White-Box Cryptography

Matthieu Rivain

CARDIS 2017

CryptoExperts

How to protect a cryptographic key?

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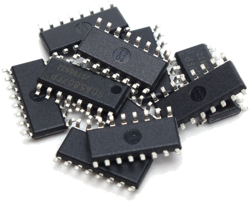


Well, put it in a **smartcard** of course!

… or any piece of **secure hardware**

But...

- Secure hardware is **expensive** (production, integration, infrastructures...)
- Long lifecycle, limited updates
- Bugs, security flaws might occur
  - *e.g.* ROCA vulnerability (October 2017)

COMPLETELY BROKEN —

Millions of high-security crypto keys crippled by newly discovered flaw

Factorization weakness lets attackers impersonate key holders and decrypt their data.

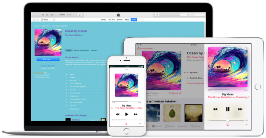
DAN GOODIN (US) - 16/10/2017, 16:30

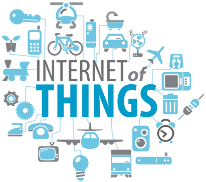
Pure software applications

- Advantages: cheaper, faster time-to-market, easier to update
- Big trend in ICTs: cloud service + mobile app
- HCE-based mobile payment
  - SE not available
  - Emulated SE in software
  - Short-term keys (tokens)
  - Regular authentication to server ("always on" paradigm)

Pure software applications

- IoT (without SE)
- Content protection, DRM
- OS / firmwares





Protecting keys in software?

- Potential threats:
  - malwares
  - co-hosted applications
  - users themselves
  - ...
- White-box adversary model
  - analyse the code
  - access the memory
  - tamper with execution
  - ...
- Ex: scan the memory for secret keys

Illustration: Shamir, van Someren. *Playing hide and seek with stored keys*.

White-box cryptography

General idea: hide the secret key in an obfuscated cryptographic implementation

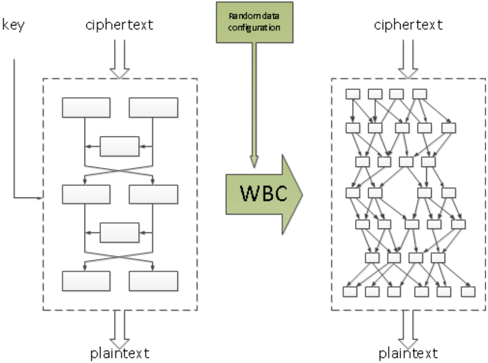
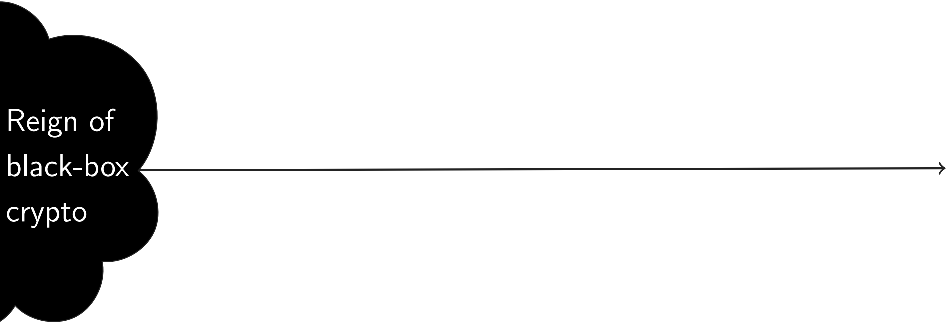


Illustration: `http://www.whiteboxcrypto.com/`

A scientific timeline



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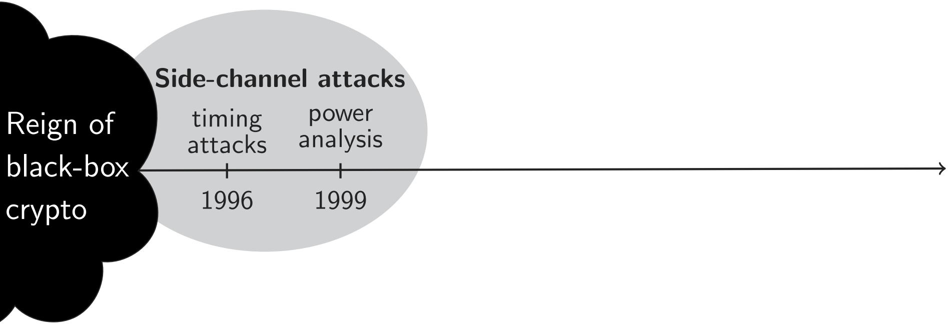

