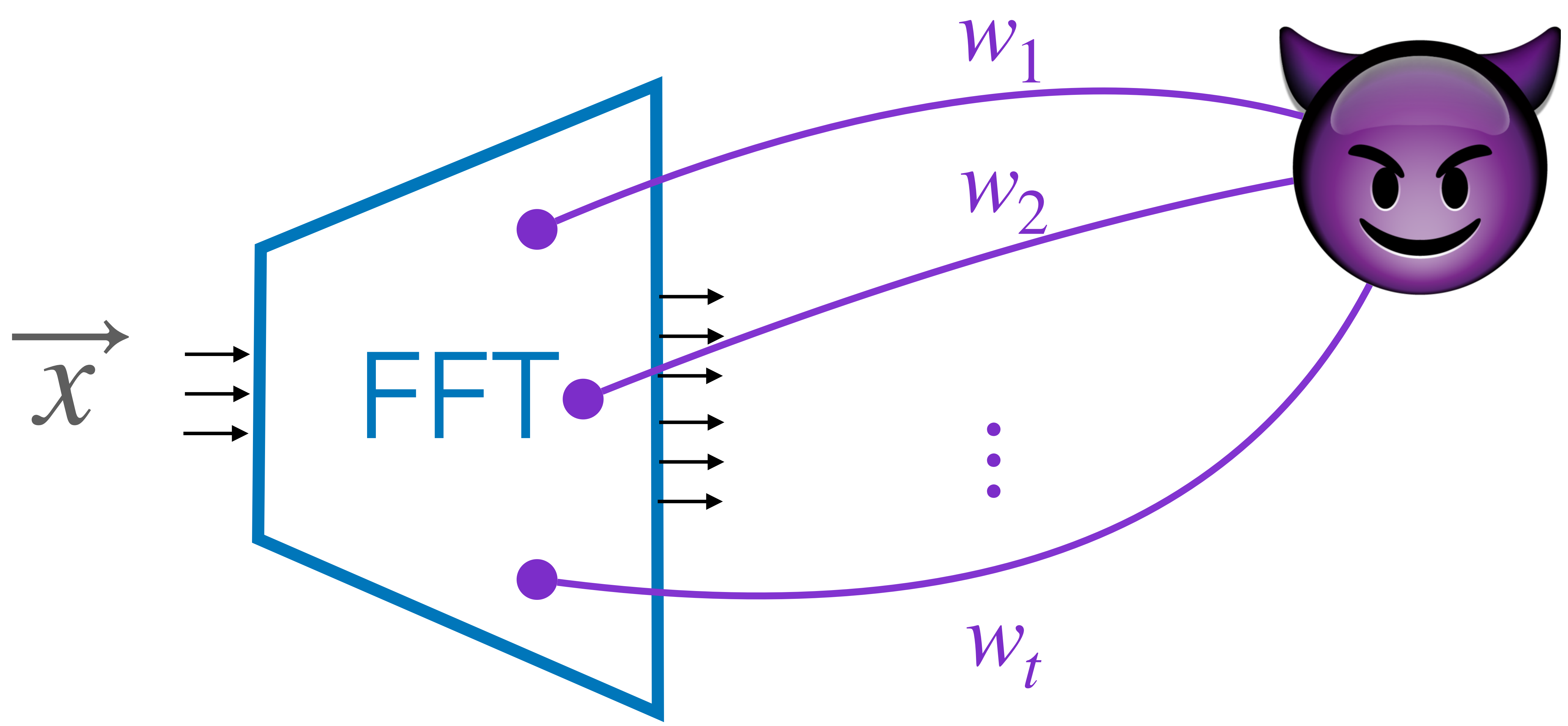
$P_{\vec{y}}$ in $2n$ points

point-wise multiplication of the evaluations

sharewise operations

$\Rightarrow$ probing secure

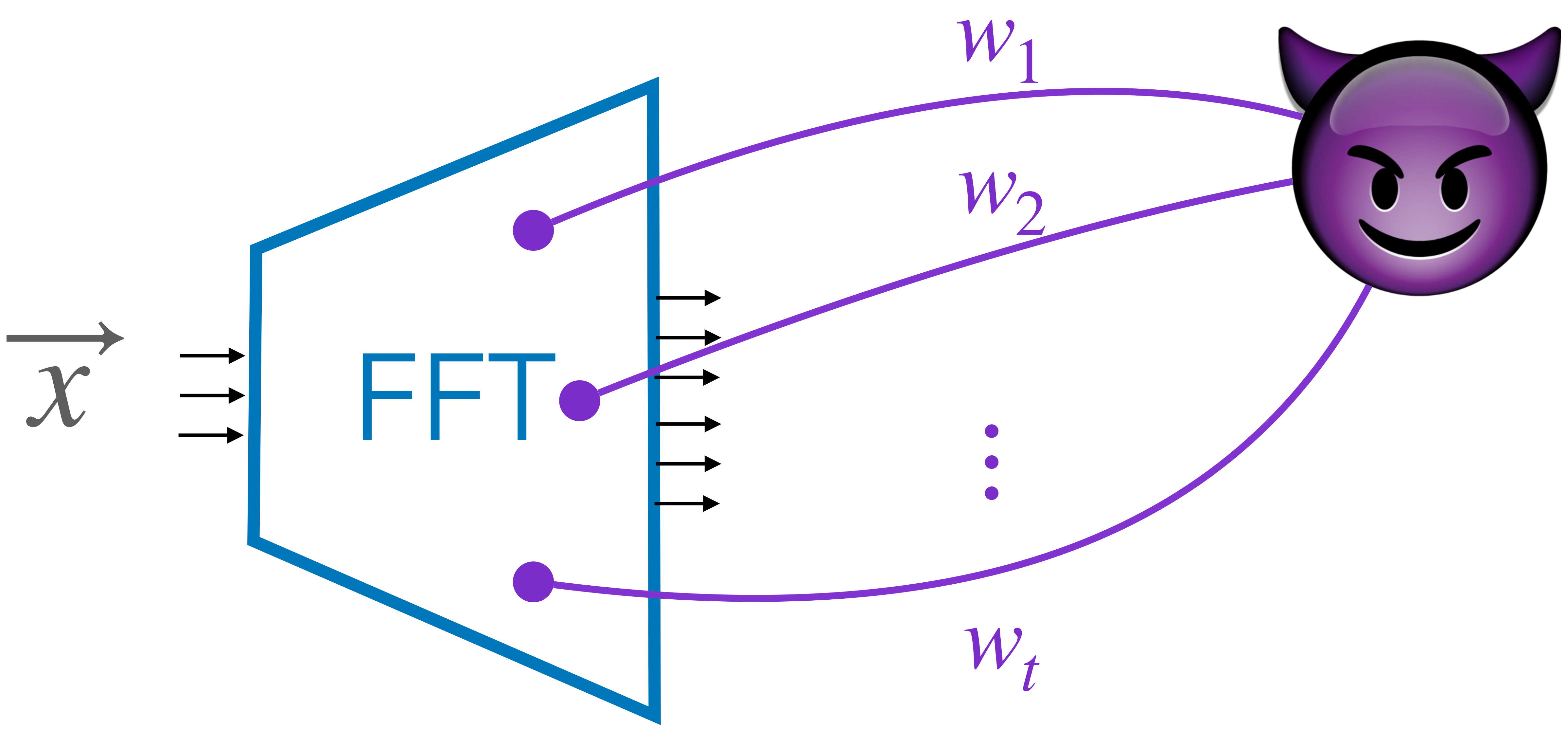
Probing security



💡 *FFT computes linear combinations of the x_i's*

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_t \end{pmatrix} = \left[\ A \ \right] \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{pmatrix}$$

Probing security



FFT computes linear combinations of the x_i's

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_t \end{pmatrix} = \begin{bmatrix} & A & \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{pmatrix}$$

Lemma 1

If $\vec{v} = \begin{pmatrix} \omega^0 \\ \omega^1 \\ \vdots \\ \omega^{n-1} \end{pmatrix} \notin \langle \begin{bmatrix} & A & \end{bmatrix} \rangle$ then $\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_t \end{pmatrix} \sim \mathcal{U}(\mathbb{F}^t)$ *(assuming A full rank wlog)*

Probing security

Lemma 2

$\exists$ at most t values of $\omega \in \mathbb{F}$ s.t. $\vec{v} = \begin{pmatrix} \omega^0 \\ \omega^1 \\ \vdots \\ \omega^{n-1} \end{pmatrix} \in \langle \left[\; A \; \right] \rangle$

Probing security

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$\exists$ at most t values of $\omega \in \mathbb{F}$ s.t. $\vec{v} = \begin{pmatrix} \omega^0 \\ \omega^1 \\ \vdots \\ \omega^{n-1} \end{pmatrix} \in \langle [\ A\ ] \rangle$

