High Order Side-Channel Security for Elliptic-Curve Implementations

Sonia Belaïd and Matthieu Rivain

CHES 2023, September 13, Prague

Roadmap

- Case study: SCA on Montgomery ladder & countermeasures
- Our solution for high-order side-channel security
- Formal model and security proof
- Application and performances

Case study: Montgomery ladder

Algorithm 1 Montgomery ladder

Input: $\boldsymbol{P}$, $\boldsymbol{k} = (k_0, k_1, \ldots, k_{n-1})_2$

Output: $\boldsymbol{Q} = [\boldsymbol{k}]\boldsymbol{P}$

$R_0 \leftarrow \boldsymbol{\mathcal{O}}$

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for $i = n - 1$ **downto** 0 **do**

    $b \leftarrow k_i$

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Algebraic variables (EC points)

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Index variables (scalar bits)

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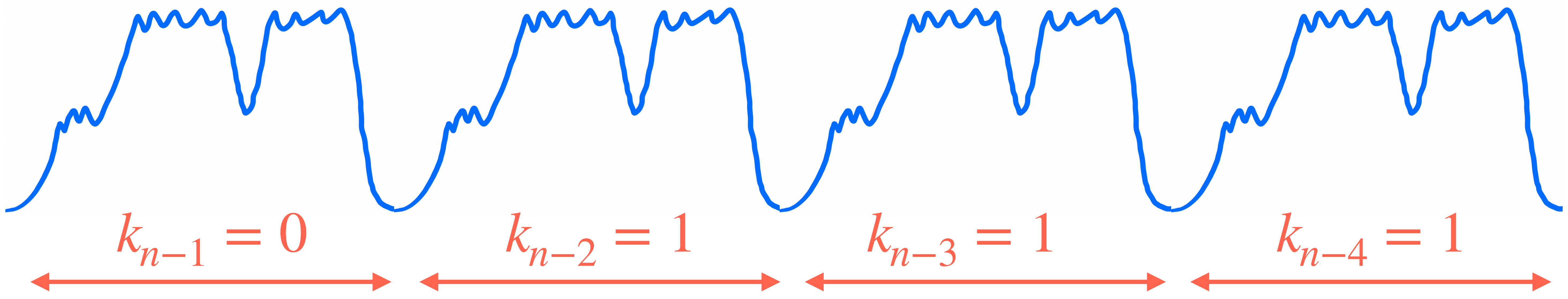

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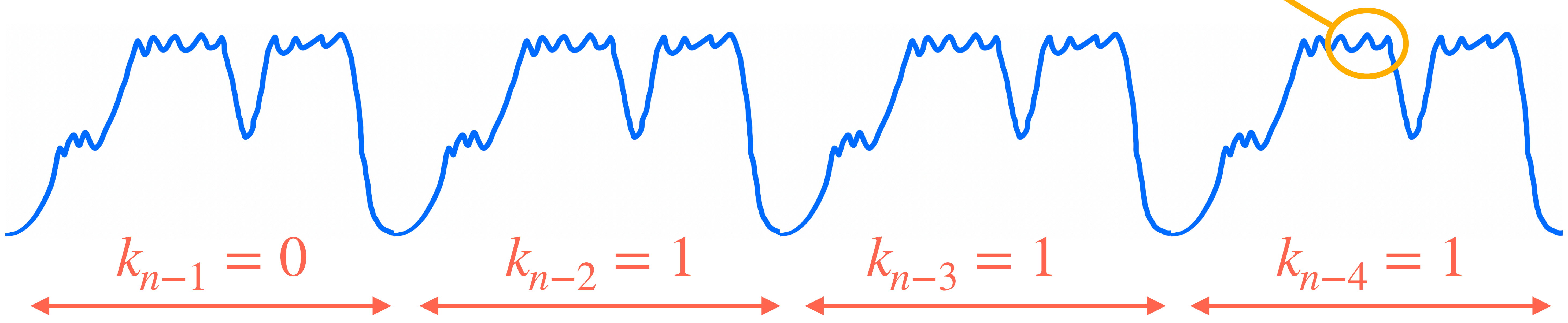
Leakage on $R_0 = [7]P$



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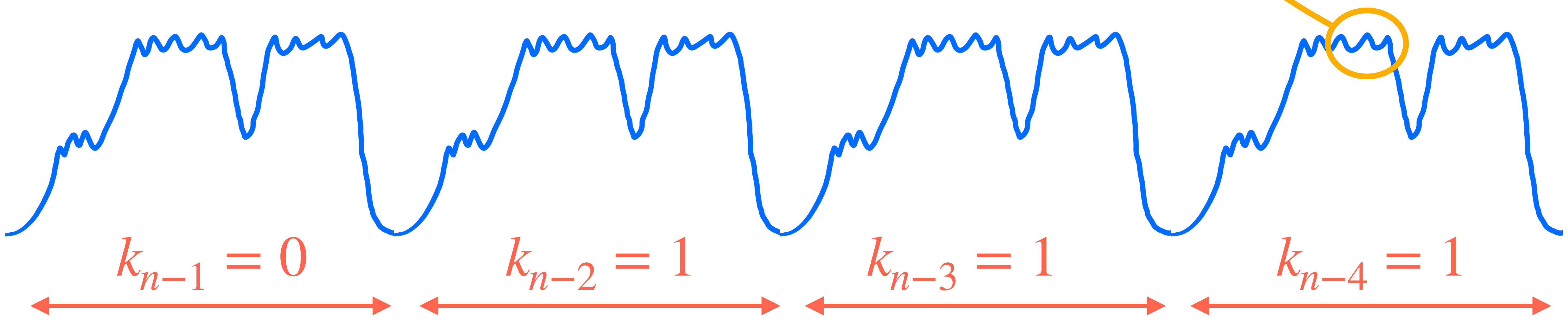

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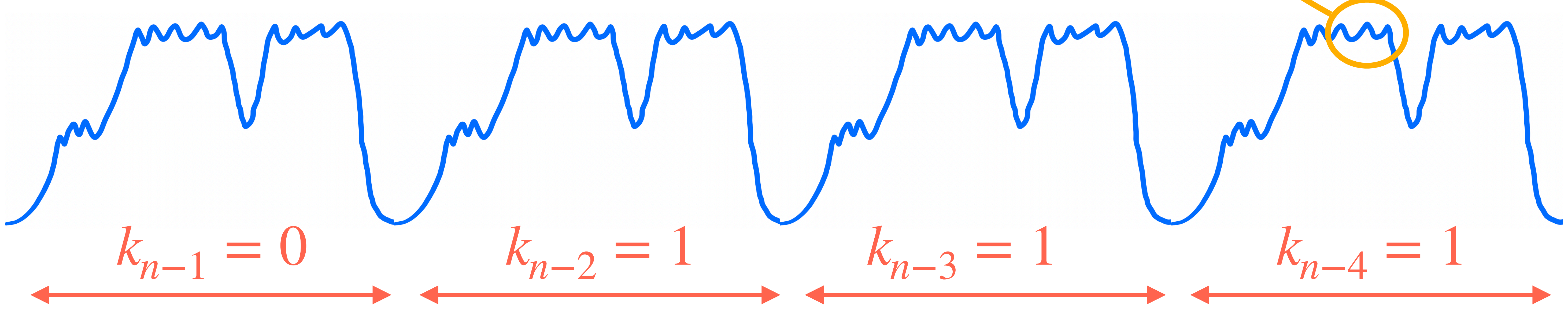

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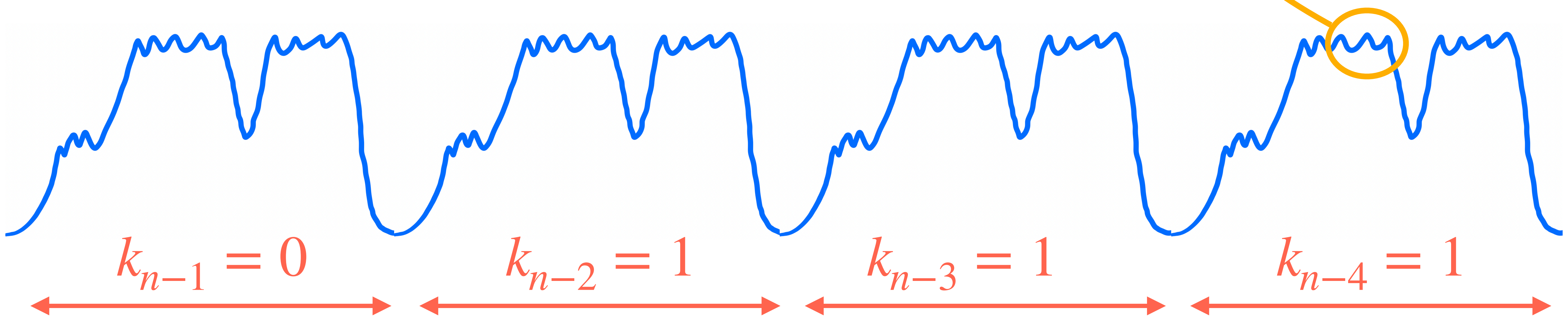