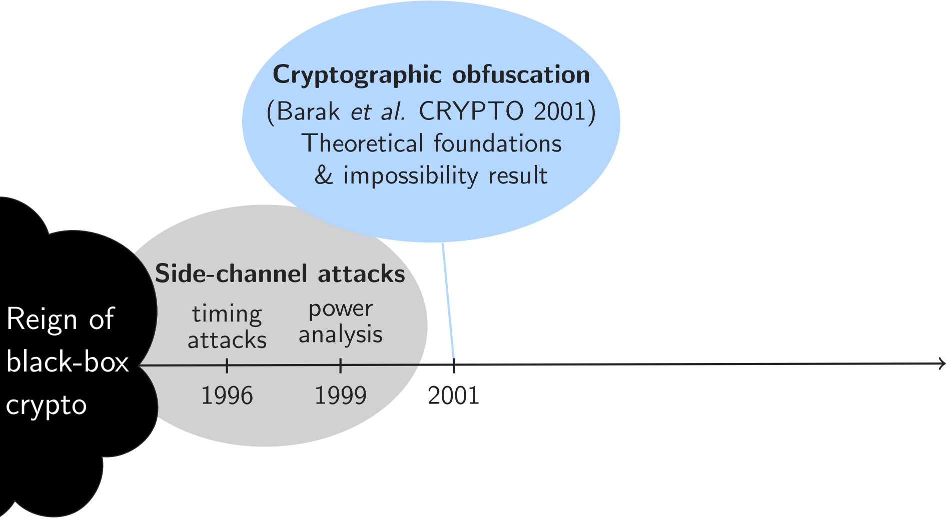
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& impossibility result
Side-channel attacks
timing attacks
power analysis
Reign of black-box crypto
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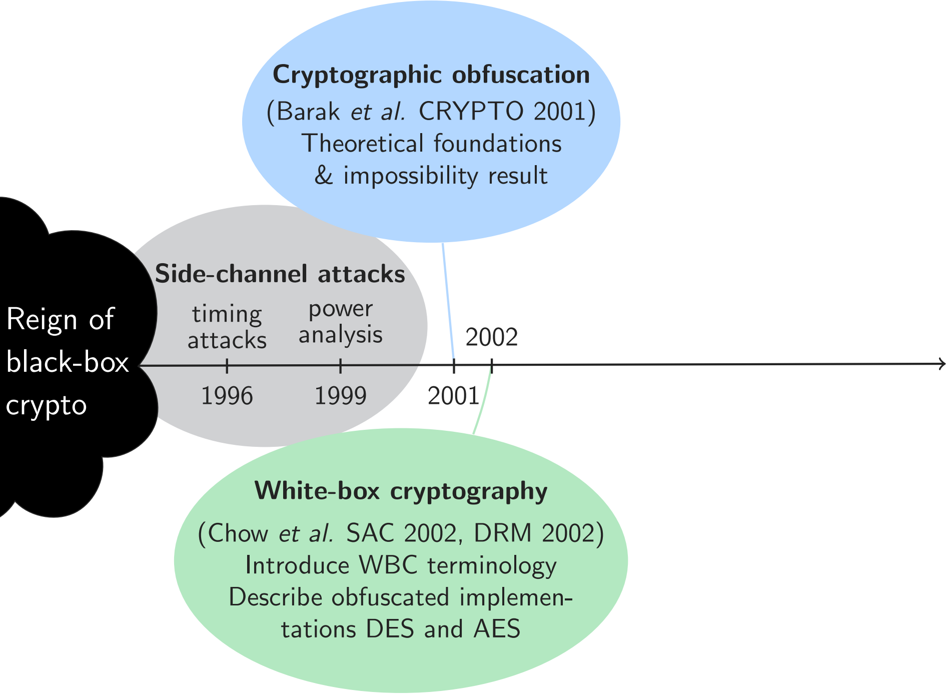
1999
2001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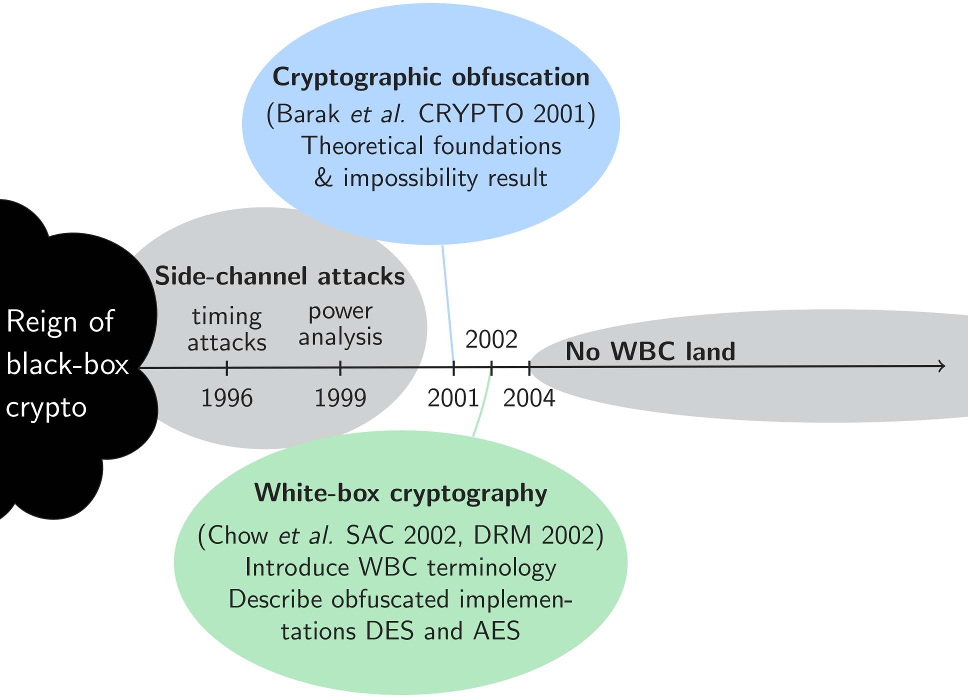
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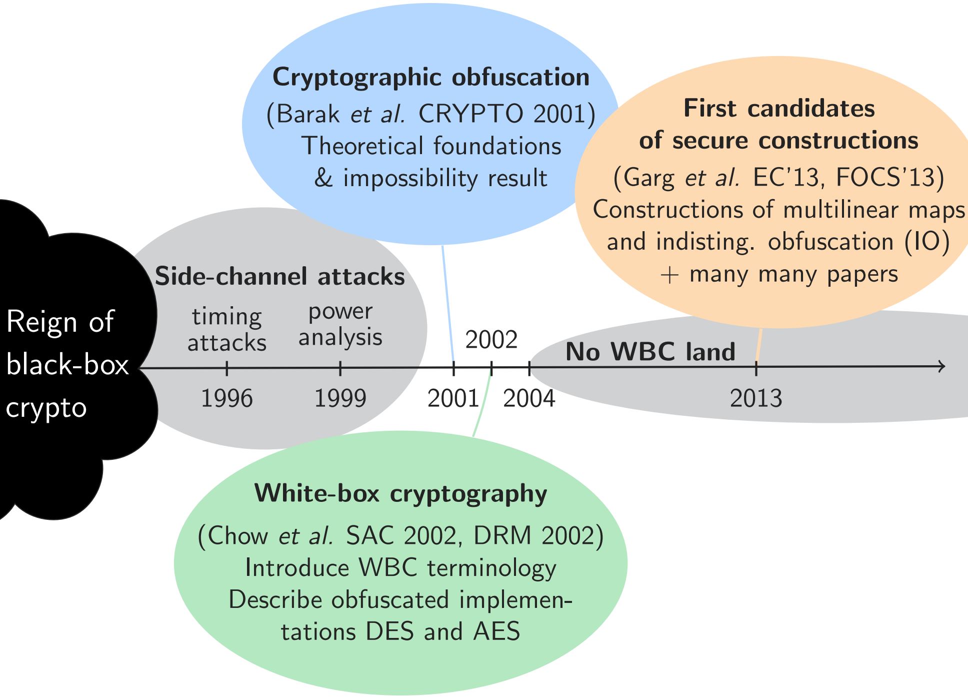
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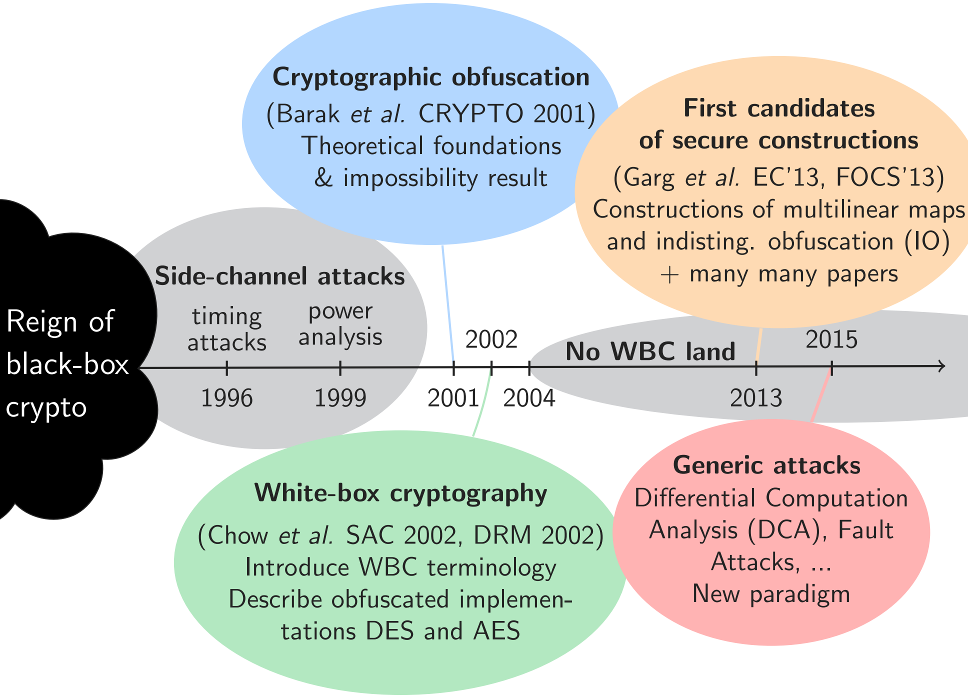
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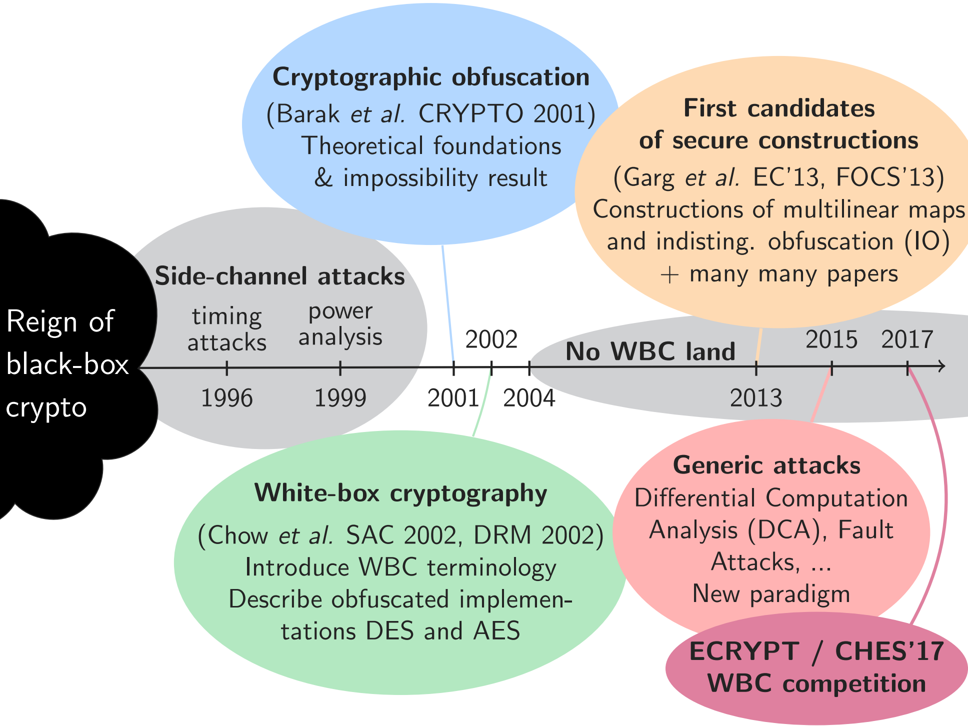
Generic attacks
Differential Computation Analysis (DCA), Fault Attacks, ...
New paradigm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New paradigm
ECRYPT / CHES'17
WBC competition

Overview of this talk

- White-box crypto theory
  - Formal definitions & security notions
- White-box crypto practice
  - Practical constructions & attacks
- White-box crypto competition
  - Wrap-up, break of challenge 777

White-Box Crypto Theory

What is a program?