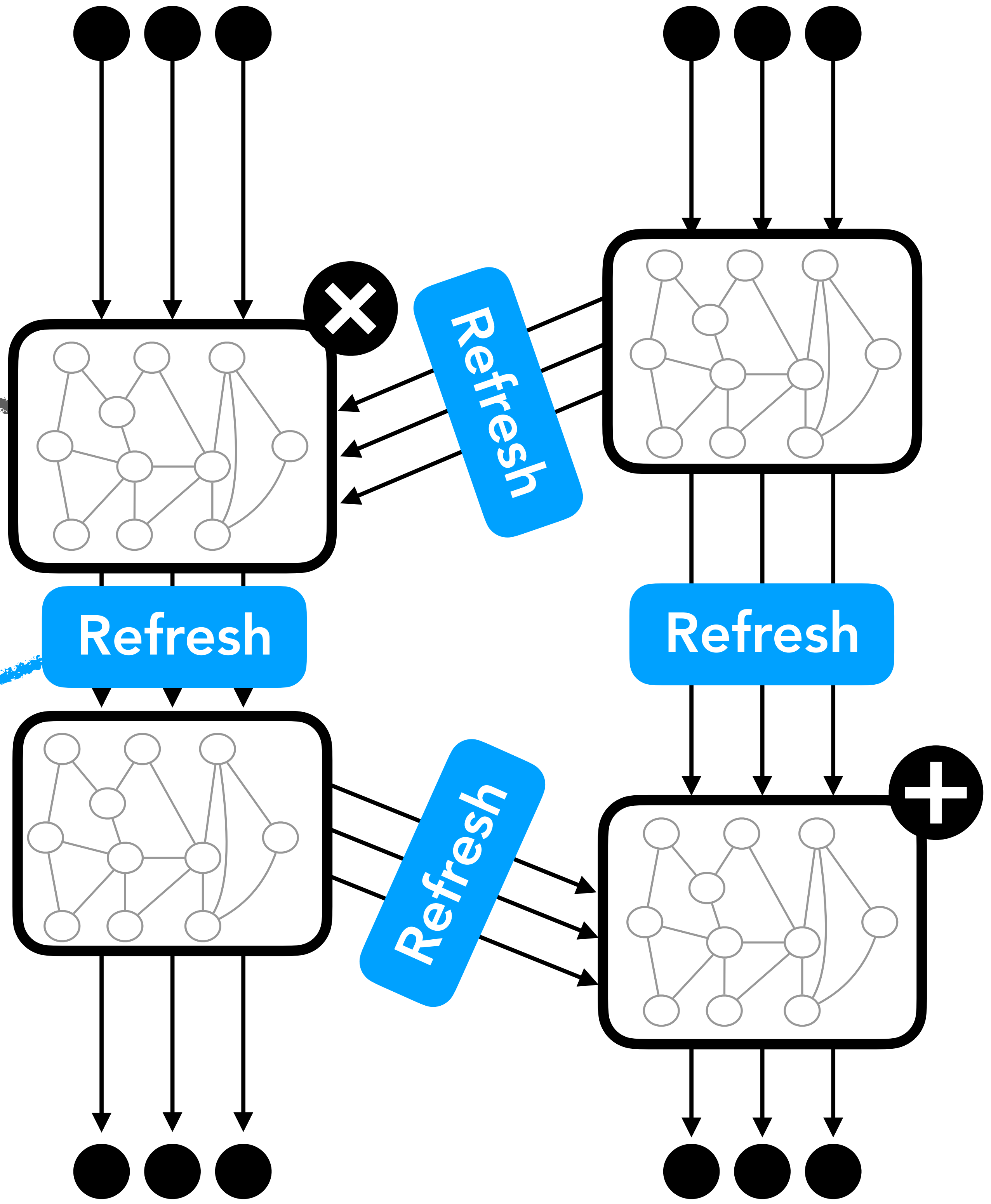
Lemma 1 + Lemma 2

$P\left[\,(w_1, \ldots, w_t) \text{ cannot be simulated}\,\right] \leq \dfrac{t}{|\mathbb{F}|} < \dfrac{n}{|\mathbb{F}|}$

Composition security

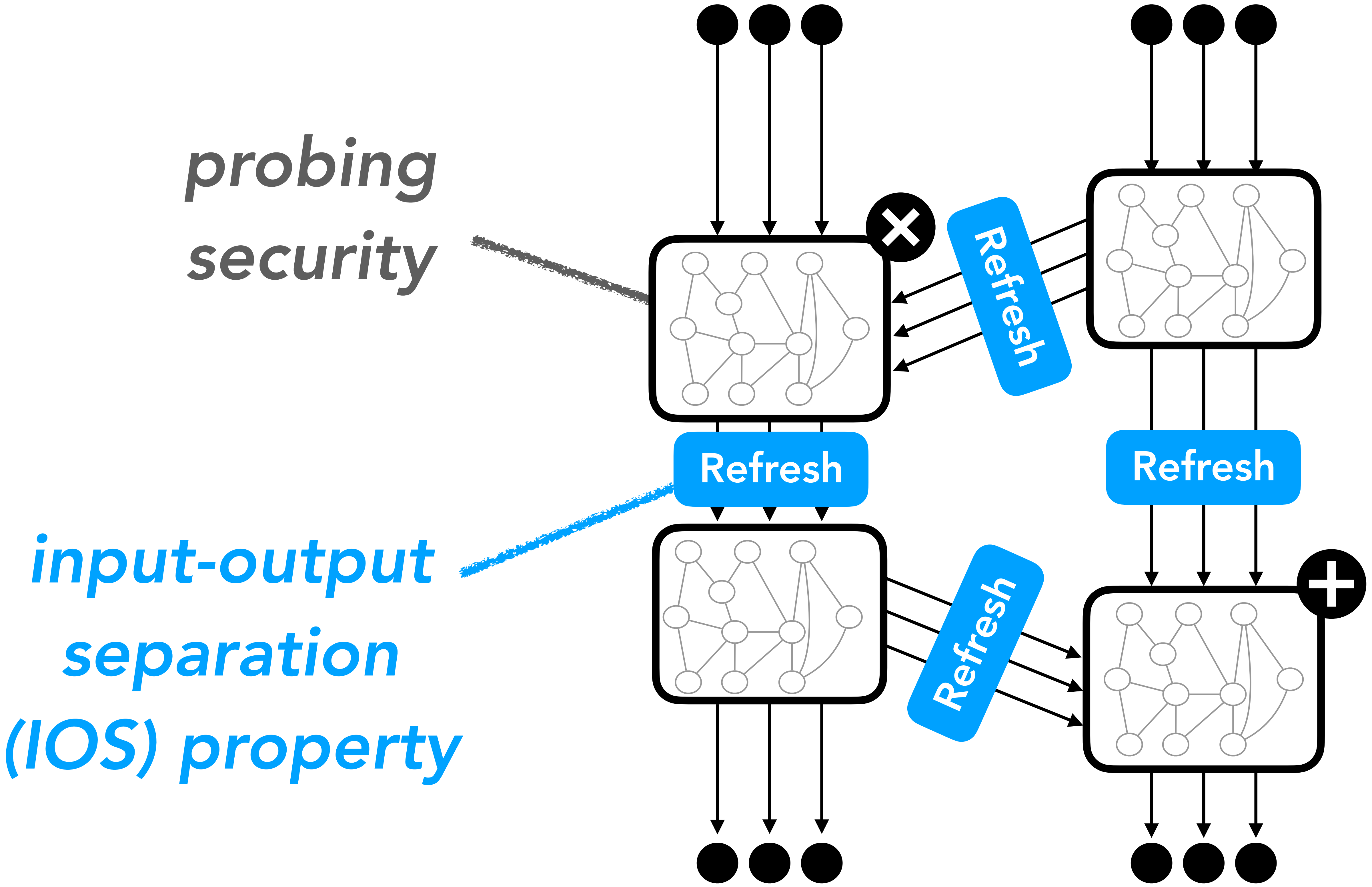
probing security

input-output separation (IOS) property



⇒ region probing security

Composition security



⇒ region probing security

Wrapping up:

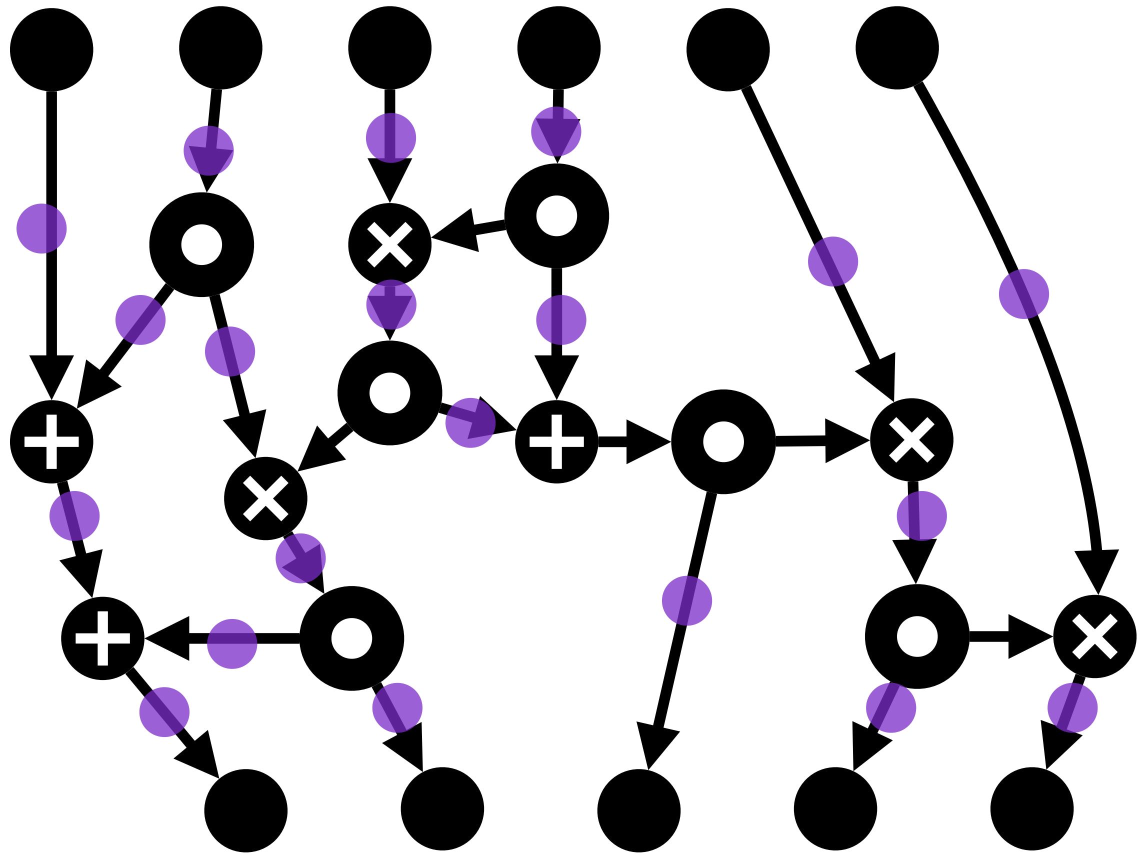
- Gadget complexity: $\Theta(n \log n)$
- Probes per gadget: $\Theta(n)$
- Leakage rate: $\Theta(1/\log n)$