$k_{n-1} = 0$ $\quad k_{n-2} = 1$ $\quad k_{n-3} = 1$ $\quad k_{n-4} = 1$

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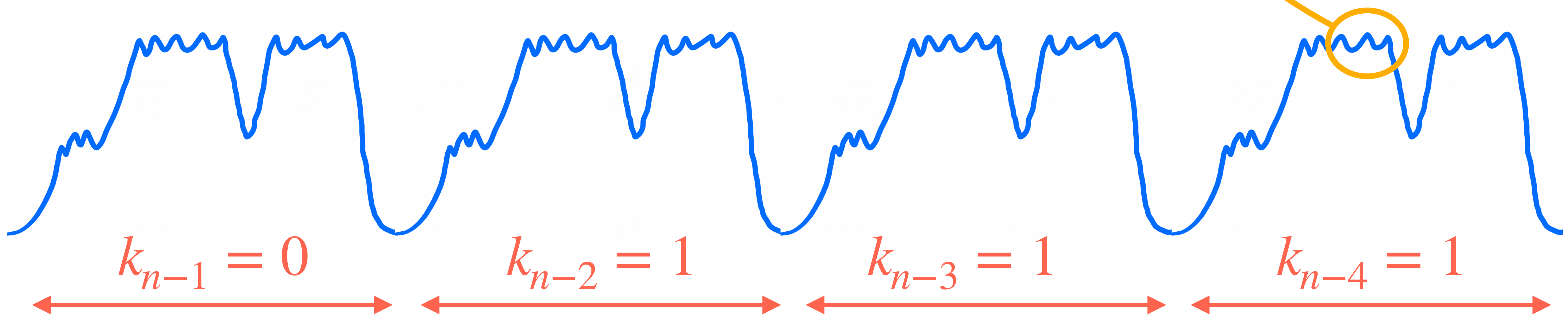

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💡 Solution: Randomizing algebraic variables

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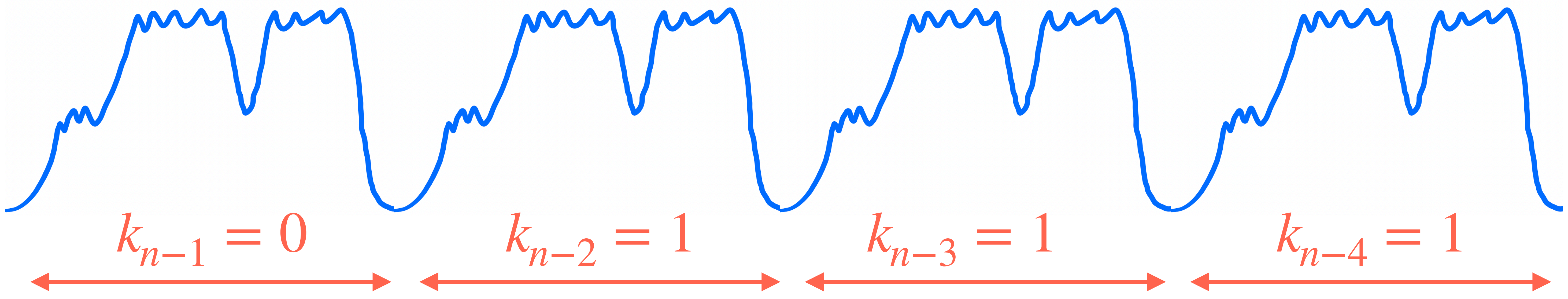

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$\boxed{R_0 \leftarrow \mathcal{O}}$ Initial randomization
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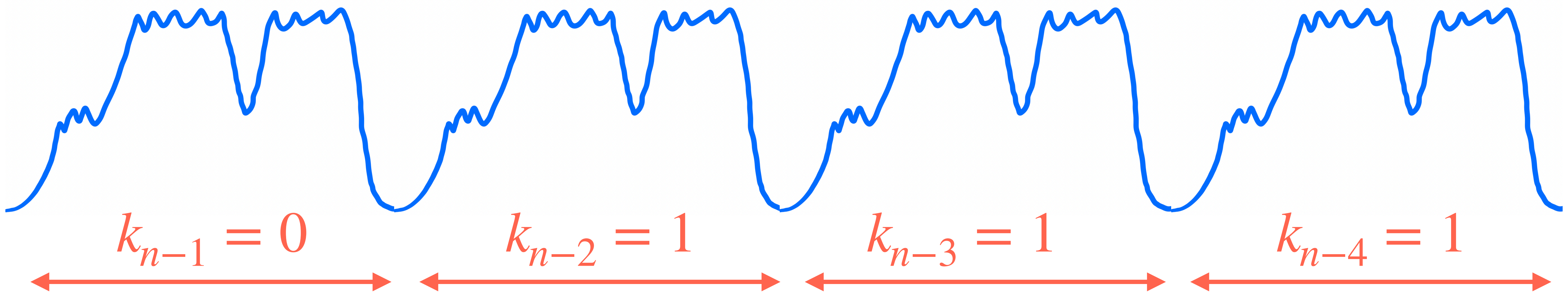

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Initial randomization

Propagation of
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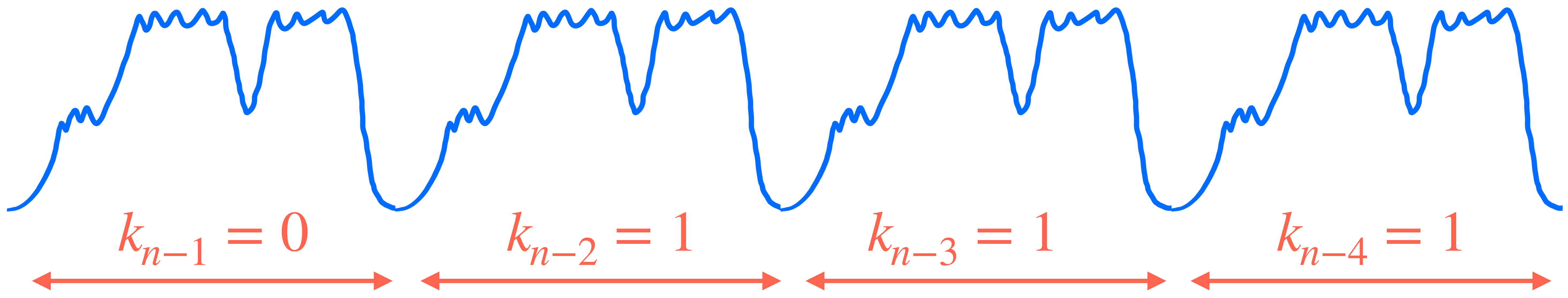
(or re-randomization)



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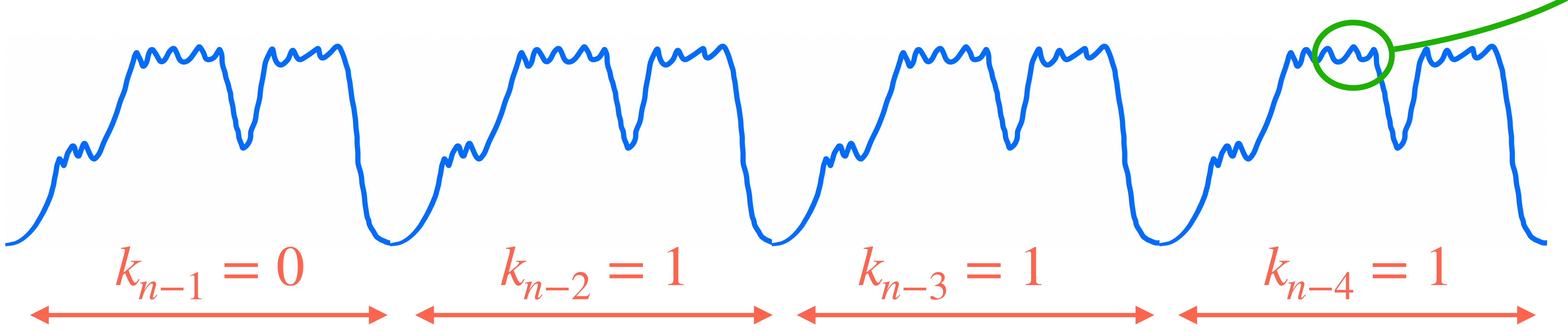