- A word in a formal language $P \in \mathcal{L}$

$$\begin{aligned} \text{execute}: \mathcal{L} \times \{0,1\}^* &\to \{0,1\}^* \\ (P, input) &\mapsto output \end{aligned}$$

(Universal Turing Machine)

- $|P|$: size of $P \in \mathcal{L}$
- time(P): # operations for execute$(P, \cdot)$

What is a program?

- $P \equiv f$ *(P implements f)*

$$\forall x \ : \ \text{execute}(P, x) = f(x)$$

- $P_1 \equiv P_2$ *(functional equivalence)*

$$\forall x \ : \ \text{execute}(P_1, x) = \text{execute}(P_2, x)$$

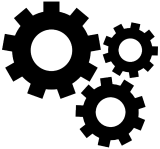
- Straight-line programs
  - no conditional statements, no loops
  - $|P| = \text{time}(P)$

What is an obfuscator?

- An algorithm:

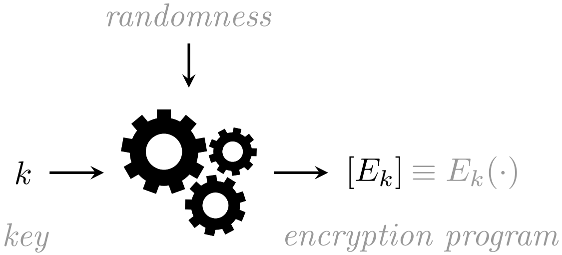
$$randomness$$

$$\downarrow$$

$P \longrightarrow$  $\longrightarrow O(P) \equiv P$

- Size and execution time increase
(hopefully not too much)

What is a white-box compiler?

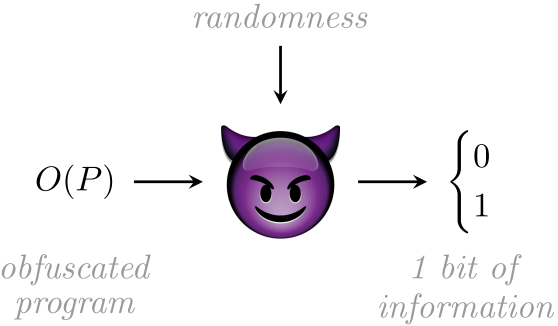


- Specific to an encryption function E
- Can be constructed from an obfuscator

$$k \to P \equiv E_k(\cdot) \xrightarrow{O} [E_k]$$

What is an adversary?

- An algorithm:



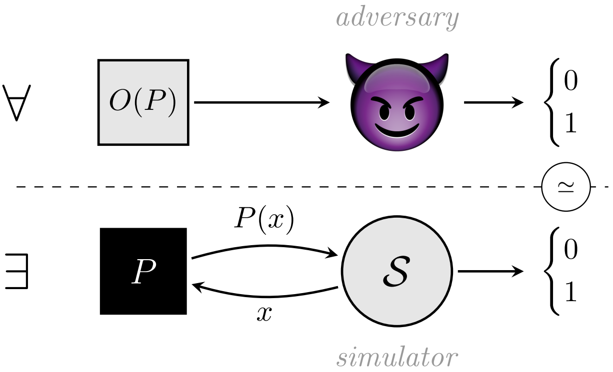
- Ex: msb of k if $P \equiv \mathrm{AES}_k(\cdot)$
- Wlg: $\nexists$ 1-bit ♉ $\Rightarrow$ $\nexists$ multi-bit ♉

[Barak *et al.* – CRYPTO 2001]

On the (Im)possibility of Obfuscating Programs

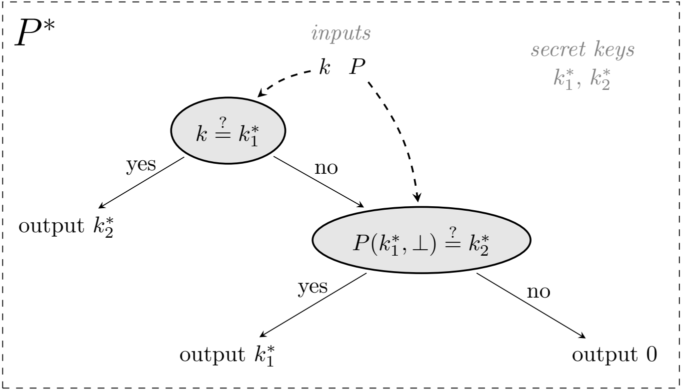
- Virtual Black Box (VBB) security notion
- Impossibility result: VBB cannot be achieved for all programs (counterexample)
- Indistinguishability Obfuscation (IO)

VBB security notion



- $O(P)$ reveals nothing more than the I/O behavior of P

Impossibility result

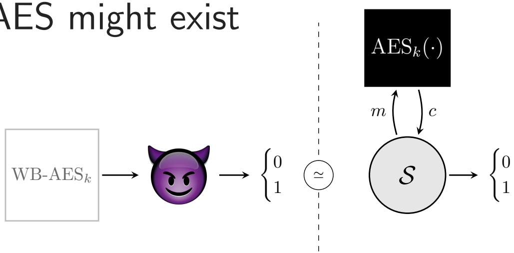


P^* cannot be VBB obfuscated:

- BB access to P^* reveals nothing
- But $O(P^*)\big(0, O(P^*)\big) = k_1^*$

The good news

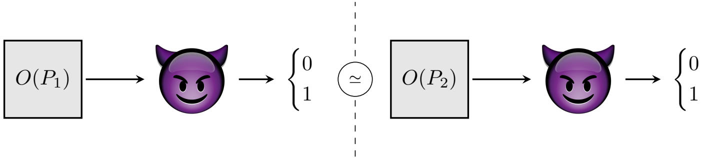
- The impossibility result does not apply to a given encryption algorithm
- VBB AES might exist



- The bad news: seems very hard to achieve

Indistinguishability Obfuscation (IO)

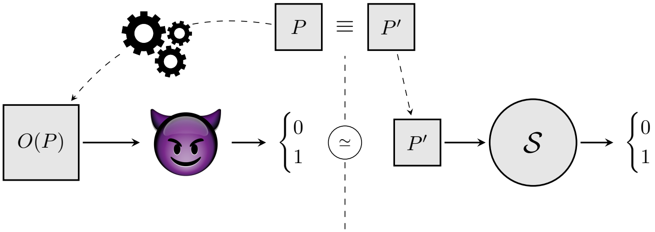
- Notion restricted to straight-line programs
- For any (P_1, P_2) st $P_1 \equiv P_2$ and $|P_1| = |P_2|$



- *i.e.* $O(P_1)$ and $O(P_2)$ are indistinguishable

Why is IO meaningful?