Security with constant leakage rate

Simulation with abort

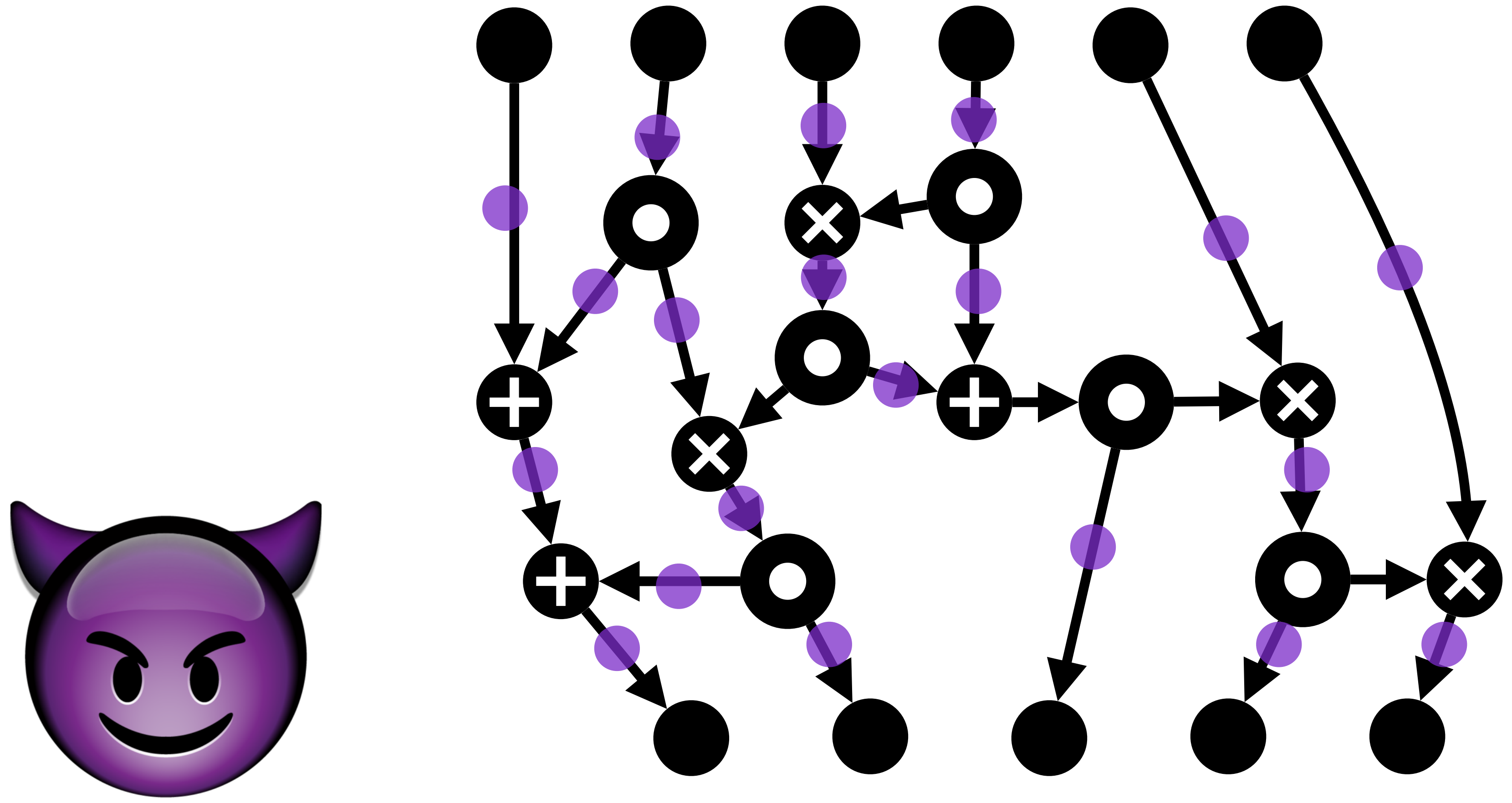
1. Sample a set of leaking wires

$$W \leftarrow \text{LeakingWires}(\hat{C}, p)$$



$$\begin{cases} w & \text{with proba } p \\ \perp & \text{with proba } 1-p \end{cases}$$

Simulation with abort



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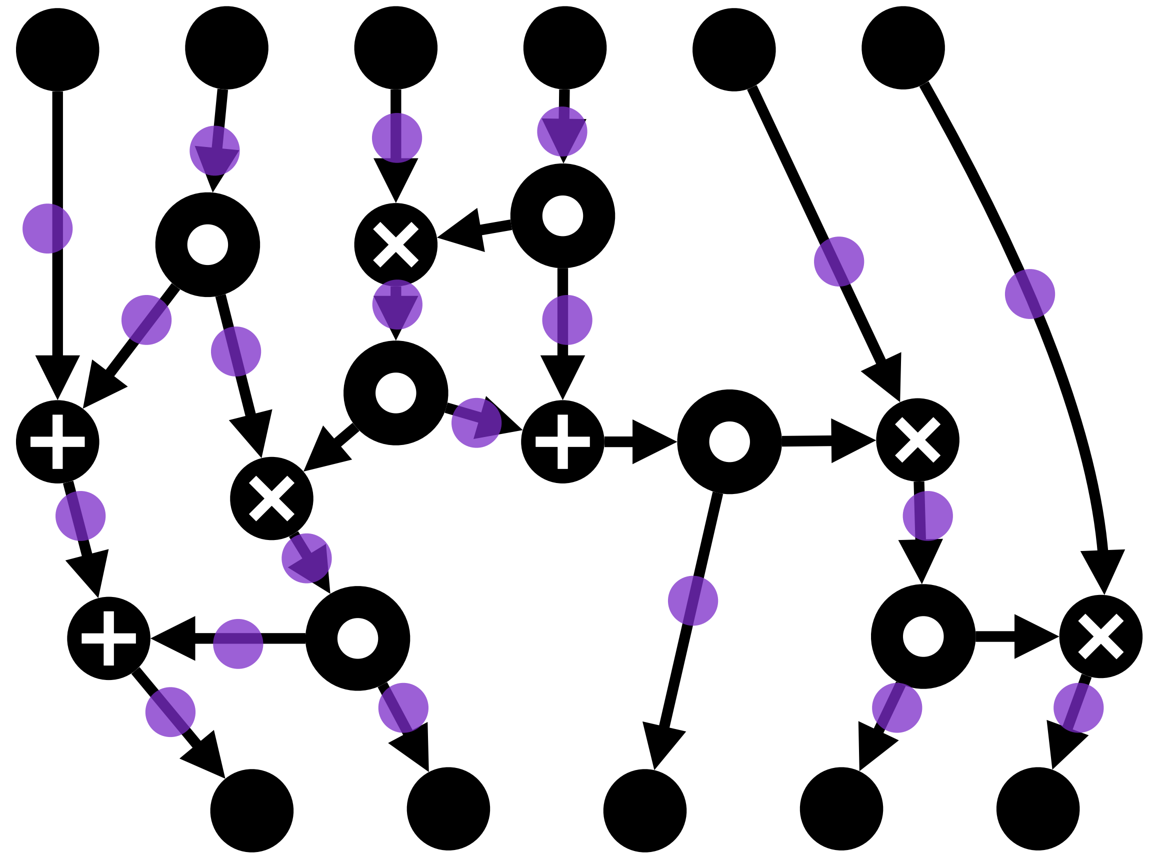
2. Simulate the corresponding wire values

$$\mathsf{Sim} : W \mapsto \begin{cases} \text{perfect simulation} \\ \perp \text{ (abort)} \end{cases}$$

Simulation with abort



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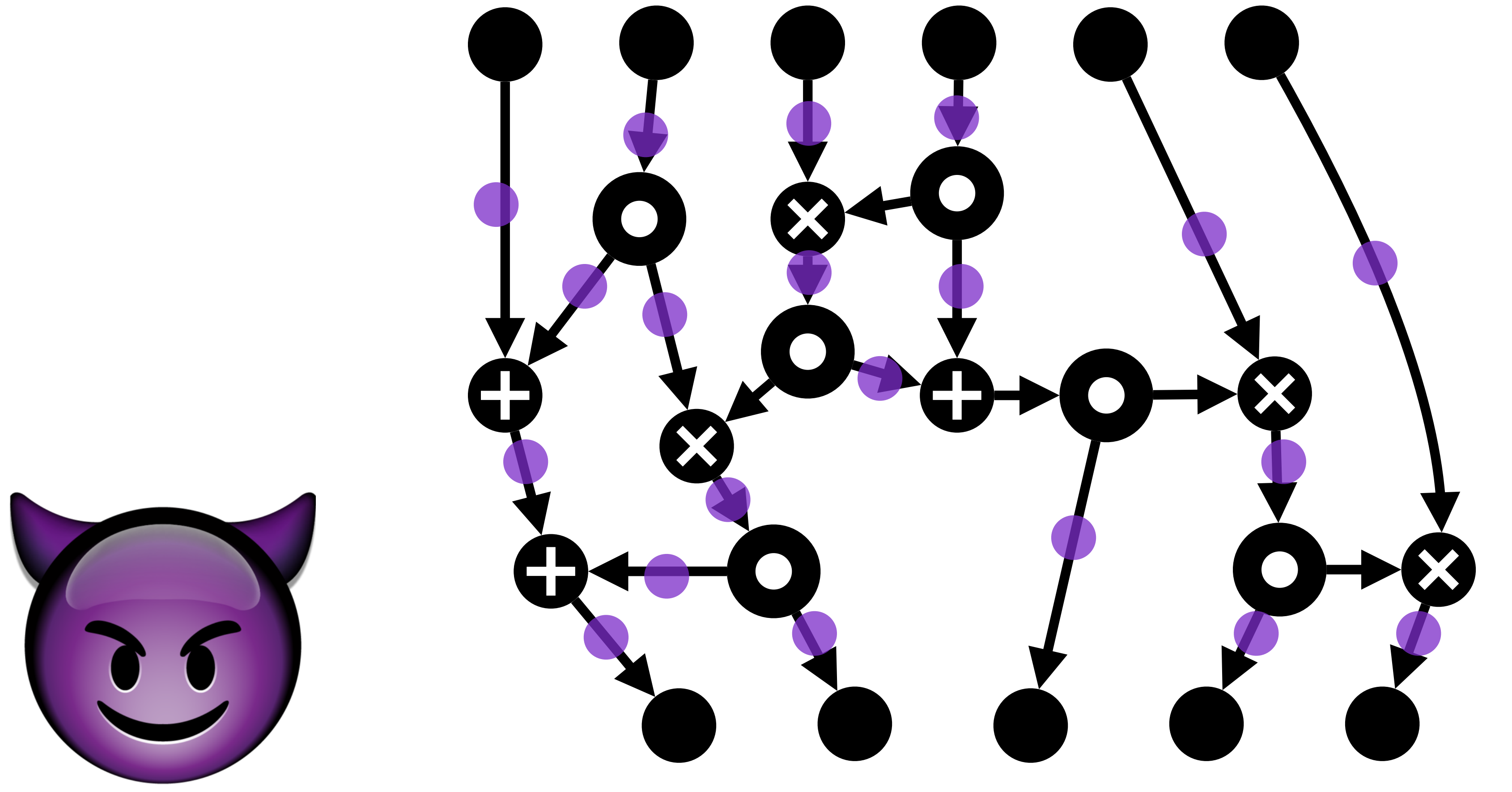