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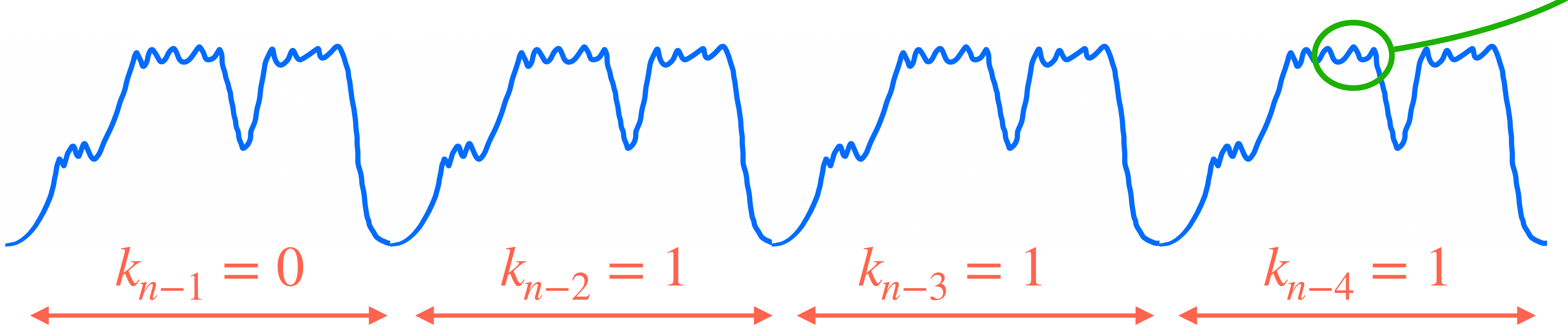
😇 No correlation anymore

Leakage on randomized R_0



Randomization techniques

- Randomization of the projective / Jacobian coordinates:
  - Point $P = (x, y)$ represented as $P \equiv (X : Y : Z)$ s.t. $x = X/Z$ and $y = Y/Z$
  - Random $r \leftarrow \mathbb{F}$, $\begin{cases} X' := r \cdot X \\ Y' := r \cdot Y \\ Z' := r \cdot Z \end{cases} \implies (X' : Y' : Z') \equiv P$

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💡 Intuition: hard to break with common SC leakage

Back to Montgomery ladder

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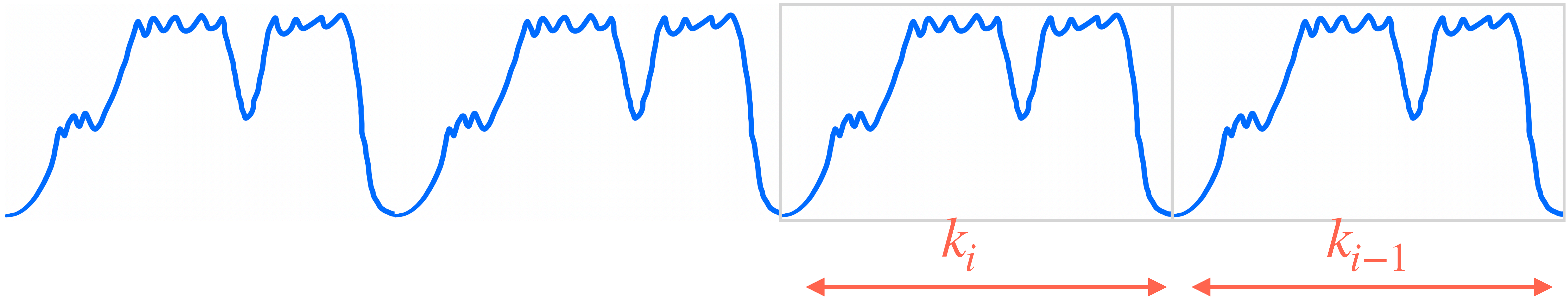

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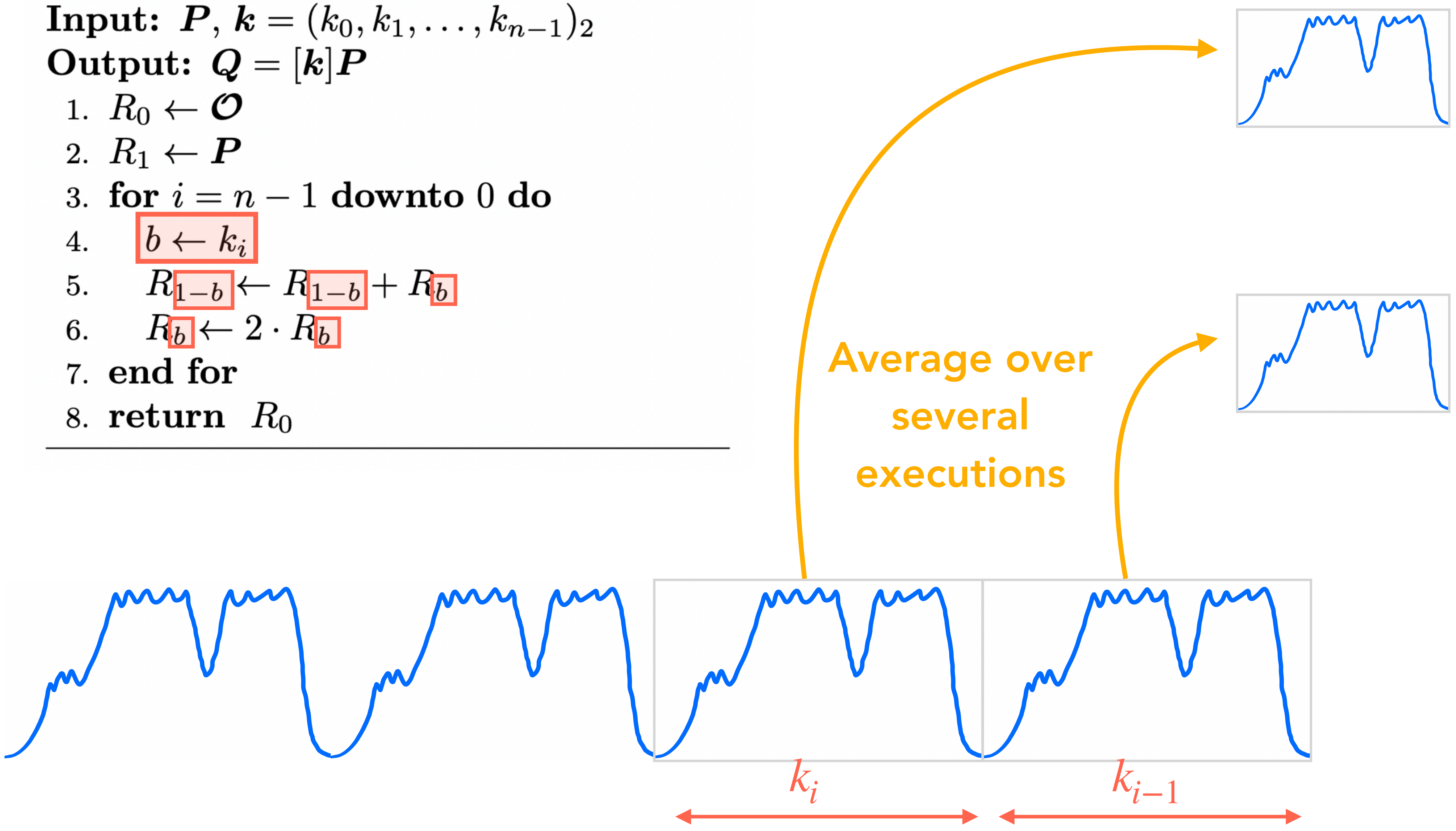

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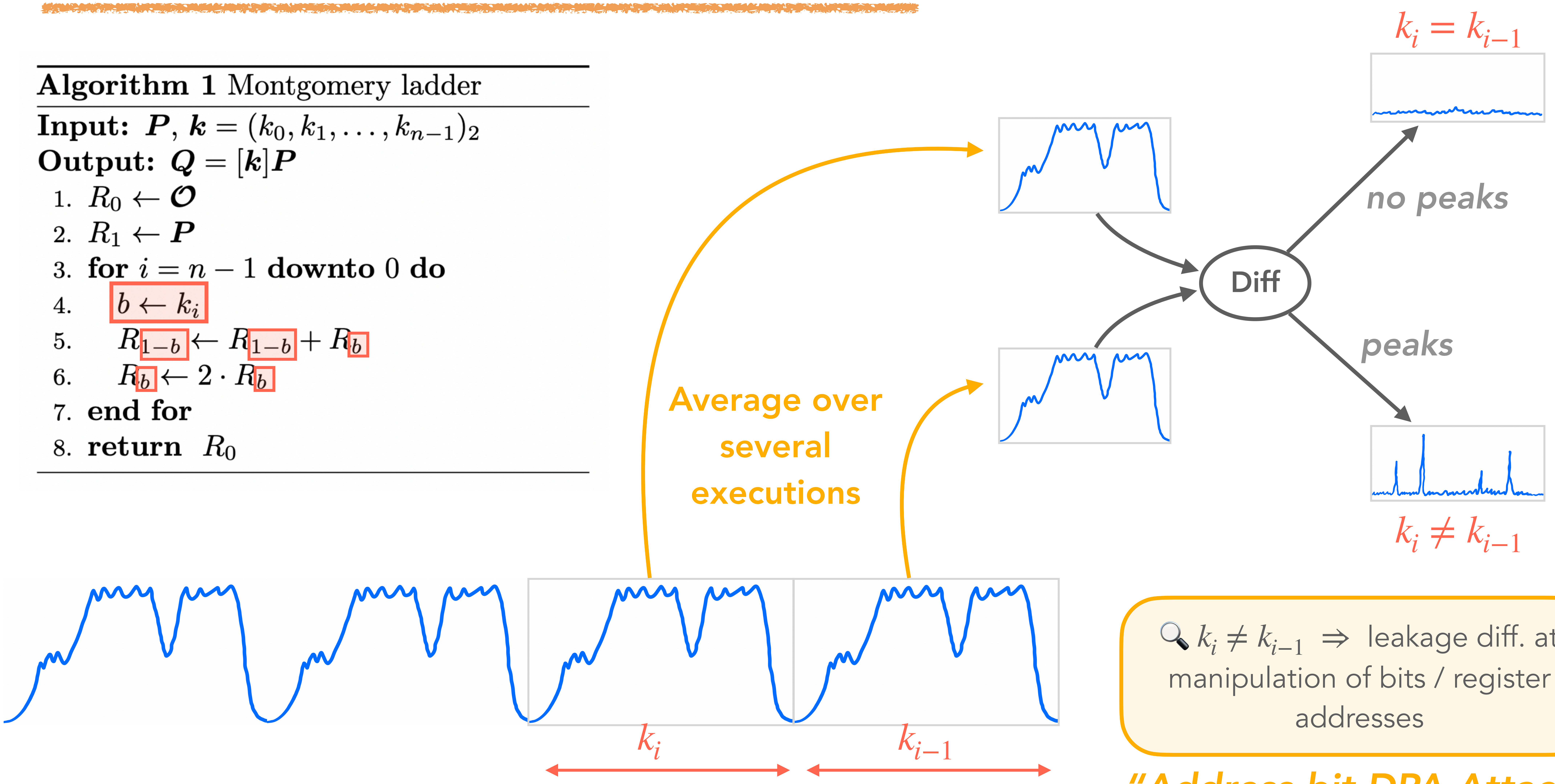