- IO $\Leftrightarrow$ Best Possible Obfuscation
- For any P':



- $O(P)$ doesn't reveal anything more than the *best obfuscated* program P'

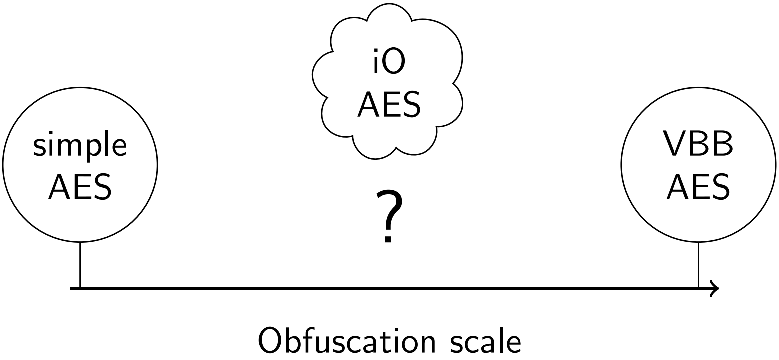
Is IO meaningful for WBC?

- IO does not imply resistance to key extraction
- For instance

$$\text{Any prog } P \equiv \mathsf{AES}_k(\cdot) \longmapsto \text{Ref implem of } \mathsf{AES}_k(\cdot)$$

- Nevertheless

$$\exists P^* \equiv \mathsf{AES}_k(\cdot) \text{ secure}$$
$$\Rightarrow$$
$$\forall P \equiv \mathsf{AES}_k(\cdot) \text{ with } |P| \geq |P^*|\text{: } IO(P) \text{ secure}$$


iO
AES
simple
AES
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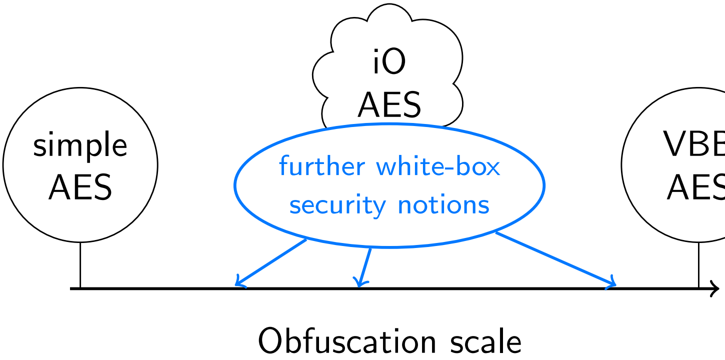
Obfuscation scale


iO
AES
simple
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further white-box
security notions
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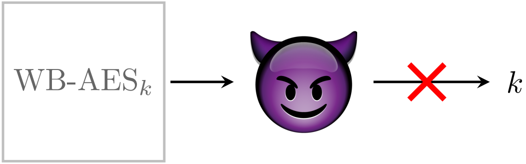
White-box security notions

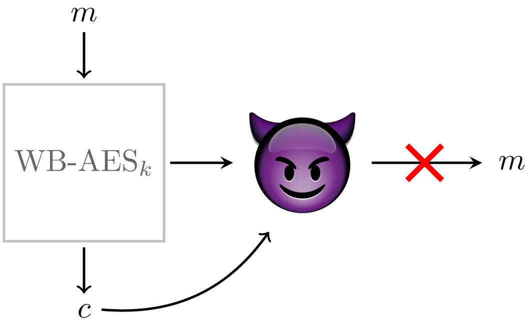
- **Unbreakability**: resistance to key extraction



- Basic requirement but insufficient in practice
- Other security notions
  - [SWP09] *Towards Security Notions for White-Box Cryptography* (ISC 2009)
  - [DLPR13] *White-Box Security Notions for Symmetric Encryption Schemes* (SAC 2013)

One-wayness

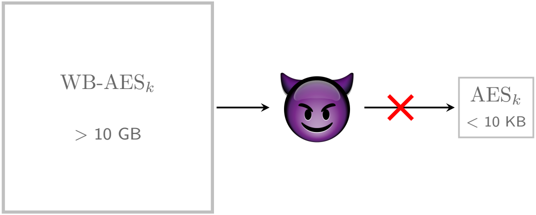
- **One-wayness**: hardness of inversion



- Turns AES into a public-key cryptosystem
- PK crypto with light-weight private operations

Incompressibility

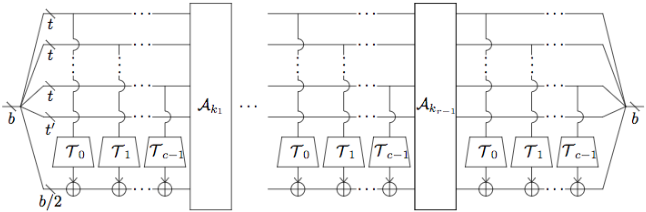
- **Incompressibility**: hardness of compression



- Makes the implementation less convenient to share at a large scale

Incompressibility

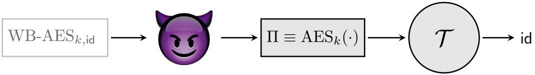
- Incompressible primitives recently proposed
  - Bogdanov *et al.* (CCS 2015, Asiacrypt 2016)
  - Fouque *et al.* (Asiacrypt 2016)



- But no white-box implementations of a standard cipher (*e.g.* AES)

Security features

- **Traceability**: WB implem traceable



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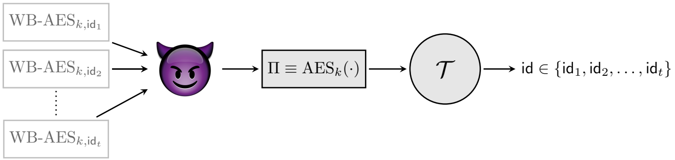

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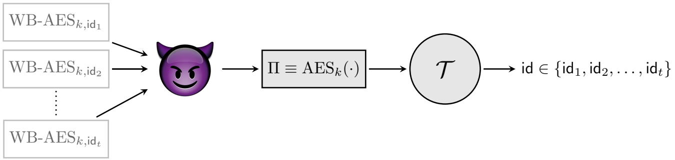
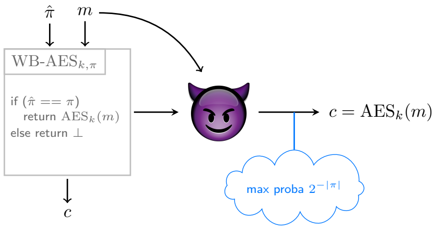


- **Password**: WB implem locked by password



Some relations

- [DLPR13] Perturbation-Value Hiding notion:

$$\text{PVH} \Rightarrow \text{traceability}$$

- If the underlying encryption scheme is secure:

$$\begin{array}{ccccc} & & \text{INC} & & \\ & & \Downarrow & & \\ \text{OW} & \Rightarrow & \text{UBK} & \Leftarrow & \text{PVH} \end{array}$$

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- No UBK construction known for AES

  $\Rightarrow$ no OW/INC/PVH/VBB construction neither

IO constructions

- Very active research field
  - 18 papers in 2017 (IACR conferences)
  - 22 papers in 2016 (IACR conferences)
- Most constructions rely on *multilinear maps*

$$e : (g_1^{e_1}, g_2^{e_2}, \ldots, g_d^{e_d}) \longmapsto g_T^{e_1 \cdot e_2 \cdots e_d}$$

(or noisy variants)

- Many breaks, security still under investigation
- Performances far beyond practical applications

White-Box Crypto Practice

Original white-box AES

- SAC 2002: *"White-Box Cryptography and an AES Implementation"* (Chow *et al.*)
- First step: network of look-up tables
- Each round split in 4 *sub-rounds*

$$(x_0, x_5, x_{10}, x_{15}) \mapsto \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \otimes \begin{pmatrix} S(x_0 \oplus k_0) \\ S(x_5 \oplus k_5) \\ S(x_{10} \oplus k_{10}) \\ S(x_{15} \oplus k_{15}) \end{pmatrix}$$

Original white-box AES

- Computed as

$$T_0[x_0] \oplus T_5[x_5] \oplus T_{10}[x_{10}] \oplus T_{15}[x_{15}]$$

- Tables T_i : 8 bits → 32 bits

$$\begin{aligned}
T_0[x] &= S(x \oplus k_0) \times (02\ 01\ 01\ 03)^T \\
T_5[x] &= S(x \oplus k_5) \times (03\ 02\ 01\ 01)^T \\
T_{10}[x] &= S(x \oplus k_{10}) \times (01\ 03\ 02\ 01)^T \\
T_{15}[x] &= S(x \oplus k_{15}) \times (01\ 01\ 03\ 02)^T
\end{aligned}$$