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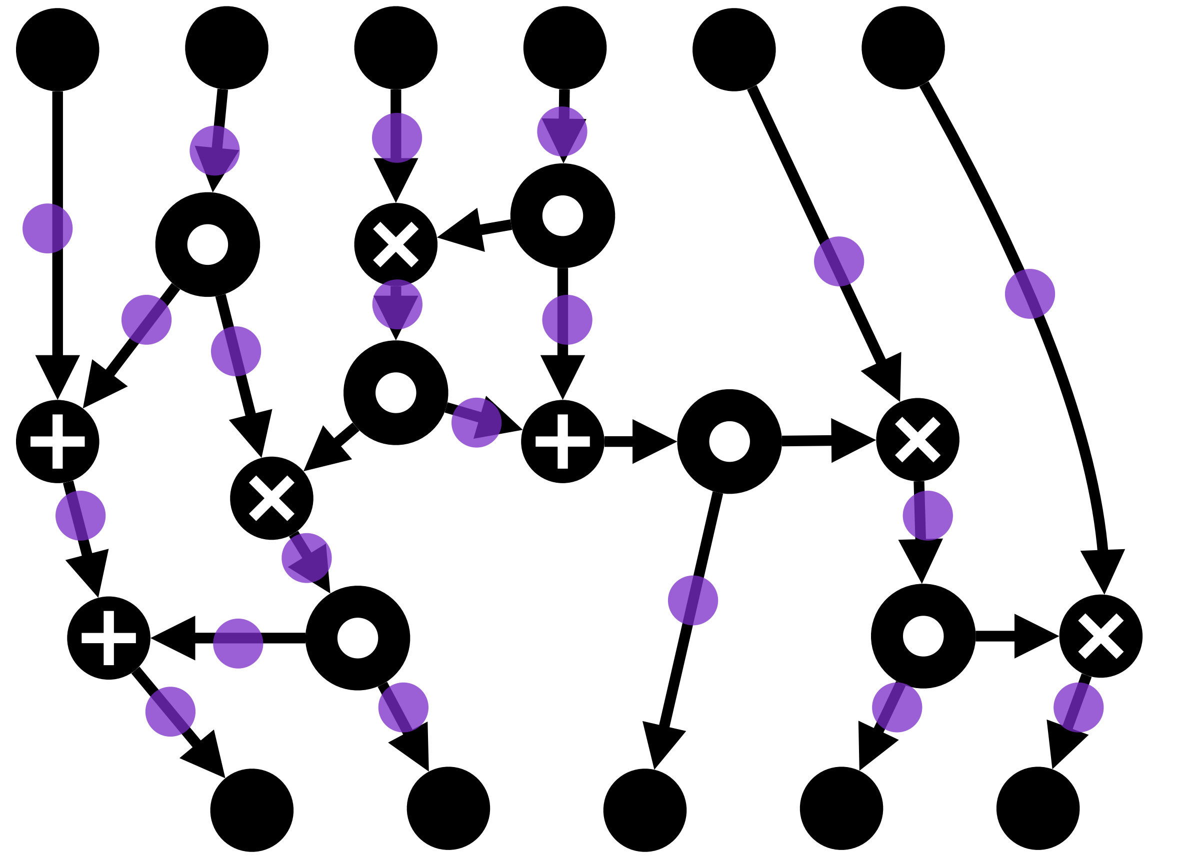
$$f(p) = \sum_W \delta_W p^{|W|}(1-p)^{s-|W|}$$

Simulation with abort



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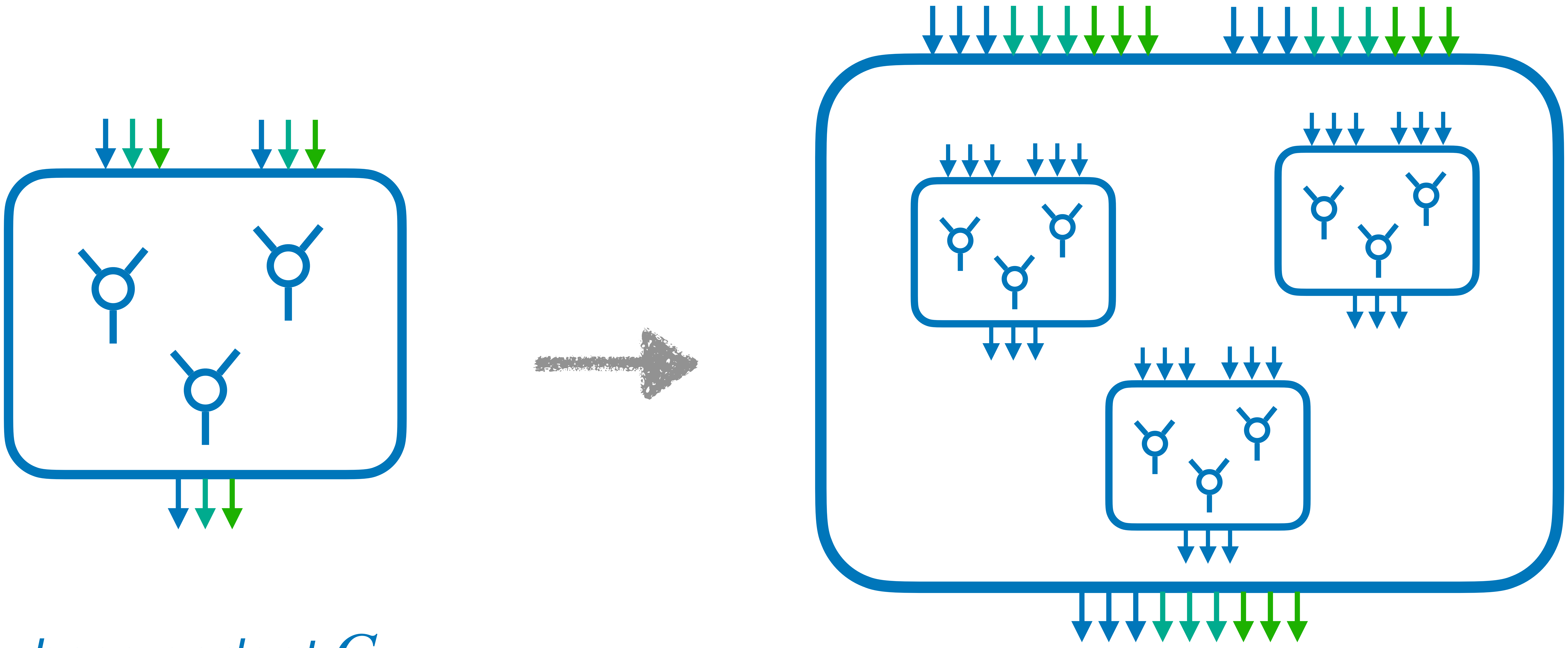
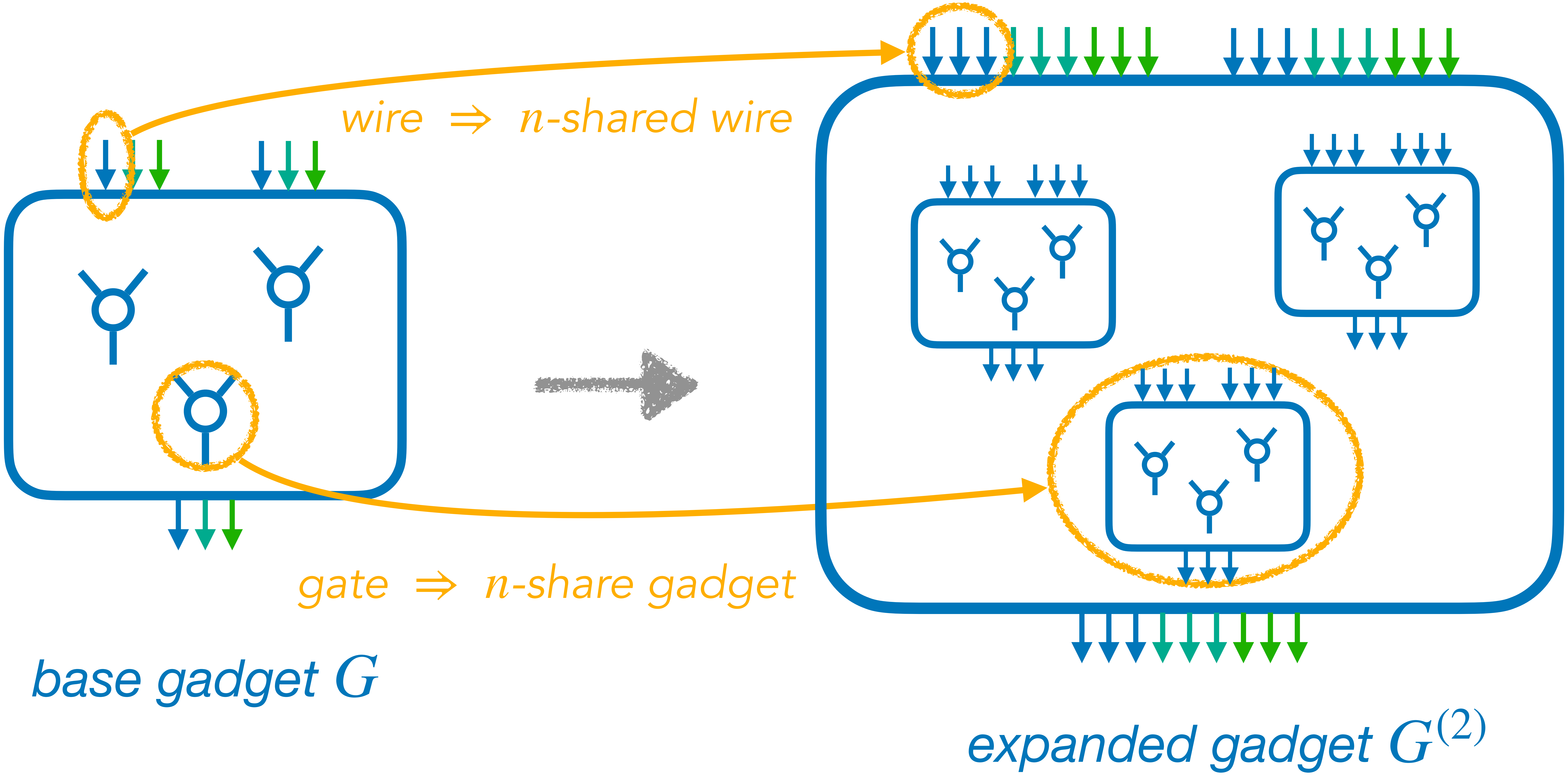
The expansion strategy