🔍 $k_i \neq k_{i-1} \Rightarrow$ leakage diff. at manipulation of bits / register addresses

"Address-bit DPA Attack"

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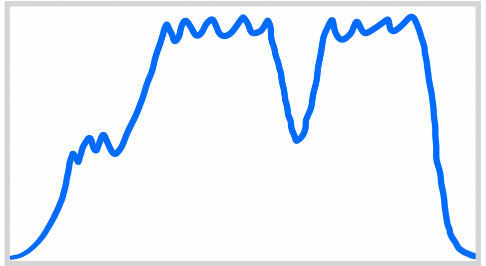
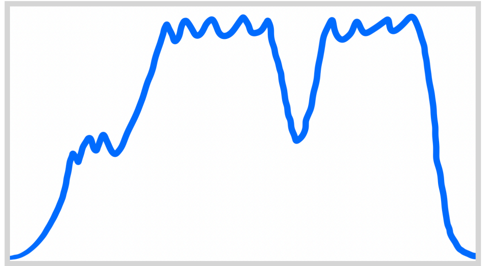
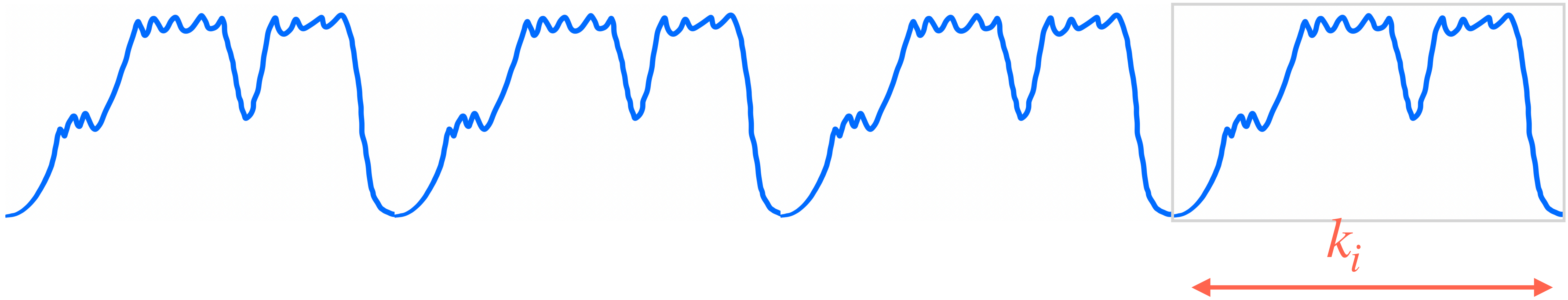
Precomputed template for $k_i = 0$



Precomputed template for $k_i = 1$





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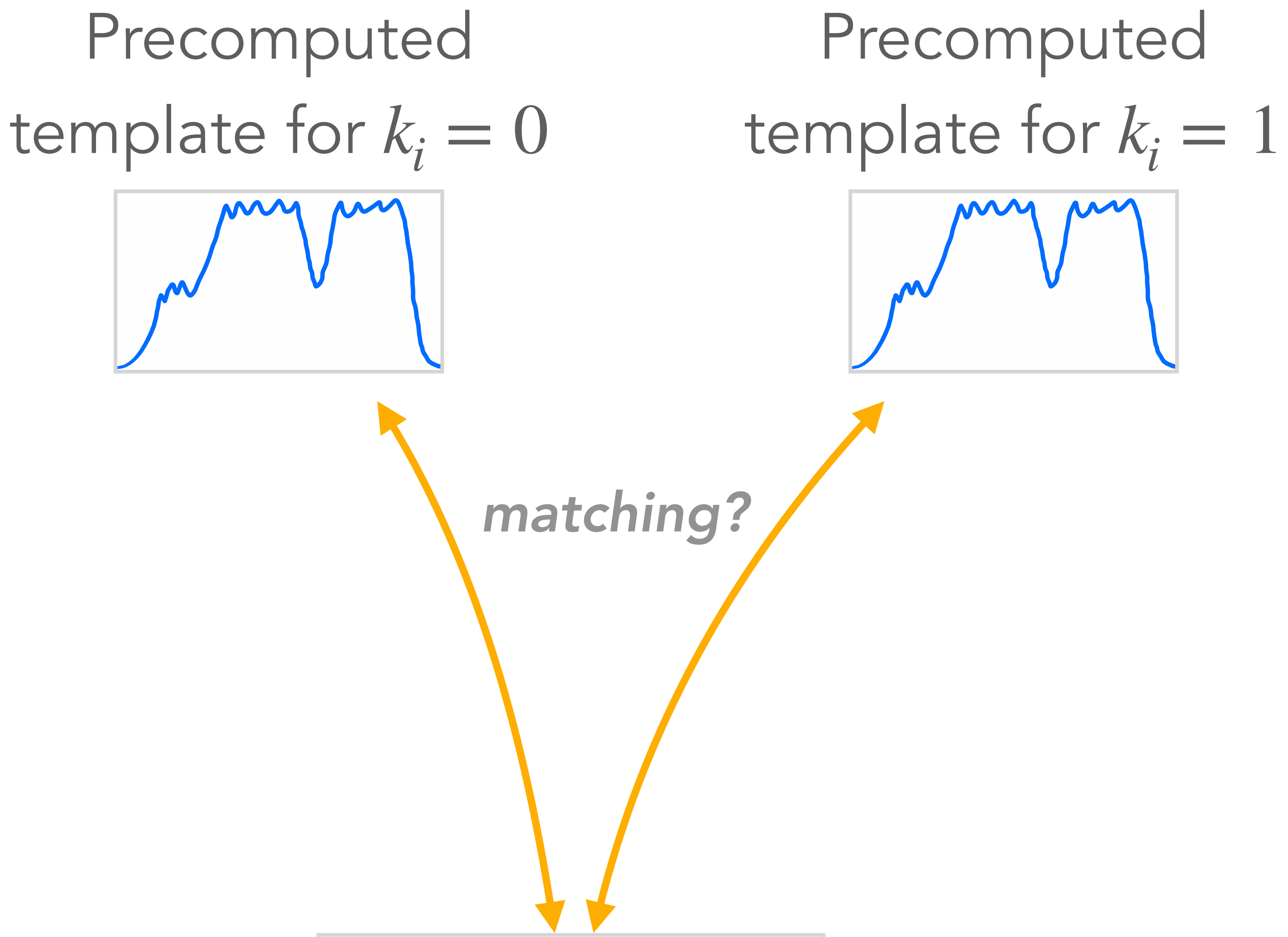

