Attack and Improvement of a Secure S-box Calculation Based on the Fourier Transform

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August 11, 2008

Outline

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3. Differential Power Analysis *vs.* Biased Masking
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Differential Power Analysis (DPA)

DPA Basics

- **Physical leakage** dependent on **intermediate variables**
- **Sensitive variable** depends on both the input plaintext and on a guessable part of the secret key
- **DPA** exploits the physical leakage on a sensitive variable for key recovery

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DPA Security

Every intermediate variable is independent of any sensitive variable.

Masking & S-box protection issue

Masking Countermeasure

- Every sensitive variable Z is masked with a random value R
- **masked variable** $\widetilde{Z} = Z \oplus R$ and **mask** R both independent of Z
- Masked variables and masks processed separately
- Completeness: $Z = \widetilde{Z} \oplus R$

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Key addition

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Substitution box

Issue: From $Z \oplus R$ and R, compute $F(Z) \oplus R'$.
All intermediate var. must be independent of Z.

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$$\widehat{F}(Z) = \sum_{a \in \mathbb{F}_2^n} F(a)(-1)^{a \cdot Z} .$$

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- It satisfies $\widehat{\widehat{F}} = 2^n F$, that is:

$$F(Z) = \frac{1}{2^n} \widehat{\widehat{F}}(Z) = \frac{1}{2^n} \sum_{a \in \mathbb{F}_2^n} \widehat{F}(a)(-1)^{a \cdot Z}$$

S-box Masking Based on the Fourier Transform

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INPUTS: a masked var. $\widetilde{Z} = Z \oplus R_1$, a mask R_1, a look-up table $\widehat{F}$
OUTPUTS: a masked output $F(Z) \oplus R_3$, a mask R_3

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$$(-1)^{(\widetilde{Z} \oplus R_2) \cdot R_1} F(Z) + R_3 \bmod 2^n =$$
$$\frac{1}{2^n}\left(2^n R_3 + R_4 + \sum_{a \in \mathbb{F}_2^n} \widehat{F}(a)(-1)^{a \cdot \widetilde{Z} \oplus R_1 \cdot (a \oplus \widetilde{Z} \oplus R_2)} \bmod 2^{2n}\right)$$

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Remark

The sum is implemented by a loop on 2^n elements.

$\Rightarrow$ *Of interest for S-boxes with small dimensions (e.g. $n = 4$).*

The Flaw

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- R_1 and $(\widetilde{Z}\oplus R_2)$ are independently and uniformly distributed (iud)
- *The scalar product of two iud r. v. $X\cdot Y$ is not a uniform r. v.:*

$$\mathrm{P}\left[X\cdot Y = 0\right] = \frac{1}{2} + \frac{1}{2^{n+1}} .$$

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DPA Principle

- Let $b_{k^*} = f(X, k^*)$ be a bit of the computation, where
  - X is a public variable (uniformly distributed)
  - k^* is a guessable part of the secret key
- Let L be the leakage on b_{k^*}

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- Assuming $\alpha > 0$ we have $|(1 - 2\alpha)\Delta| < |\Delta|$

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  - $\Delta_k \to \left(\frac{1}{2} + \varepsilon\right)\Delta + \left(\frac{1}{2} - \varepsilon\right)(-\Delta)$

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- If $k \neq k^*$ then $\Delta_k \to 2\varepsilon(1 - 2\alpha)\Delta$
- The convergence requires about $(\frac{1}{2\varepsilon})^2$ times more leakage measurements

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DPA against the FT-Based S-box Masking

- Targeted bit: $a \cdot Z \oplus R_1 \cdot (\widetilde{Z} \oplus R_2)$
  - Z: sensitive n-bit S-box input
  - a: loop index
  - $R_1 \cdot (\widetilde{Z} \oplus R_2)$: biased mask

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- Mask bias: $\varepsilon = \frac{1}{2^{n+1}}$
  - Number of required measurements multiply by $(\frac{1}{2\varepsilon})^2 = 2^{2n}$
  - If $n = 4$ then $(\frac{1}{2\varepsilon})^2 = 256$

DPA against the FT-Based S-box Masking
Practical Experiment

- Masked AES implementation
- S-box implemented with the *composite field method*
- F is defined as : $F(x) = \begin{cases} x^{-1} & \text{if } x \in GF(16)\backslash\{0\} \\ 0 & \text{if } x = 0 \end{cases}$

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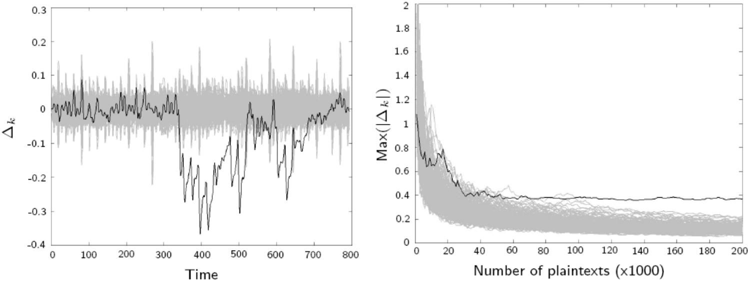

(a) Value of Δ_k. (b) Convergence of $|\Delta_k|$.

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Improved FT-Based S-box Masking

- The exponent is masked with one random bit R_2

$$
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&(-1)^{R_2} F(Z) + R_3 \bmod 2^n \\
&\quad = \left\lfloor \frac{1}{2^n} \left(2^n R_3 + R_4 + \sum_{a \in \mathbb{F}_2^n} \widehat{F}(a)(-1)^{a \cdot Z \oplus R_2} \bmod 2^{2n} \right) \right\rfloor ,
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Improved FT-Based S-box Masking

- The exponent is masked with one random bit R_2

$$(-1)^{R_2} F(Z) + R_3 \bmod 2^n = \left\lfloor \frac{1}{2^n} \left(2^n R_3 + R_4 + \sum_{a \in \mathbb{F}_2^n} \widehat{F}(a)(-1)^{a \cdot Z \oplus R_2} \bmod 2^{2n} \right) \right\rfloor ,$$

- For every a:

$$\begin{array}{ll} Tmp \leftarrow a \cdot \widetilde{Z} & [Tmp = a \cdot \widetilde{Z}] \\ Tmp \leftarrow Tmp \oplus R_2 & [Tmp = a \cdot \widetilde{Z} \oplus R_2] \\ Tmp \leftarrow Tmp \oplus a \cdot R_1 & [Tmp = a \cdot Z \oplus R_2] \end{array}$$

- DPA Security: exponent masked by R_2, sum masked by (R_3, R_4)

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- Efficiency: 2^{n+1} look-ups avoided

Outline

Conclusion

- The FT-based DPA countermeasure of CHES 2006 has a flaw
- The flaw makes an efficient DPA attack possible
- Our attack has been practically validated
- We propose an improved version of the countermeasure
  - provably secure against DPA
  - more efficient than the original countermeasure