base gadget G

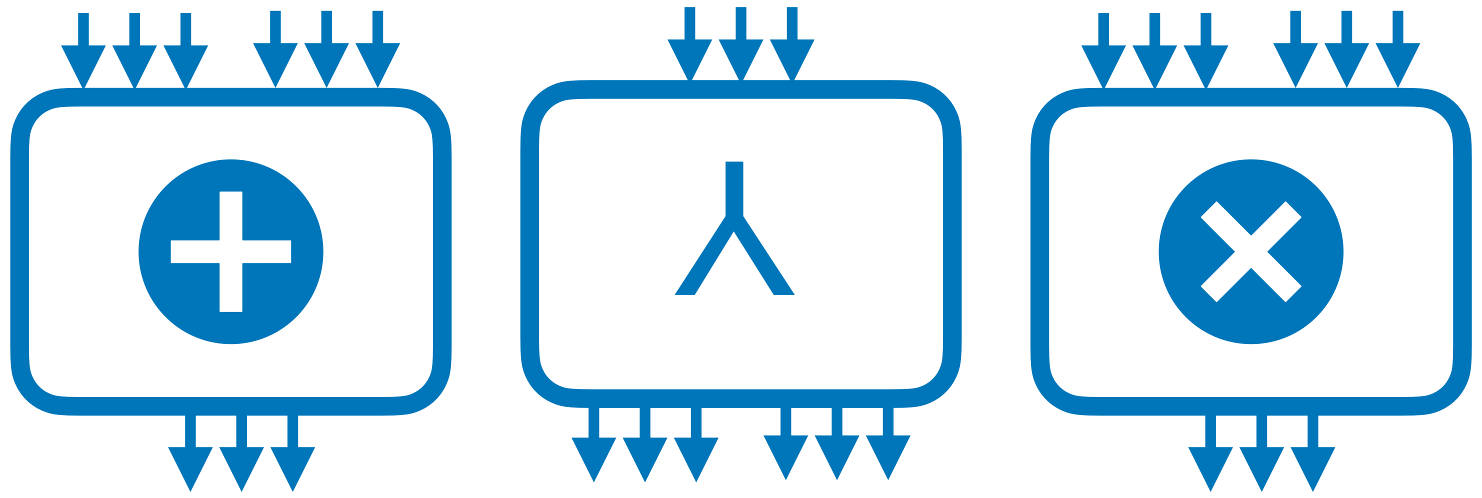
expanded gadget $G^{(2)}$

The expansion strategy



The expansion strategy

💡 Idea: bootstrap constant-size (small) gadgets

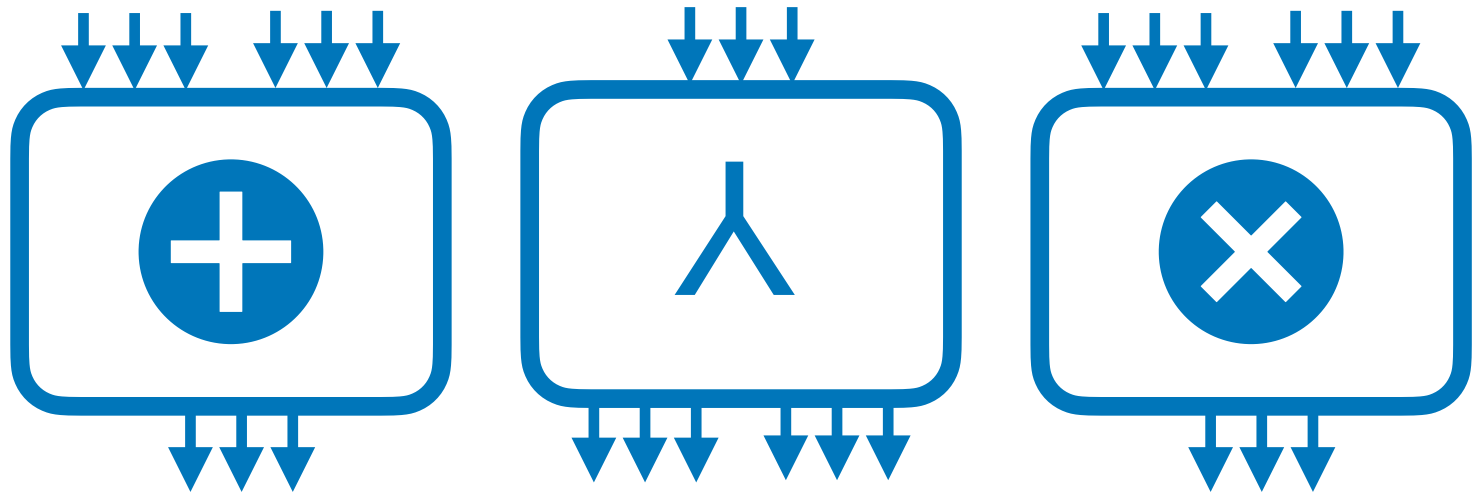


base gadgets

$$\{G\} \to \{G^{(2)}\} \to \cdots \to \{G^{(k)}\}$$

The expansion strategy

Idea: bootstrap constant-size (small) gadgets



base gadgets

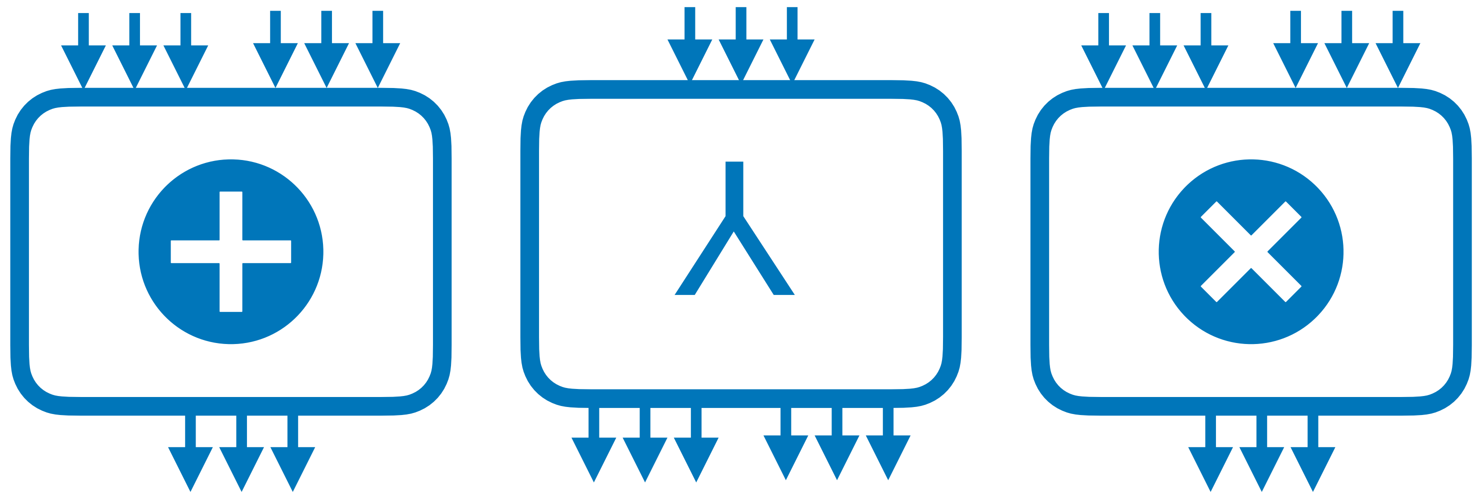
$$\{G\} \to \{G^{(2)}\} \to \cdots \to \{G^{(k)}\}$$

Goal: amplification of random probing security

$$p \longrightarrow f(p)$$

The expansion strategy

Idea: bootstrap constant-size (small) gadgets



base gadgets

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The expansion strategy

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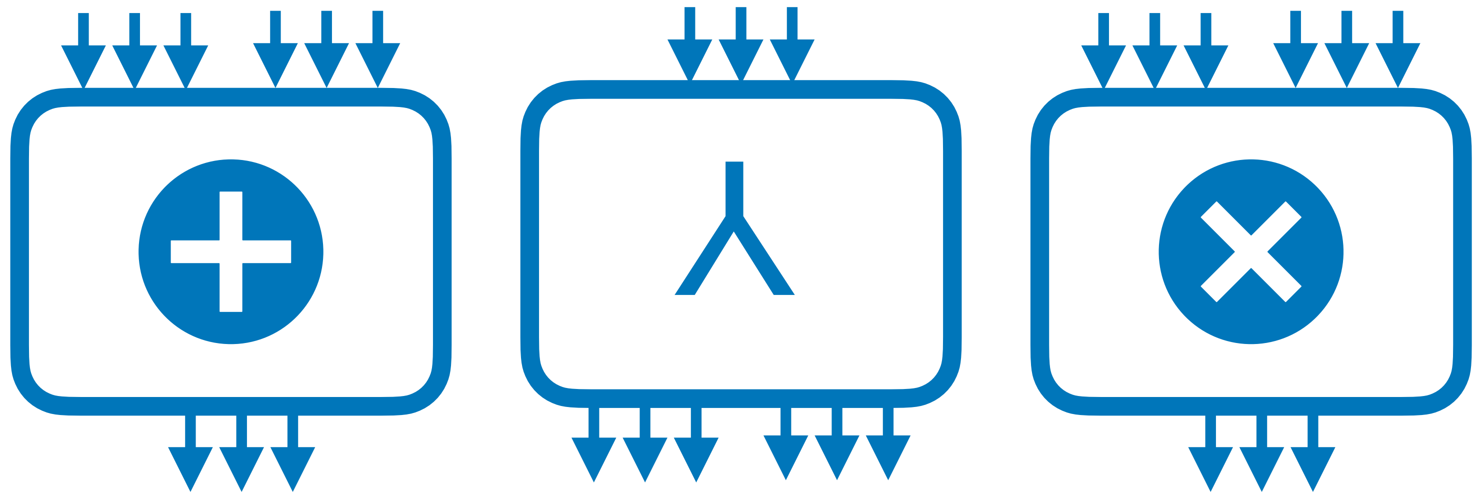