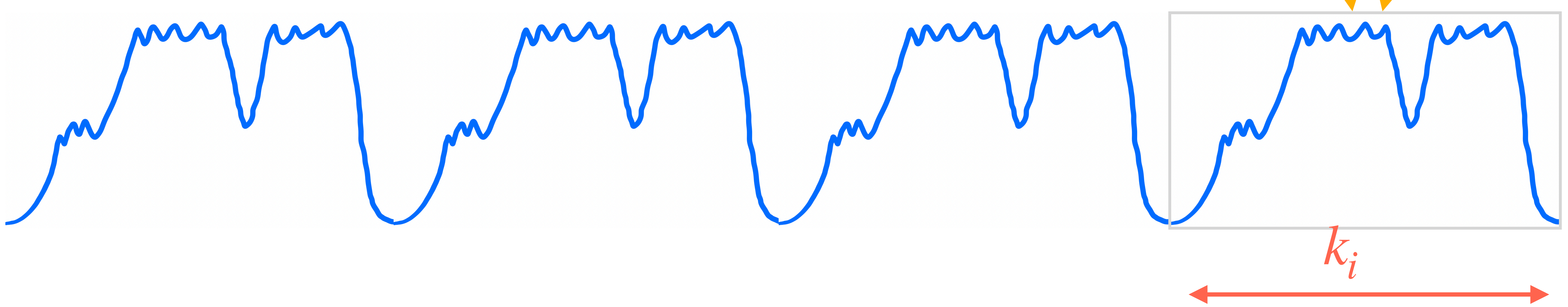

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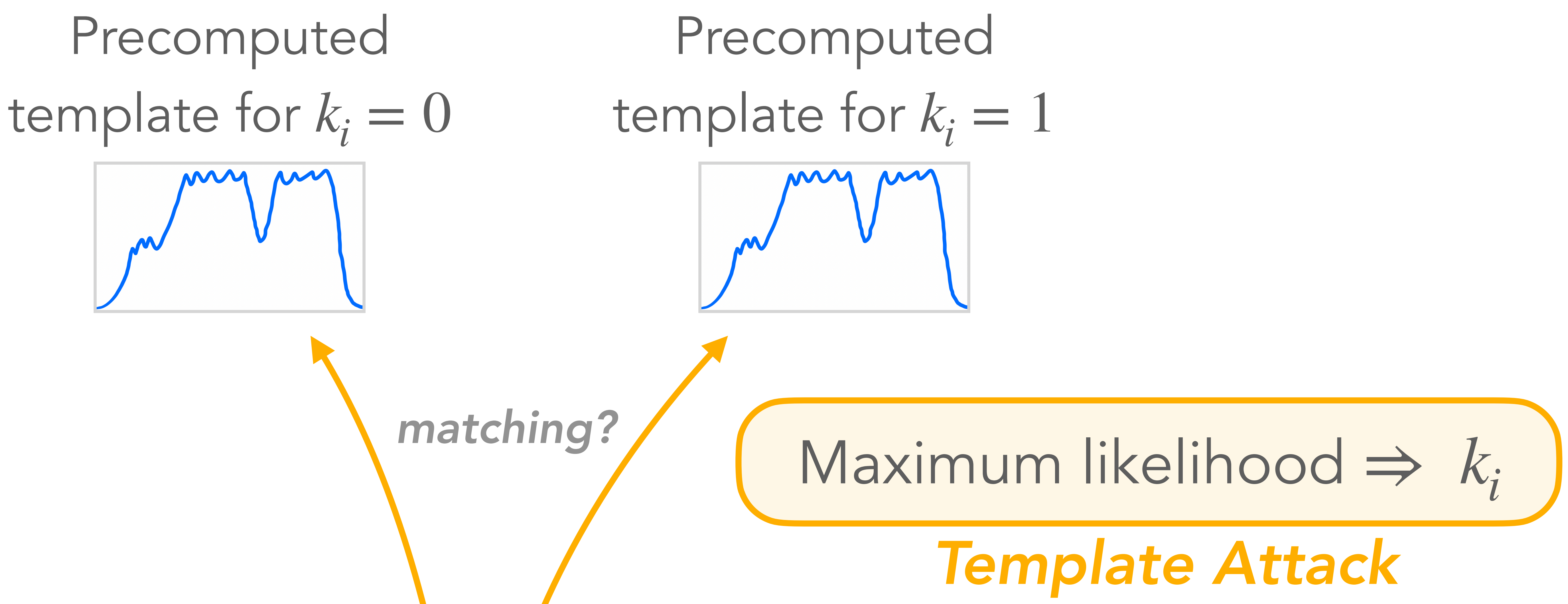

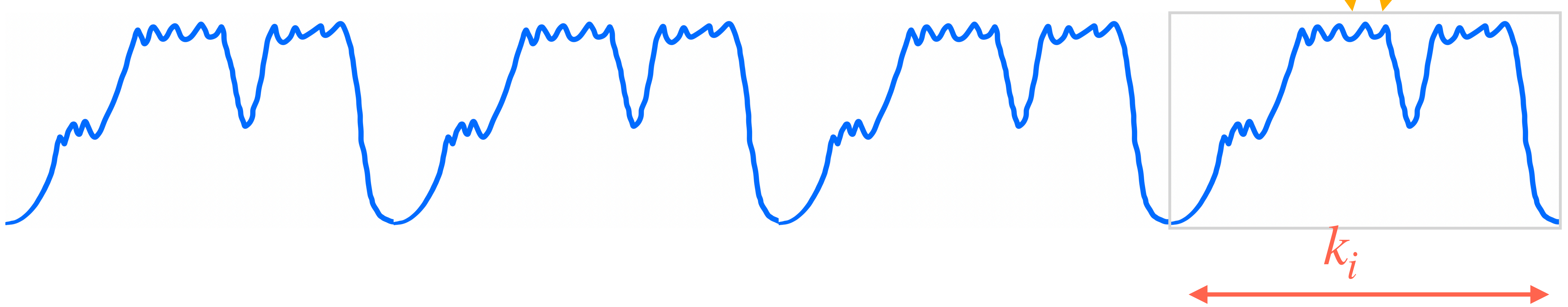

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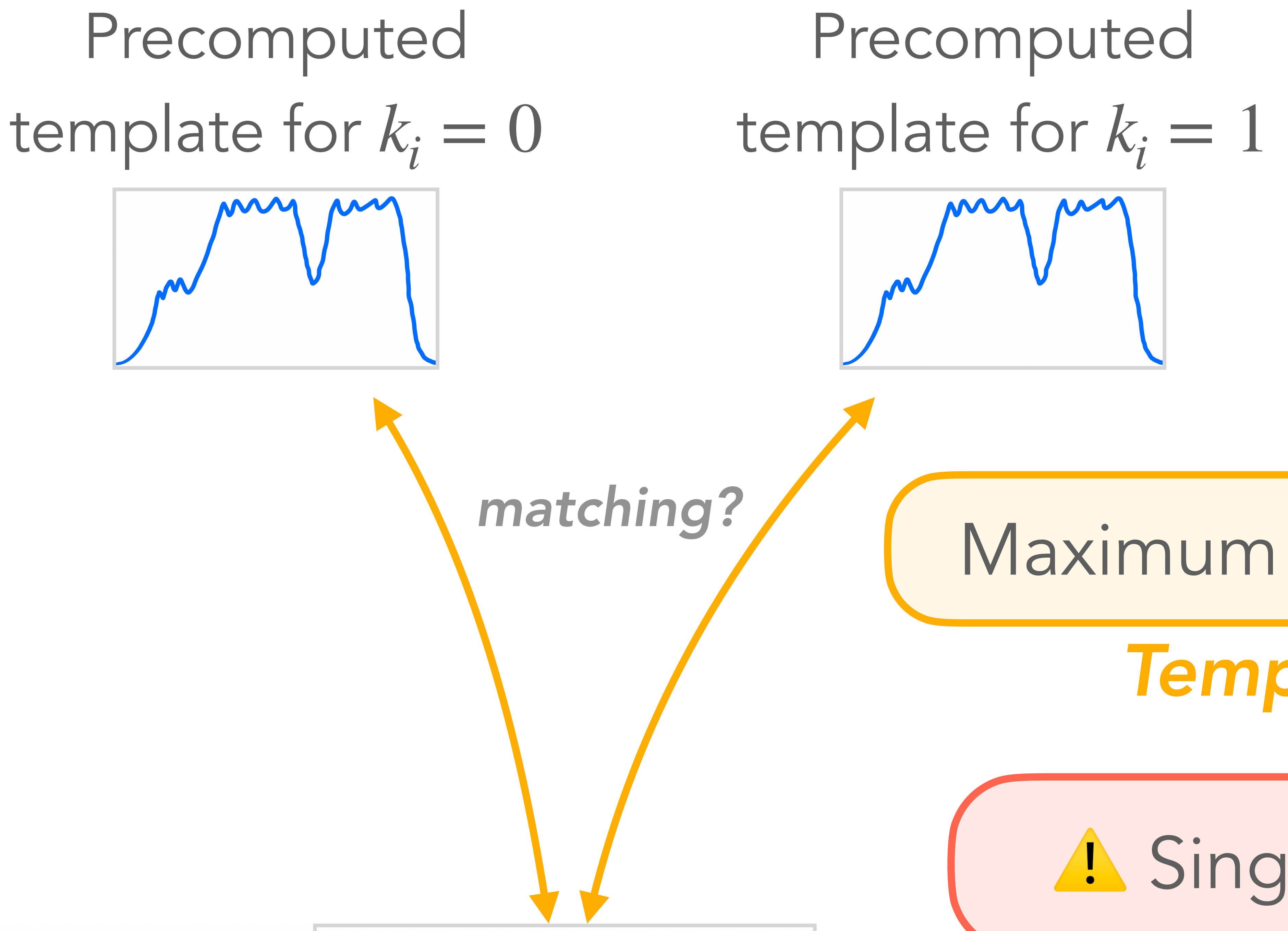