- XOR table: 8 bits → 4 bits

$$T_{\text{xor}}[x_0 || x_1] = x_0 \oplus x_1$$

Original white-box AES

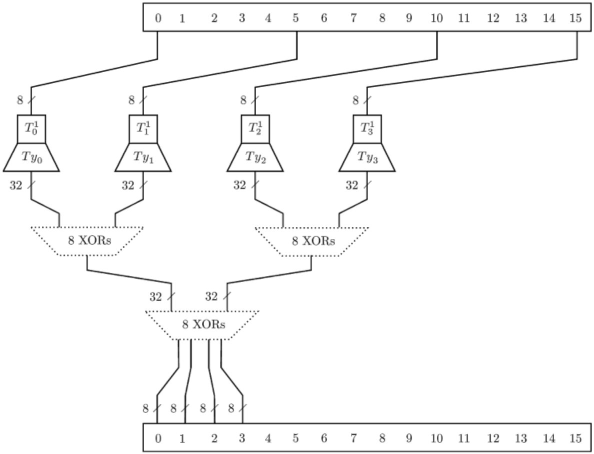


Illustration: J. Muir *"A Tutorial on White-box AES"* (ePrint 2013)

Original white-box AES

- Second step: randomize look-up tables
- Each table T is replaced by

$$T' = g \circ T \circ f^{-1}$$

where f, g are **random encodings**

- For two *connected* tables T, R

$$\begin{aligned} T' &= g \circ T \circ f^{-1} \\ R' &= h \circ R \circ g^{-1} \end{aligned} \quad \Rightarrow \quad R' \circ T' = h \circ (R \circ T) \circ f^{-1}$$

Original white-box AES

- Intuition: encoded tables bring no information
- True for a single (bijective) table $g \circ T \circ f^{-1}$
- Not for the large picture

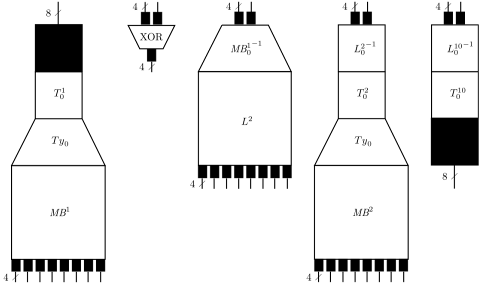
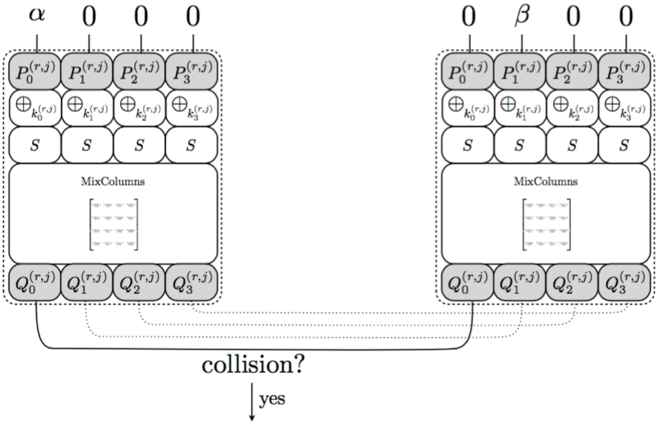


Illustration: J. Muir *"A Tutorial on White-box AES"* (ePrint 2013)

Many breaks

- First break: BGE attack
  - Billet *et al. Cryptanalysis of a White Box AES Implementation* (SAC 2004)
- Generic attack on WB SPN ciphers
  - Michiels *et al. Cryptanalysis of a Generic Class of White-Box Implementations* (SAC 2008)
- Collision attack & improved BGE attack
  - Lepoint *et al. Two Attacks on a White-Box AES Implementation* (SAC 2013)
- Attack complexity ~ 2^{22}

Example: collision attack



$$02 \cdot S_0(\alpha) \oplus 03 \cdot S_1(0) = 02 \cdot S_0(0) \oplus 03 \cdot S_1(\beta)$$

where $S_0(x) = S(P_0(x) \oplus k_0)$ and $S_1(x) = S(P_1(x) \oplus k_1)$

Patches and variants

- Perturbed WB-AES using MV crypto (Bringer *et al.* ePrint 2006)
  ⇒ broken (De Mulder *et al.* INDOCRYPT 2010)

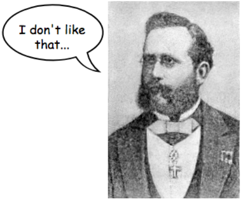
- WB-AES based on wide linear encodings (Xiao-Lai, CSA 2009)
  ⇒ broken (De Mulder *et al.* SAC 2012)

- WB-AES based on dual AES ciphers (Karroumi, ICISC 2010)
  ⇒ broken (Lepoint *et al.* SAC 2013)

- Same situation with DES

Secret design paradigm

- Industrial need
  - home-made solutions
  - mix of several obfuscation techniques
  - secret designs

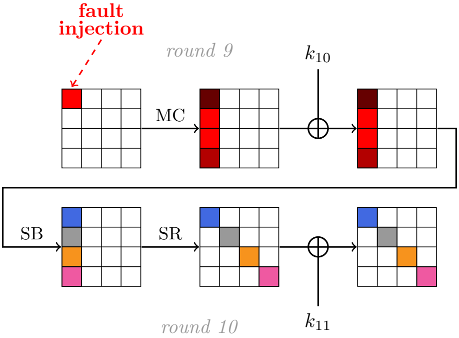


Auguste Kerckhoffs

- Security evaluations by ITSEF labs
- Development of generic attacks
  - Fault attacks, DCA
  - Avoid costly reverse engineering effort

Fault attacks

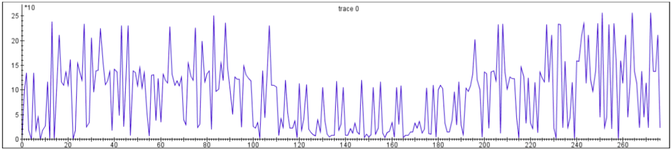
- Easy fault injection in the white-box context
- Plenty of efficient FA techniques (on *e.g.* AES)



- Original white-box AES vulnerable to this attack

Differential Computation Analysis

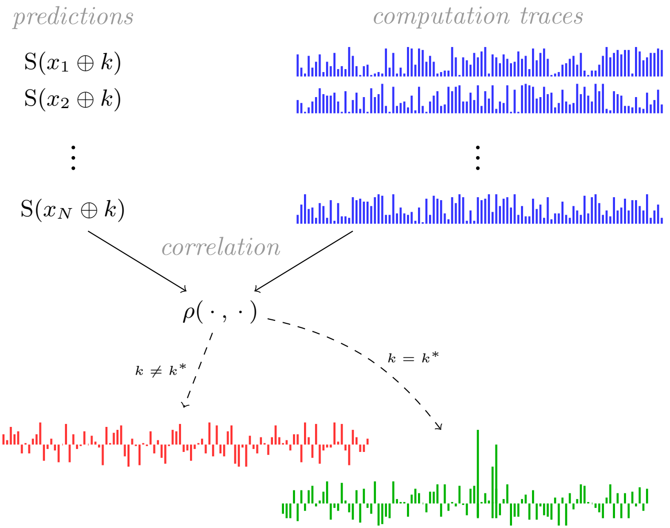
- Suggested by NXP / Riscure
  - Presentation at BalckHat 2015
  - Best paper award CHES 2016
- Record data-dependent information at execution ⇒ *computation trace*



Trace: J. Bos (presentation CHES 2016)

- Apply DPA techniques to computation traces

Differential Computation Analysis



DCA in presence of encodings

- DCA can break the original white-box AES
  - [Bos *et al.* CHES 2016] *Differential Computation Analysis*

- Why?
  - random encodings are hardcoded
  - for some Enc, we might have

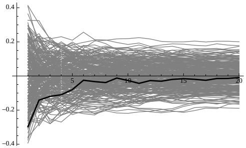
$$\rho(x_i, \mathsf{Enc}(x)_j) \gg 0$$

  - especially with 4-bit encodings

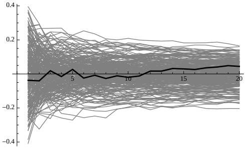
$$\mathsf{Enc}(x_0 \,\|\, x_1) = \mathsf{Enc}(x_0) \,\|\, \mathsf{Enc}(x_1)$$