base gadgets

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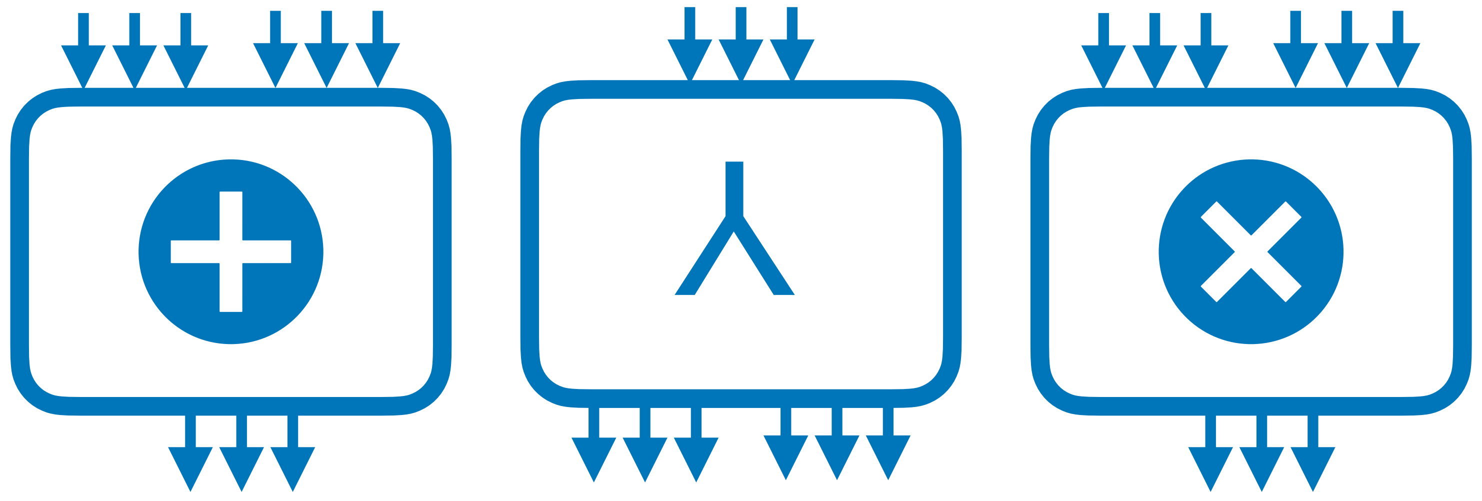

base gadgets

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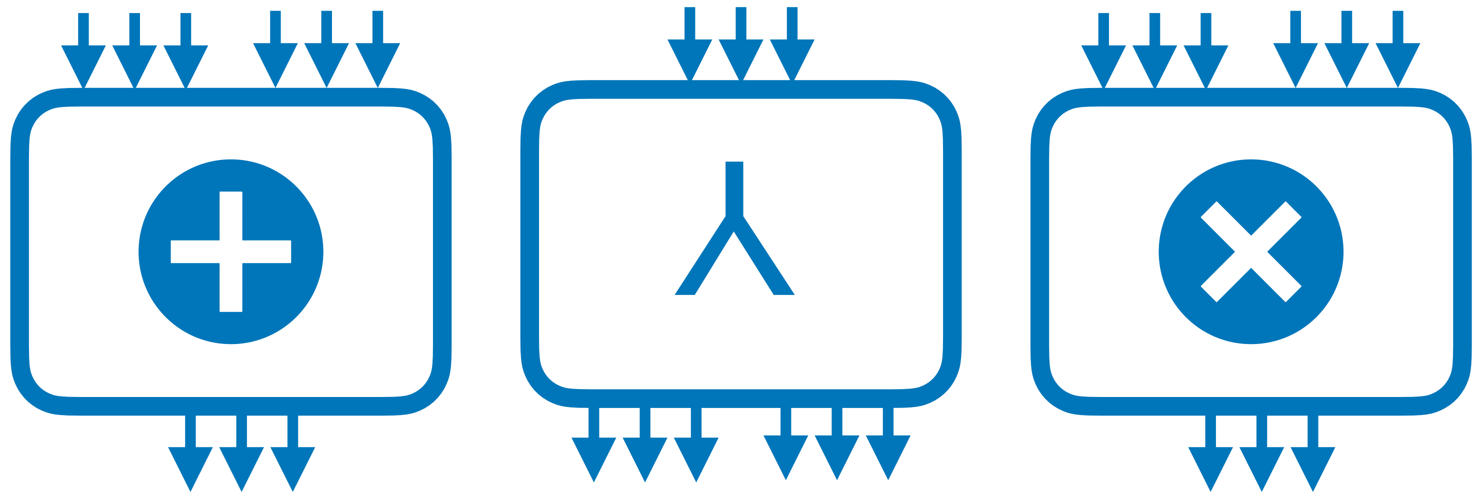
Goal: amplification of random probing security

$$p \longrightarrow f(p) \longrightarrow f(f(p)) \longrightarrow \cdots \longrightarrow f^{(k)}(p)$$

$$f(p) \leq \mathcal{O}(p^d)$$

The expansion strategy

Idea: bootstrap constant-size (small) gadgets



base gadgets

$$\{G\} \rightarrow \{G^{(2)}\} \rightarrow \cdots \rightarrow \{G^{(k)}\}$$

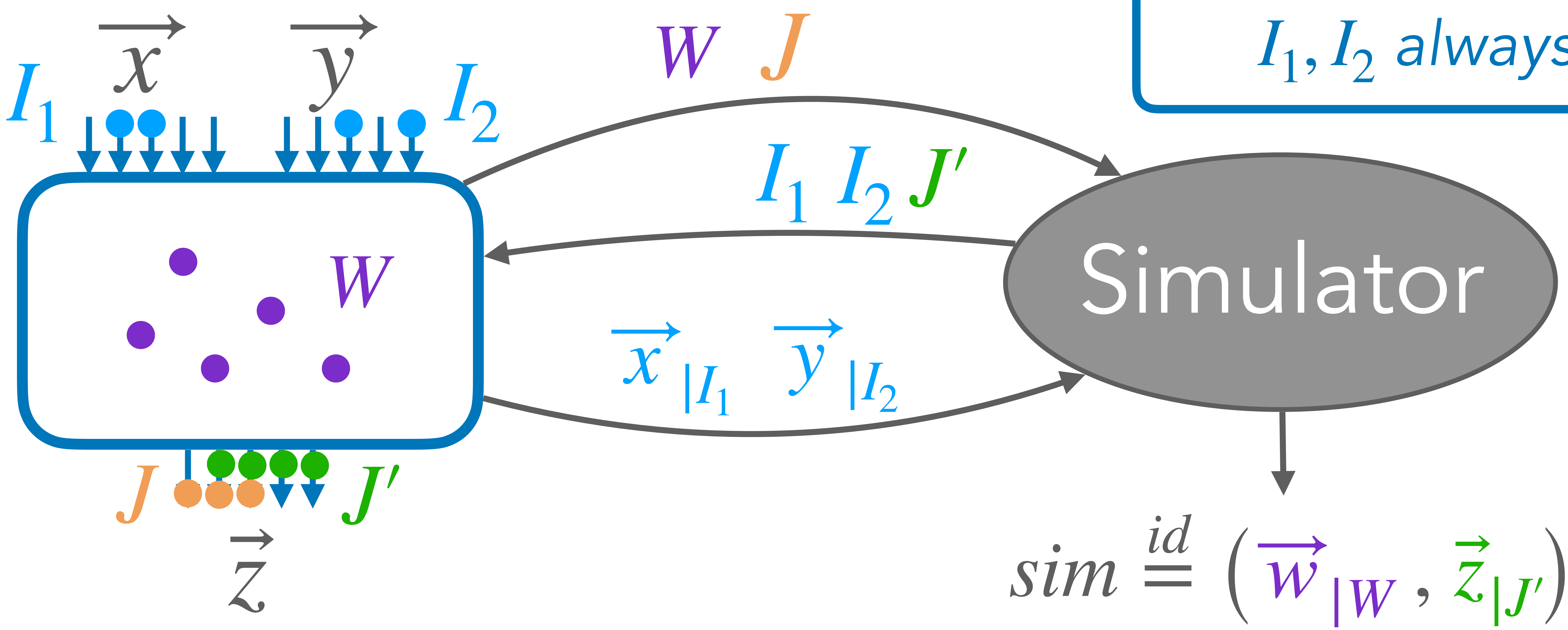
Goal: amplification of random probing security

$$p \longrightarrow f(p) \longrightarrow f(f(p)) \longrightarrow \cdots \longrightarrow f^{(k)}(p)$$

$$f(p) \leq \mathcal{O}(p^d)$$

$$\mathcal{O}\left(p^{d^k}\right)$$

Random probing expandability (RPE)



I_1, I_2 *always exist*

$sim \stackrel{id}{=} (\vec{w}_{|W}, \vec{z}_{|J'})$

RPE threshold t: $|J| \leq t$,
$(|I_1| > t$ *or* $|I_2| > t)$ = *simulation failure*

if $|J| > t$*, sim. can choose*
J' *s.t.* $|J'| = n - 1$

Random probing expandability (RPE)

Base gadgets $\{G\}$ f-RPE $\Rightarrow$ expanded gadgets $\{G^{(2)}\}$ $f^{(2)}$-RPE

$\Rightarrow$ expanded gadgets $\{G^{(k)}\}$ $f^{(k)}$-RPE

Random probing expandability (RPE)