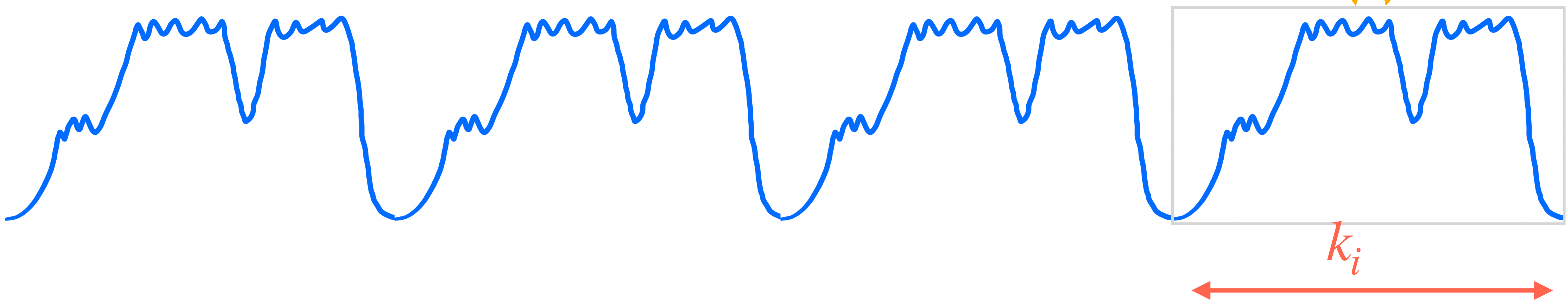

Maximum likelihood $\Rightarrow$ k_i

Template Attack

⚠️ Single trace attack

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💡 Solution: Randomizing the scalar

Scalar randomization

- Scalar blinding:

$$k' \leftarrow k + r \cdot |E(\mathbb{F}_p)| \quad \Longrightarrow \quad [k]P = [k']P$$

with $|E(\mathbb{F}_p)|$ the order of the EC

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⚠️ Still vulnerable to single trace attack

Scalar randomization

- <u>Boolean masking:</u>

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$R_0 \leftarrow \boldsymbol{\mathcal{O}}$
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for $i = n - 1$ **downto** 0 **do**
    $b \leftarrow k_i$
    $R_{1-b} \leftarrow R_{1-b} + R_b$
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end for
return R_0

Scalar randomization

- Boolean masking:

Algorithm 1 Montgomery ladder

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CSwap(T_0, T_1, b):

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- Masking the scalar $(b^0, b^1) := (b \oplus r, r)$ for random bit $r \leftarrow \{0,1\}$
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What can go wrong now?!

😼 2nd-order attack on masked scalar bits

$$\text{Leakage}(b^0) + \text{Leakage}(b^1) \text{ depends on } b$$

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😈 But 2nd order "collision" leakage remains

😇 High order security
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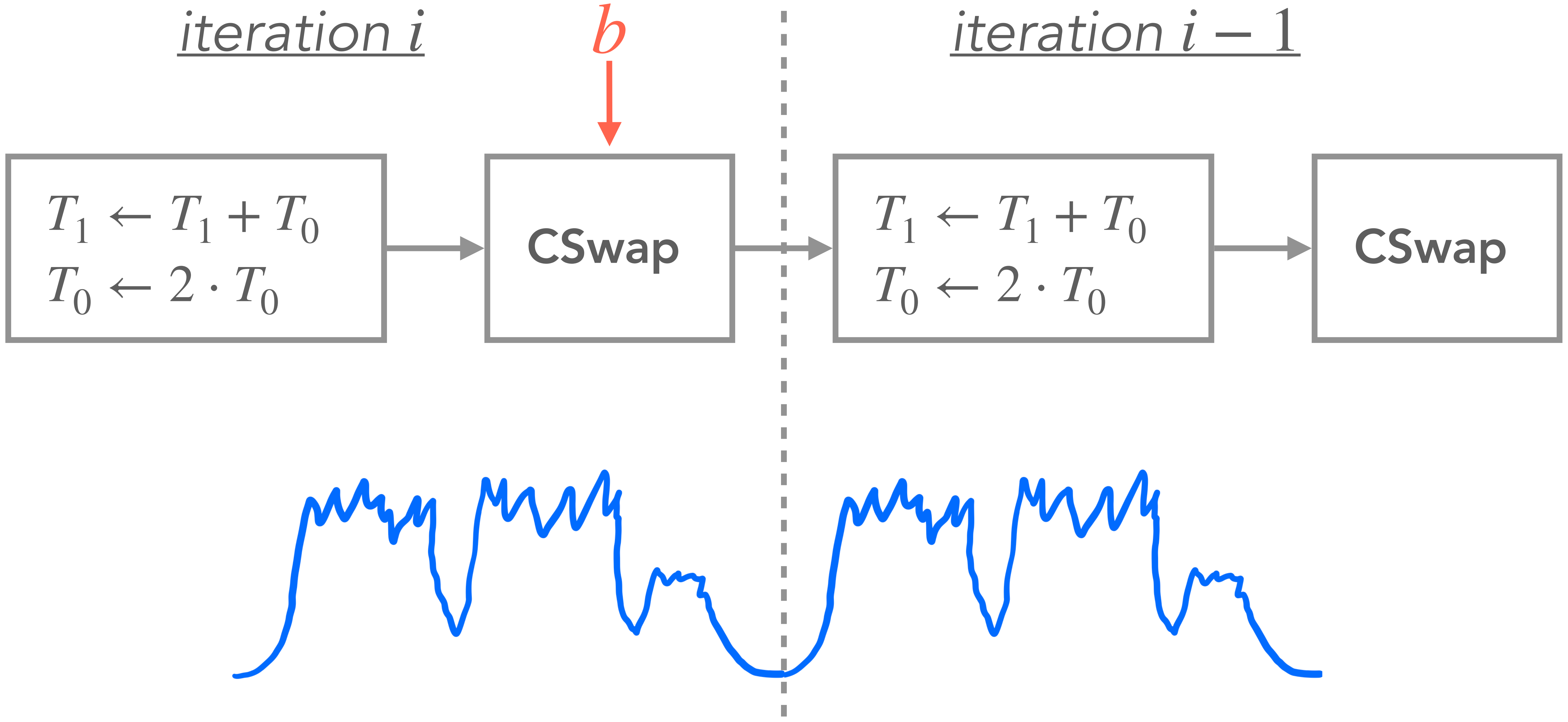
Collision attacks



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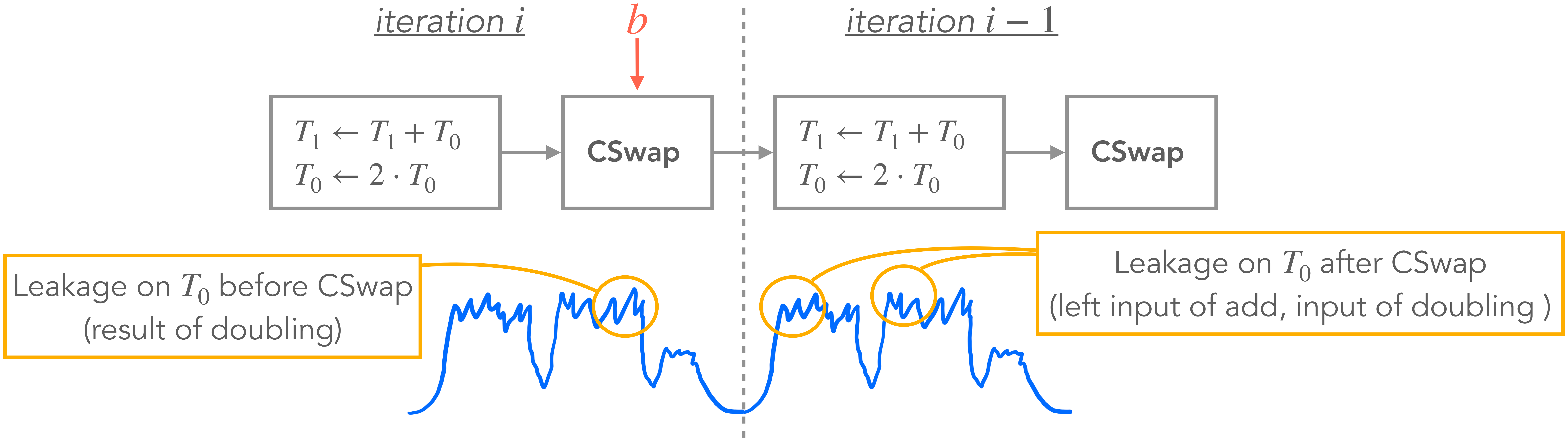