DCA experiment

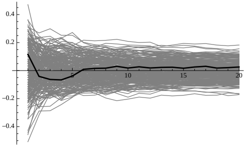
- Random 4-bit encoding Enc
- Correlation $\rho\big(\mathrm{S}(x \oplus k)_0, \, \mathsf{Enc}(\mathrm{S}(x \oplus k^*))_j\big)$



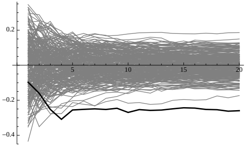
Bit 0



Bit 1



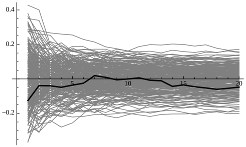
Bit 2



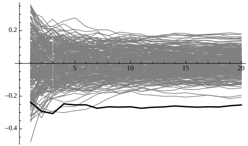
Bit 3

DCA experiment

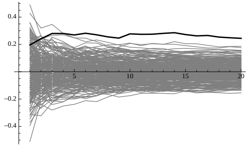
- With another (4-bit) encoding



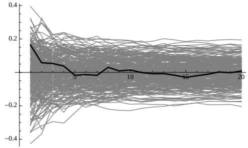
Bit 0



Bit 1



Bit 2

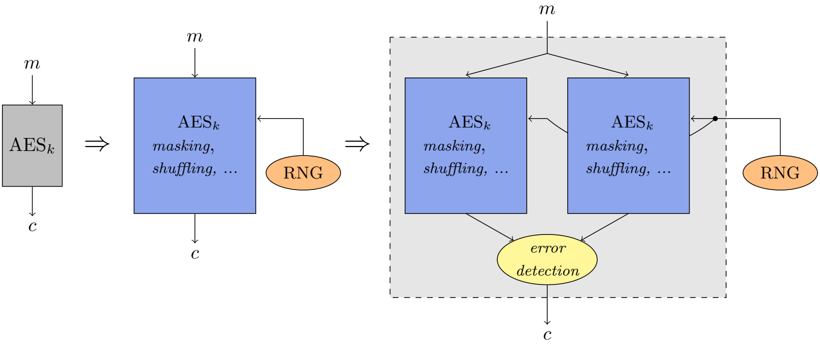


Bit 3

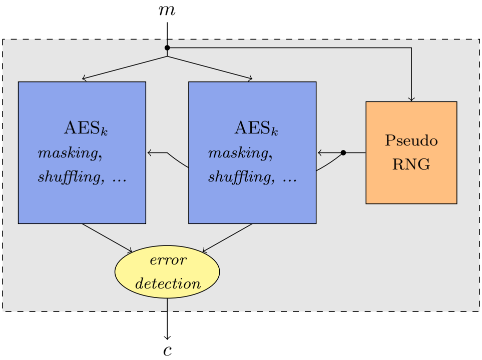
- Most of the time 1, 2, or 3 bits leak

Countermeasures?

- Natural approach: use known SCA/FA countermeasures

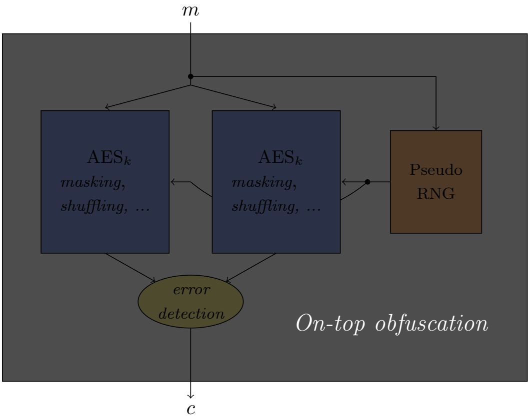


Countermeasures?



- Pseudo-randomness from m
- PRNG should be somehow secret

Countermeasures?



- Countermeasures hard to remove
- P-randomness / redundancy hard to detect

Open problems

- How to obfuscate the countermeasures?
- How to generate pseudo-randomness?
- Resistance to higher-order DCA, multiple FA?

White-Box Crypto Competition


CHES 2017
Capture the Flag
Challenge
The WhibOx Contest
An ECRYPT White-Box Cryptography Competition

WhibOx Contest

- Goal: confront designers and attackers in the **secret design paradigm**
- Designers could submit WB AES implems:
  - C source code $\leq$ 50MB
  - executable $\leq$ 20MB
  - RAM consumption $\leq$ 20MB
  - running time $\leq$ 1sc
- Attackers could try to recover the keys of submitted implems

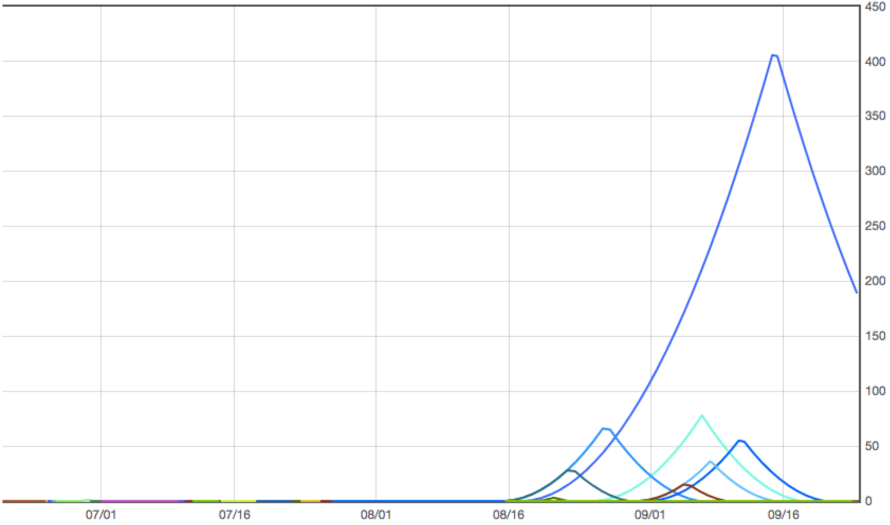
Score system

- Unbroken implem on day n

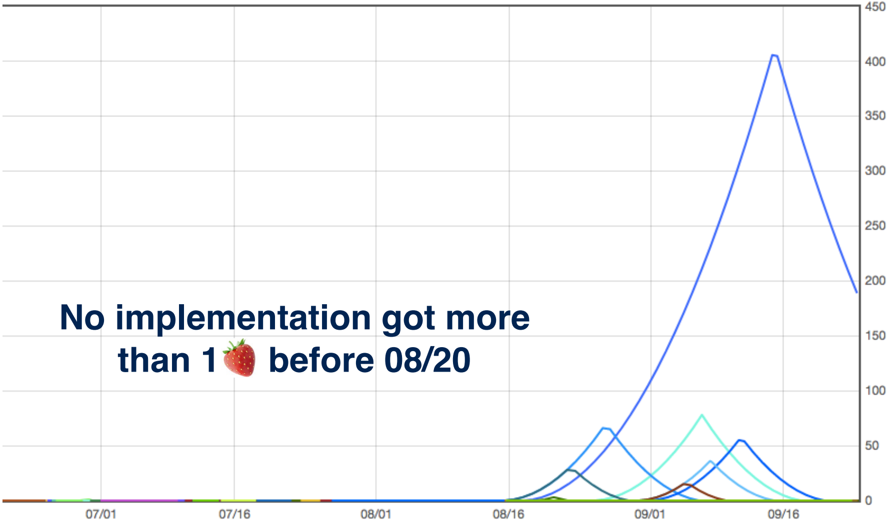
$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}\ \textbf{ST}$$

- Break on day n
  - Designer gets $\frac{n(n+1)}{2}$ **ST** points
  - Attacker gets $\frac{n(n+1)}{2}$ **BN** points
  - Challenge score starts decreasing symmetrically