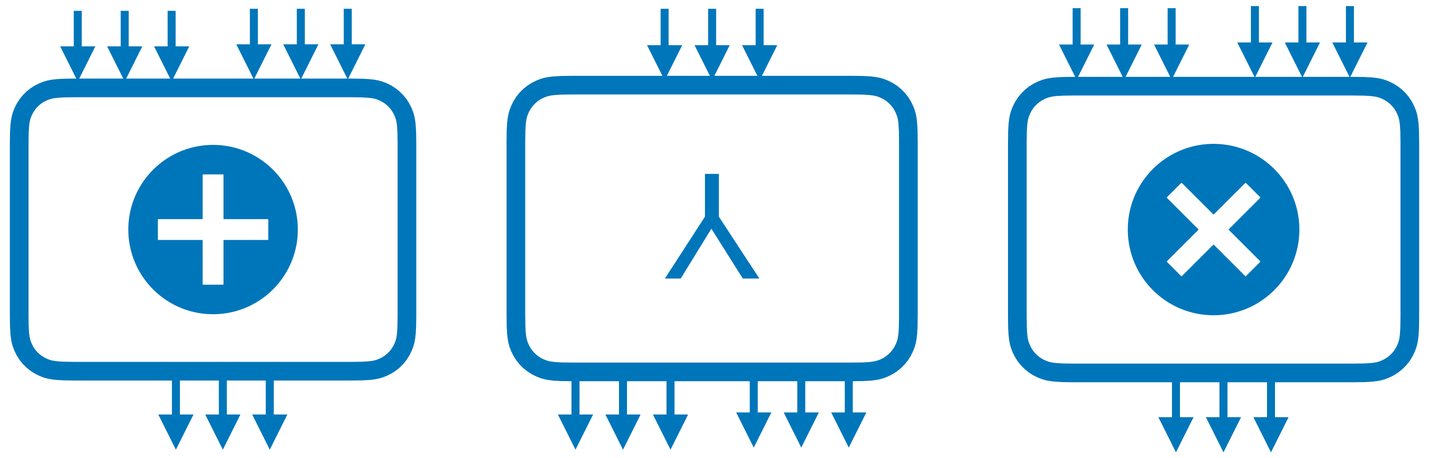
Base gadgets $\{G\}$ f-RPE $\Rightarrow$ expanded gadgets $\{G^{(2)}\}$ $f^{(2)}$-RPE

$\Rightarrow$ expanded gadgets $\{G^{(k)}\}$ $f^{(k)}$-RPE

$f^{(k)}(p)$ simulation security vs.

p-random probing leakage

Concrete instantiations

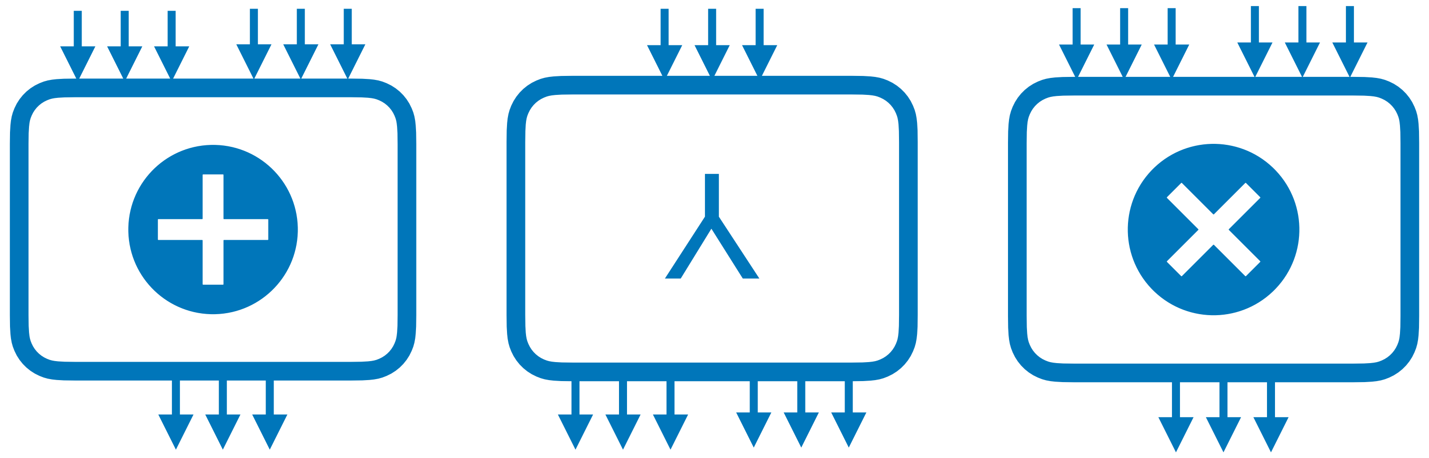


constant-size (small) gadgets

Your gadget is f-RPE

IronMask

Concrete instantiations



constant-size (small) gadgets

Your gadget is f-RPE

IronMask

Maximum tolerated leakage probability

$$p_{max} \in [0,1) \quad \text{such that} \quad f(p_{max}) < p_{max}$$

Concrete instantiations

3-share gadgets

$$G_{\mathsf{R}} : z_1 \leftarrow r_1 + x_1$$
$$z_2 \leftarrow r_2 + x_2$$
$$z_3 \leftarrow (r_1 + r_2) + x_3$$

$$\Rightarrow \quad \mathcal{O}(|C|\kappa^{3.9}) \ , \quad p_{max} = 2^{-7.5}$$

5-share gadgets

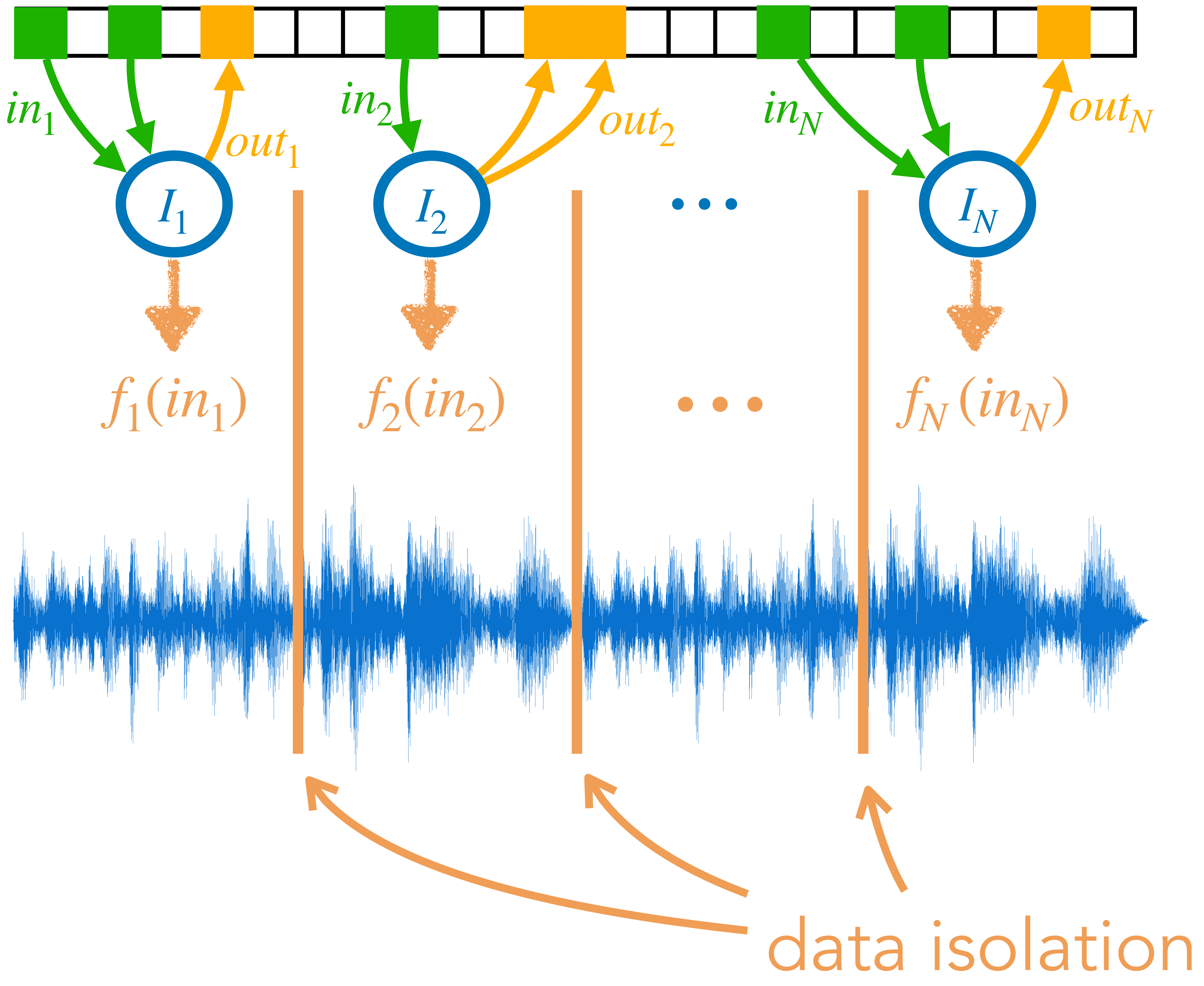
$$G_{\mathsf{R}} : z_1 \leftarrow (r_1 + r_2) + x_1$$
$$z_2 \leftarrow (r_2 + r_3) + x_2$$
$$z_3 \leftarrow (r_3 + r_4) + x_3$$
$$z_4 \leftarrow (r_4 + r_5) + x_4$$
$$z_5 \leftarrow (r_5 + r_1) + x_5$$