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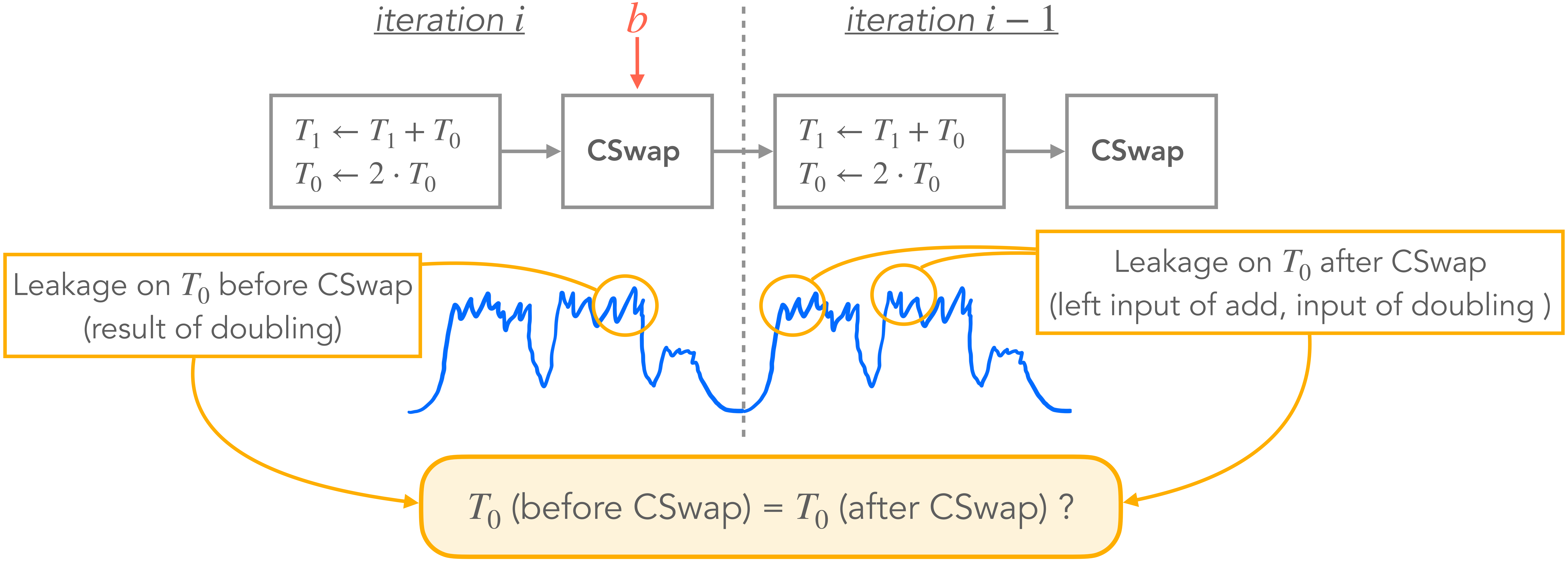

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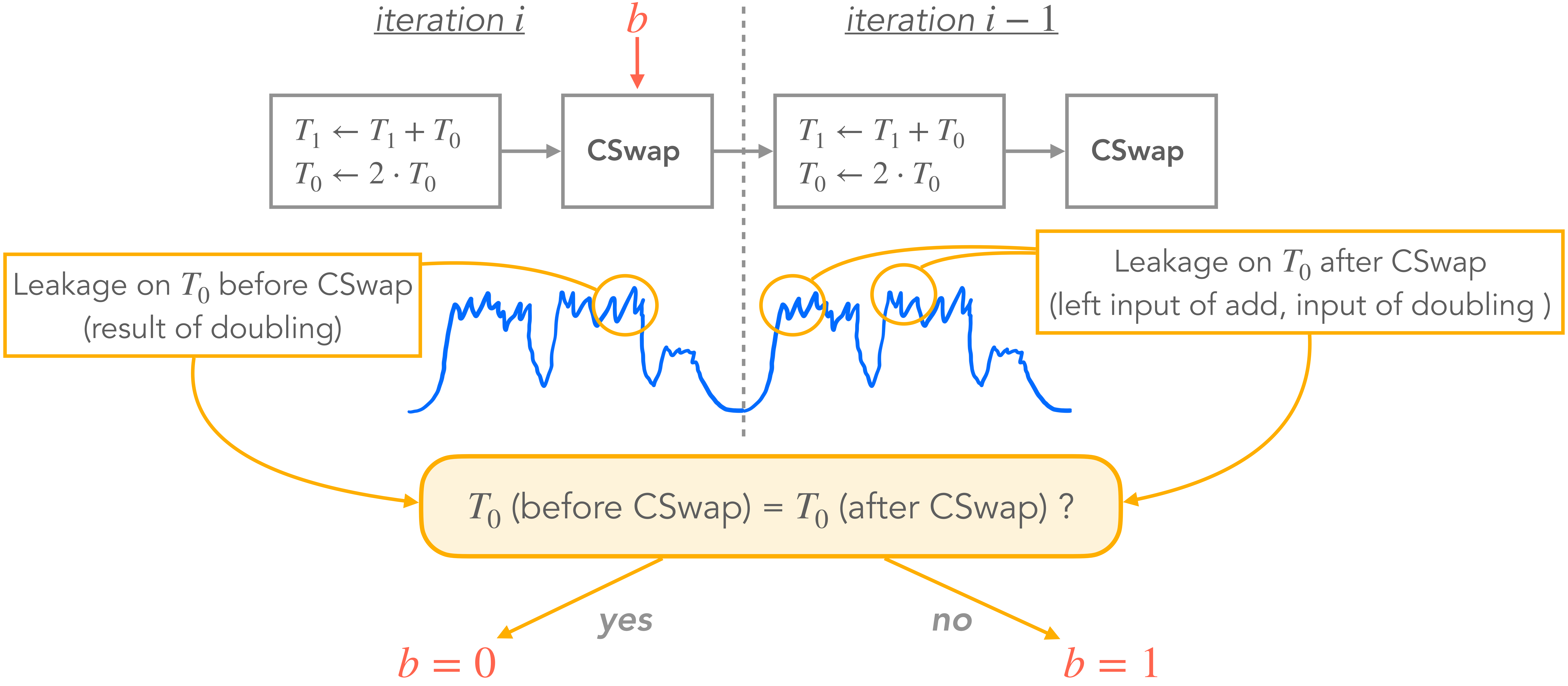

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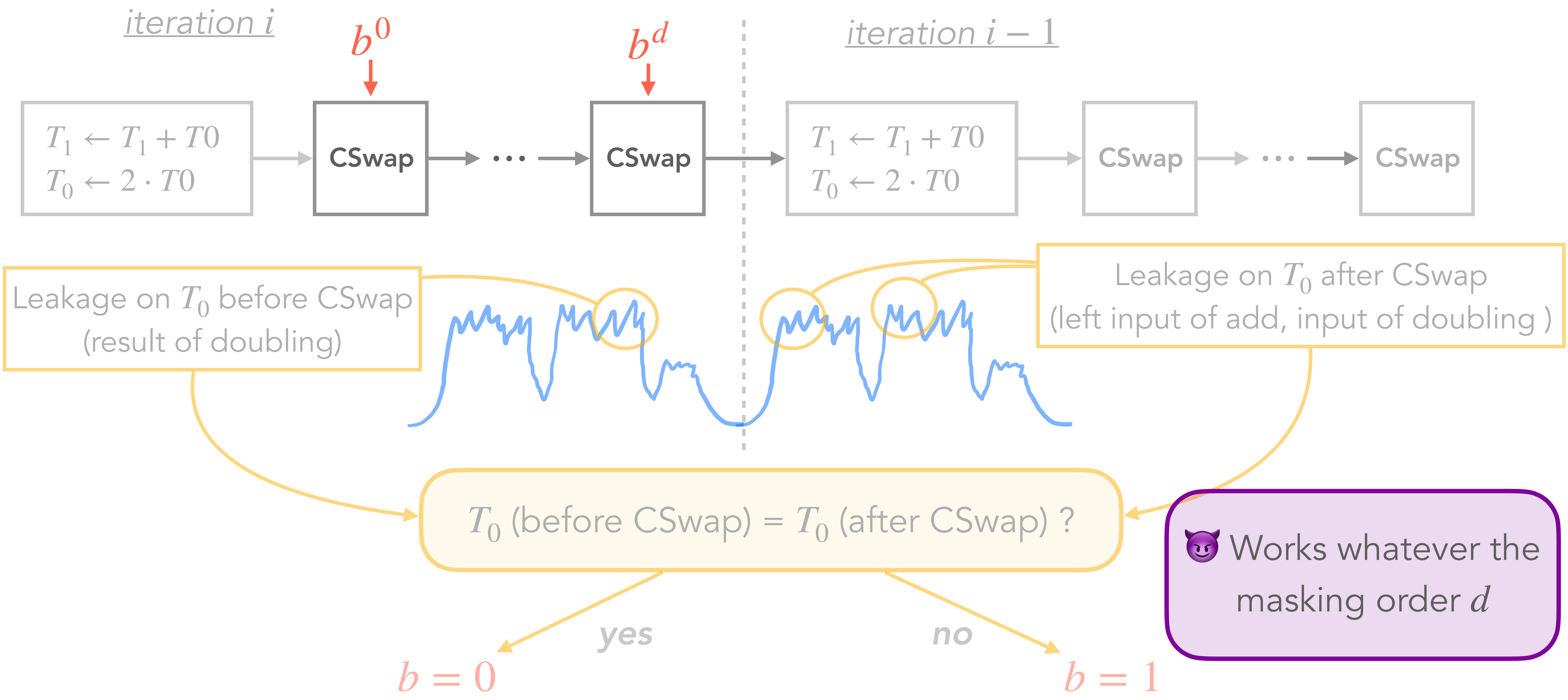
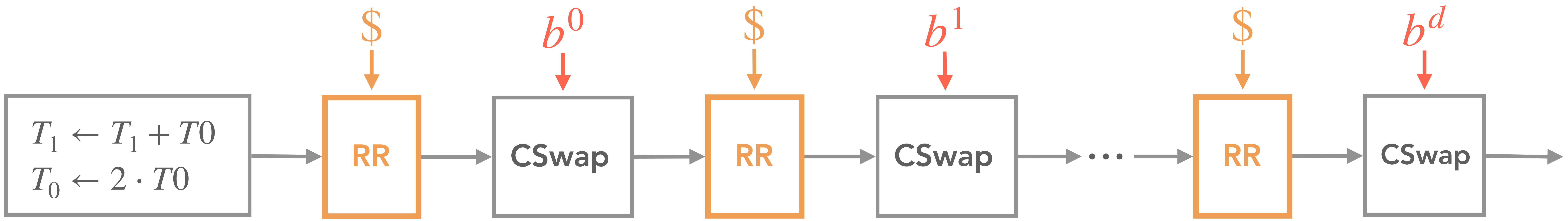