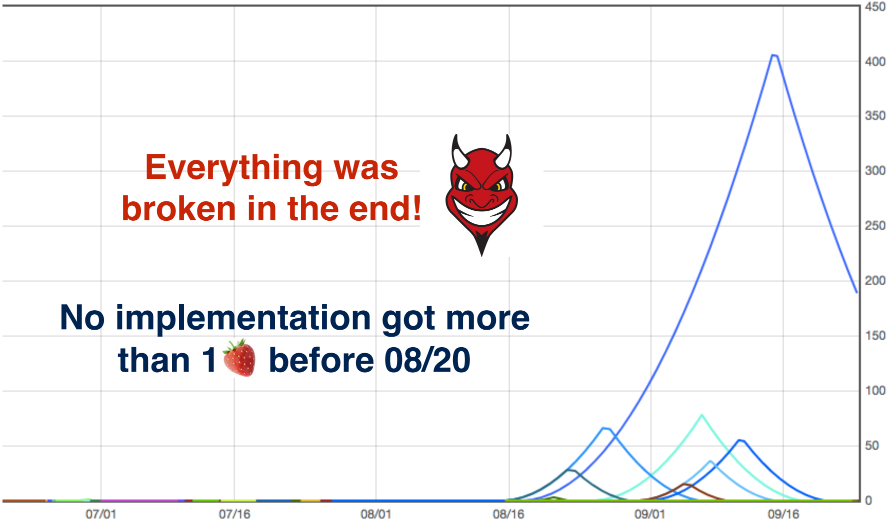
Strawberry scores over time



Strawberry scores over time



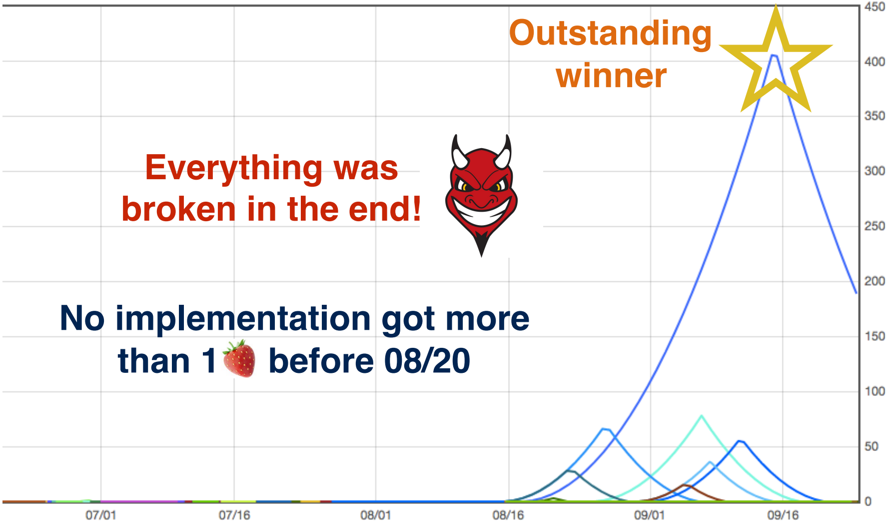
Strawberry scores over time



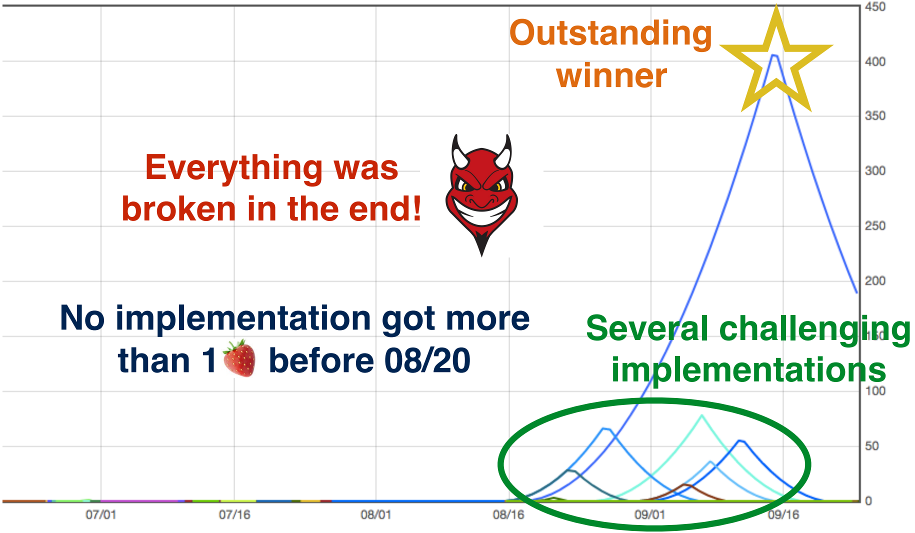
Everything was broken in the end!

No implementation got more than 1🍓 before 08/20

Strawberry scores over time



Strawberry scores over time



Results

- 94 submitted implementations
- ~ 870 breaks
- Socreboard:

id	designer	breaker	score	# days	# breaks
777	cryptolux	team_cryptoexperts	406	28	1
815	grothendieck	cryptolux	78	12	1
753	sebastien-riou	cryptolux	66	11	3
877	chaes	You!	55	10	2
845	team4	cryptolux	36	8	2

cryptolux: Biryukov, Udovenko

team_cryptoexperts: Goubin, Paillier, Rivain, Wang

Implementation 777

- Several obfuscation layers
  - Encoded Boolean circuit
  - Bitslicing, error detection, dummy operations
  - Virtualization, naming obfuscation
- Code size: 28 MB
- Code lines: 2.3 K
- 12 global variables
  - `pDeoW`: computation state (2.1 MB)
  - `JGNNvi`: program bytecode (15.3 MB)

Implementation 777

- 1020 functions of the form

```
void xSnEq (uint UMNsVLp, uint KtFY, uint vzJZq) {
  if (nIlajqq () == IFWBUN (UMNsVLp, KtFY))
    EWwon (vzJZq);
}

void rNUiPyD (uint hFqeIO, uint jvXpt) {
  xkpRp[hFqeIO] = MXRIWZQ (jvXpt);
}

void cQnB (uint QRFOf, uint CoCiI, uint aLPxnn) {
  ooGoRv[(kIKfgI + QRFOf) & 97603] =
    ooGoRv[(kIKfgI + CoCiI) | 173937] & ooGoRv[(kIKfgI + aLPxnn) | 39896];
}

uint dLJT (uint RouDUC, uint TSCaTl) {
  return ooGoRv[763216 ul] | qscwtK (RouDUC + (kIKfgI << 17), TSCaTl);
}
```

Analysis of functions

- Table of function pointers indexed by bytecode
- Only 210 functions are called (over 1020)
- Ducplicates of 21 different functions
    - memory reading/writing
    - bitwise operations, bit shifts
    - goto, conditional jump

De-virtualisation

```
PROGRAM  = ...  // bytecode 15.3 MB
FUNC_PTR = ...  // 210 function pointers

interpretor()
{
    pc = 0;
    while(pc < eop)
    {
        nb_arg = PROGRAM[pc]; pc++;
        func_index = PROGRAM[pc]; pc++;
        function = FUNC_PTR[func_index];
        for (i=0; i<nb_arg; i++)
        {
            arg[i] = PROGRAM[pc]; pc++;
        }
        function(arg[0], ...);
    }
}
```

Simulation ⇒ equivalent program with do-while loops
of arithmetic instructions

Human reverse engineering

- Remove some dummy loops
- Get sequence of 64-loops of 64-bit instructions
  - First part: 64×64 bitslice program
  - 3 instances with the input plaintext
  - rest with hardcoded values
  - Second part: (probably) error detection and extraction of the ciphertext
- Extract a Boolean circuit with $\sim$ **600K gates**

SSA form

- Put it in Static Single Assignment (SSA) form:

$$
\begin{array}{lcl}
x & = & \ldots \\
y & = & \ldots \\
t & = & \neg x \\
x & = & t \oplus y \\
y & = & y \wedge t \\
t & = & x \vee y \\
 & \vdots &
\end{array}
\quad \Rightarrow \quad
\begin{array}{lcl}
x & = & \ldots \\
y & = & \ldots \\
t_1 & = & \neg x \\
t_2 & = & t_1 \oplus y \\
t_3 & = & y \wedge t_1 \\
t_4 & = & t_2 \vee t_3 \\
 & \vdots &
\end{array}
$$