$$\Rightarrow \quad \mathcal{O}(|C|\kappa^{3.2}) \ , \quad p_{max} = 2^{-12}$$

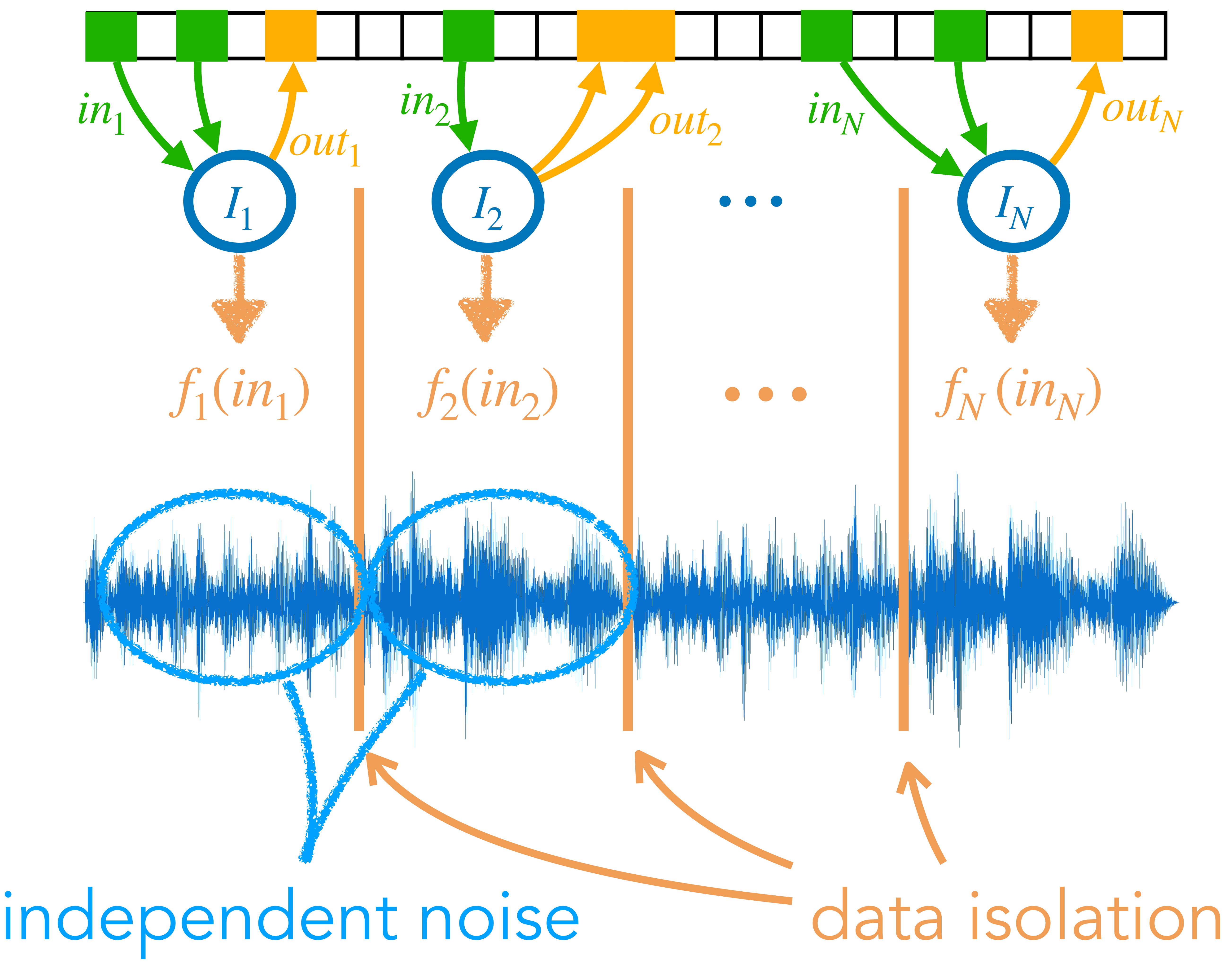
Conclusion

- [x] Provable security against side-channel attacks
- [ ] (Fully) bridging theory and practice

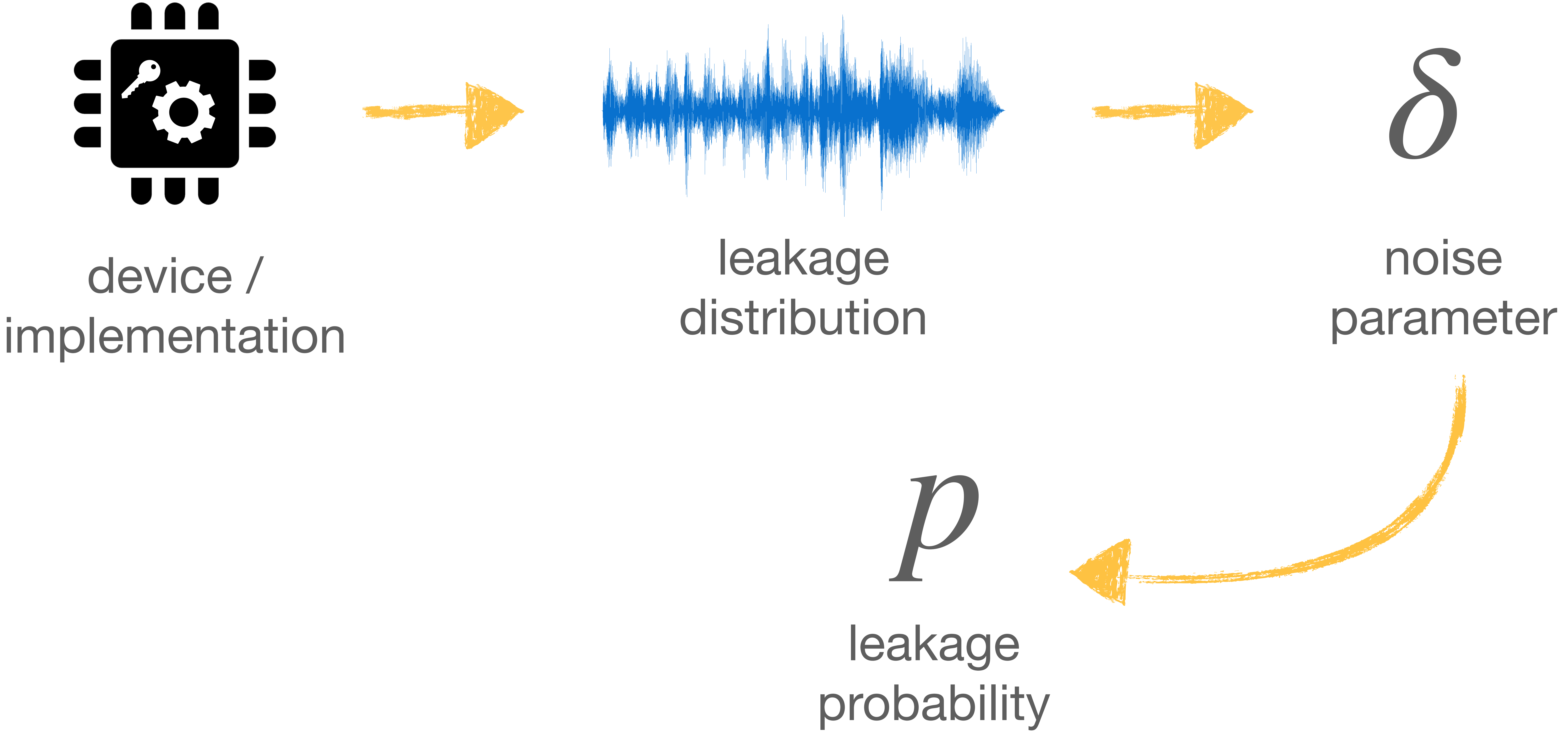
Physical assumptions



Physical assumptions



Noise parameters



Performances

3-share gadgets

$$G_R : z_1 \leftarrow r_1 + x_1$$
$$z_2 \leftarrow r_2 + x_2$$
$$z_3 \leftarrow (r_1 + r_2) + x_3$$

$\Rightarrow \mathcal{O}(|C|\kappa^{3.9})$, $p_{max} = 2^{-7.5}$

→ improved complexity

→ optimised implementations

5-share gadgets

$$G_R : z_1 \leftarrow (r_1 + r_2) + x_1$$
$$z_2 \leftarrow (r_2 + r_3) + x_2$$
$$z_3 \leftarrow (r_3 + r_4) + x_3$$
$$z_4 \leftarrow (r_4 + r_5) + x_4$$
$$z_5 \leftarrow (r_5 + r_1) + x_5$$

$\Rightarrow \mathcal{O}(|C|\kappa^{3.2})$, $p_{max} = 2^{-12}$

VeriSiCC Seminar 2022

Verification and Generation of Side-Channel Countermeasures

September 22, 2022, Paris

https://cryptoexperts.com/verisicc/seminaire-2022.html

Thank you!

🙏🤔⁉️🙋

References

- **Differential Power Analysis** — Paul C. Kocher, Joshua Jaffe, Benjamin Jun: Differential Power Analysis. CRYPTO 1999
- **Masking / soundness of masking with noise** — Suresh Chari, Charanjit S. Jutla, Josyula R. Rao, Pankaj Rohatgi: Towards Sound Approaches to Counteract Power-Analysis Attacks. CRYPTO 1999
- **Masking applied to DES** — Louis Goubin, Jacques Patarin: DES and Differential Power Analysis (The "Duplication" Method). CHES 1999
- **Probing model / ISW scheme** — Yuval Ishai, Amit Sahai, David A. Wagner: Private Circuits: Securing Hardware against Probing Attacks. CRYPTO 2003
- **"Only computation leaks" model** — Silvio Micali, Leonid Reyzin: Physically Observable Cryptography. TCC 2004

References

- **Noisy leakage model** — Emmanuel Prouff, Matthieu Rivain: Masking against Side-Channel Attacks: A Formal Security Proof. EUROCRYPT 2013
- **Unifying probing and noisy models** — Alexandre Duc, Stefan Dziembowski, Sebastian Faust: Unifying Leakage Models: From Probing Attacks to Noisy Leakage. EUROCRYPT 2014
- **Composition security for masking** — Gilles Barthe, Sonia Belaïd, François Dupressoir, Pierre-Alain Fouque, Benjamin Grégoire, Pierre-Yves Strub, Rébecca Zucchini: Strong Non-Interference and Type-Directed Higher-Order Masking. CCS 2016
- **Random probing expansion strategy** — Prabhanjan Ananth, Yuval Ishai, Amit Sahai: Private Circuits: A Modular Approach. CRYPTO 2018

References

- **Quasilinear masking**

  - Dahmun Goudarzi, Antoine Joux, Matthieu Rivain: How to Securely Compute with Noisy Leakage in Quasilinear Complexity. ASIACRYPT 2018
  - Dahmun Goudarzi, Thomas Prest, Matthieu Rivain, Damien Vergnaud: Probing Security through Input-Output Separation and Revisited Quasilinear Masking. IACR TCHES 2021

- **Random probing expandability**

  - Sonia Belaïd, Jean-Sébastien Coron, Emmanuel Prouff, Matthieu Rivain, Abdul Rahman Taleb: Random Probing Security: Verification, Composition, Expansion and New Constructions. CRYPTO 2020
  - Sonia Belaïd, Matthieu Rivain, Abdul Rahman Taleb: On the Power of Expansion: More Efficient Constructions in the Random Probing Model. EUROCRYPT 2021

References

- **IronMask tool** — Sonia Belaïd, Darius Mercadier, Matthieu Rivain, Abdul Rahman Taleb: IronMask: Versatile Verification of Masking Security. IEEE S&P 2022