Circuit minimization

Detect (over many executions) and remove:

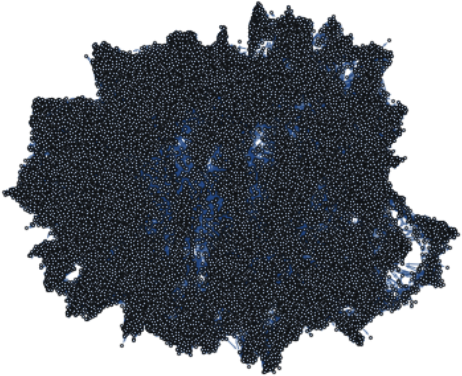
- Dummy variable: t_i never used?
- Constant: $t_i = 0$? $t_1 = 1$?
- Duplicate: $t_i = t_j$?
- Pseudo-randomness:

$$(t_i \to t_i \oplus 1) \;\Rightarrow\; \text{same result?}$$

- Several rounds: ∼600K ⇒ ∼**280K** gates

Data dependency analysis

Data dependency graph (20% of the circuit):



Data dependency analysis

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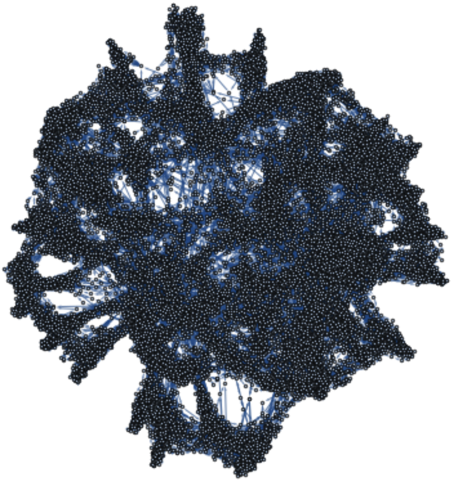
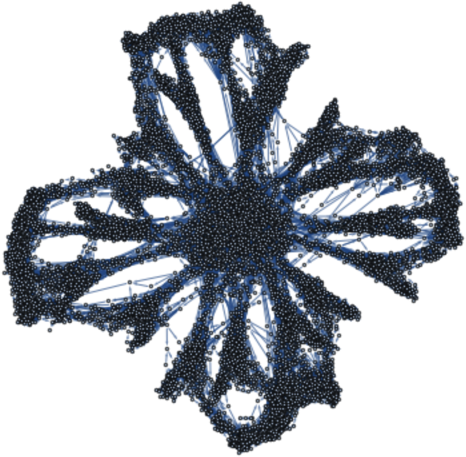


Data dependency analysis

Data dependency graph (5% of the circuit):



Data dependency analysis

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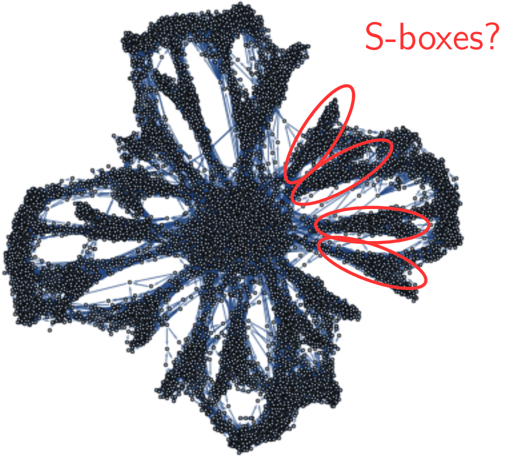

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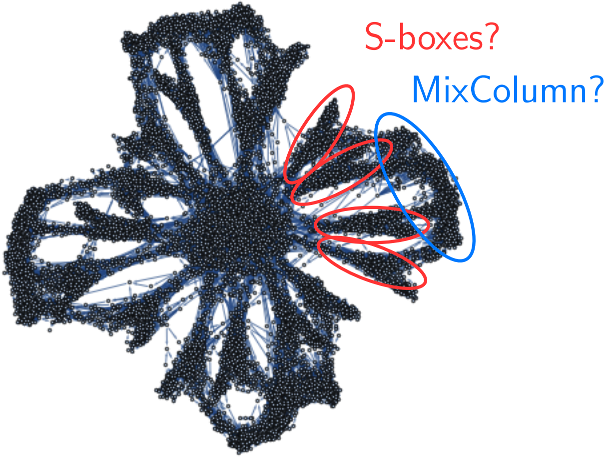

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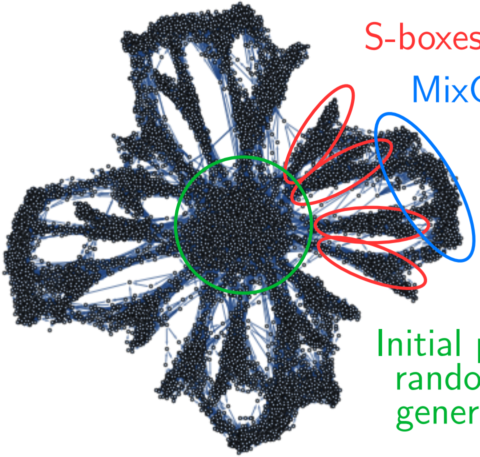

Data dependency analysis

- Cluster analysis $\Rightarrow$ gates within one "s-box"
- Identify all the outgoing variables:

$$s_1, s_2, \ldots, s_n$$

- Likely hypothesis:

$$S(x \oplus k^*) = \mathsf{Dec}(s_1, s_2, \ldots, s_m)$$

for some deterministic decoding function

Key recovery

- Hypothesis: linear decoding function
- Record the s_i's over n executions

$$\begin{bmatrix} s_1^{(1)} & s_2^{(1)} & \cdots & s_m^{(1)} \\ s_1^{(2)} & s_2^{(2)} & \cdots & s_m^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ s_1^{(n)} & s_2^{(n)} & \cdots & s_m^{(n)} \end{bmatrix} \qquad \begin{bmatrix} S_j(x^{(1)} \oplus k) \\ S_j(x^{(2)} \oplus k) \\ \vdots \\ S_j(x^{(n)} \oplus k) \end{bmatrix},$$

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- Linear system solvable for $k = k^*$

Key recovery

- And it works! For example:
  - s-box cluster with $n = 34$ outgoing variables
  - using $T = 50$ executions traces
  - one solution per S_j for $k = k^*$
  - no solutions for $k \neq k^*$

$j = 0$: 0,0,0,0,0,0,1,0,1,0,1,1,1,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 1$: 0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 2$: 0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 3$: 0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 4$: 0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 5$: 0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 6$: 0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 7$: 0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0

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j = 1: 0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
j = 2: 0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
j = 3: 0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
j = 4: 0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
j = 5: 0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
j = 6: 0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
j = 7: 0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0

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$j = 1$: 0,0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 2$: 0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 3$: 0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 4$: 0,0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 5$: 0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
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- Decoding

$$\underbrace{(s_7, s_8, \ldots, s_{21})}_{\text{15 outgoing bits}} \xrightarrow{\times \text{ Bin. Mat.}} \underbrace{(S_0(x \oplus k^*), \ldots, S_7(x \oplus k^*))}_{\text{8 s-box coordinates}}$$

Key recovery

- And it works! For example:
  - s-box cluster with $n = 34$ outgoing variables
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$j = 0$: 0,0,0,0,0,0,1,0,1,0,1,1,1,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 1$: 0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 2$: 0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 3$: 0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 4$: 0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 5$: 0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 6$: 0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$j = 7$: 0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0

- Decoding

$$\underbrace{(s_7, s_8, \ldots, s_{21})}_{\text{15 outgoing bits}} \xrightarrow{\times \text{ Bin. Mat.}} \underbrace{(S_0(x \oplus k^*), \ldots, S_7(x \oplus k^*))}_{\text{8 s-box coordinates}}$$

Conclusion

- Theory:
  - No provably secure constructions
  - More work needed on security models & notions
- Practice:
  - Everything broken in the literature
  - Moving toward a secret design paradigm
  - More work needed on generic attacks and countermeasures in the white-box context
- ECRYPT / CHES'17 competition:
  - Nothing stood > 28 days
  - Can obscurity really bring (a bit of) security?