Our solution



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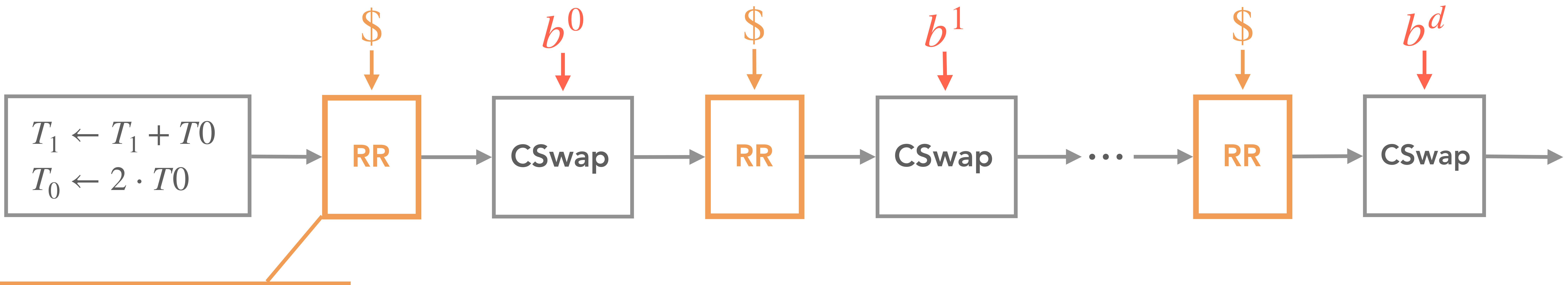

Rerandomize:

$$T_0 \leftarrow \mathsf{R}(T_0)$$
$$T_1 \leftarrow \mathsf{R}(T_1)$$

(for some randomization R)

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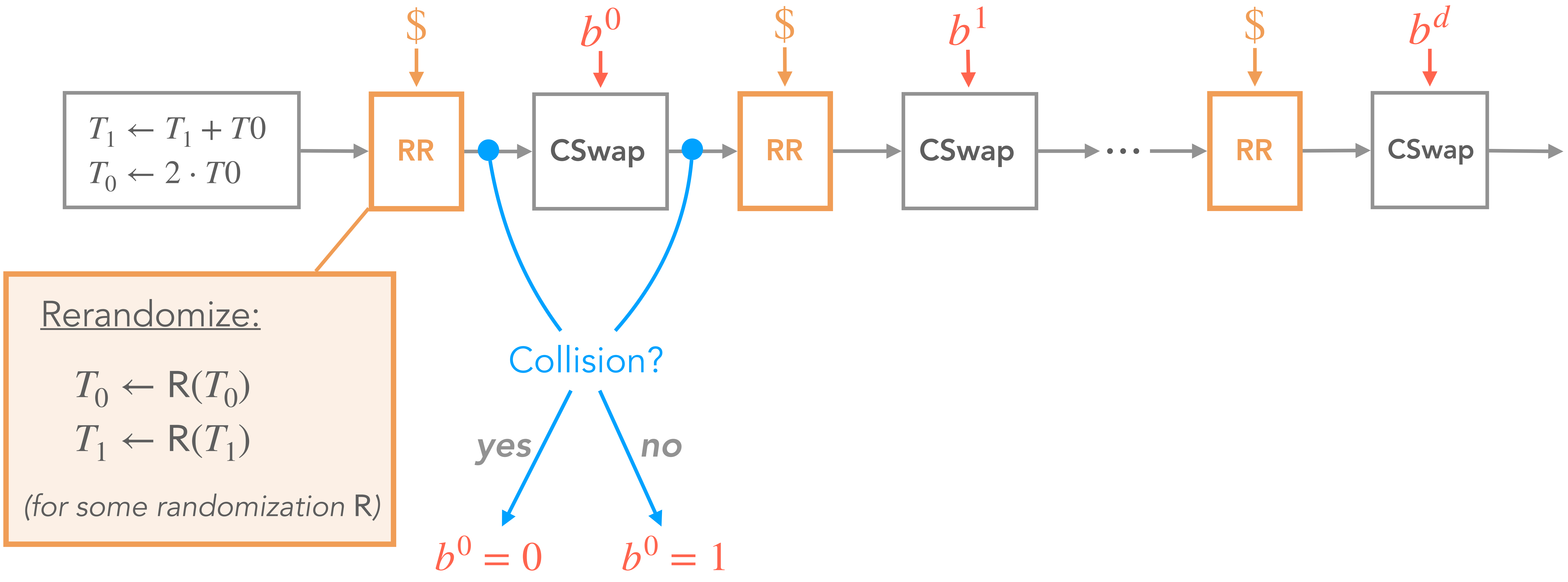

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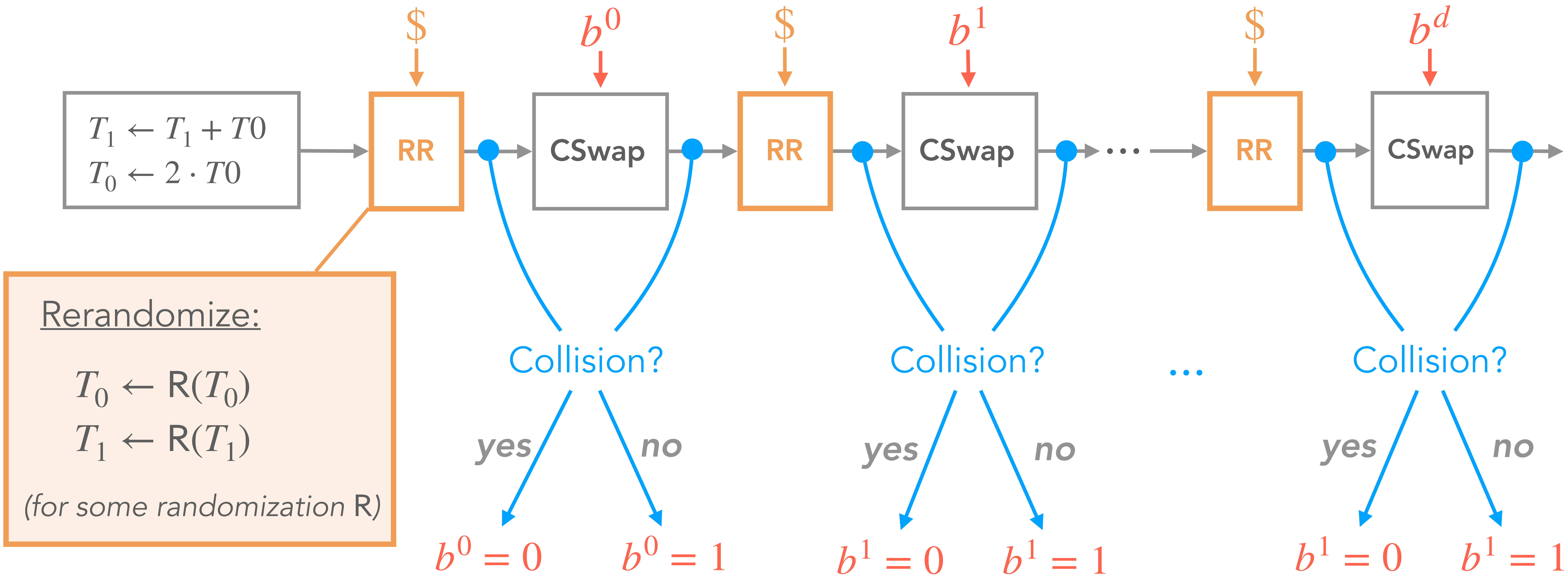

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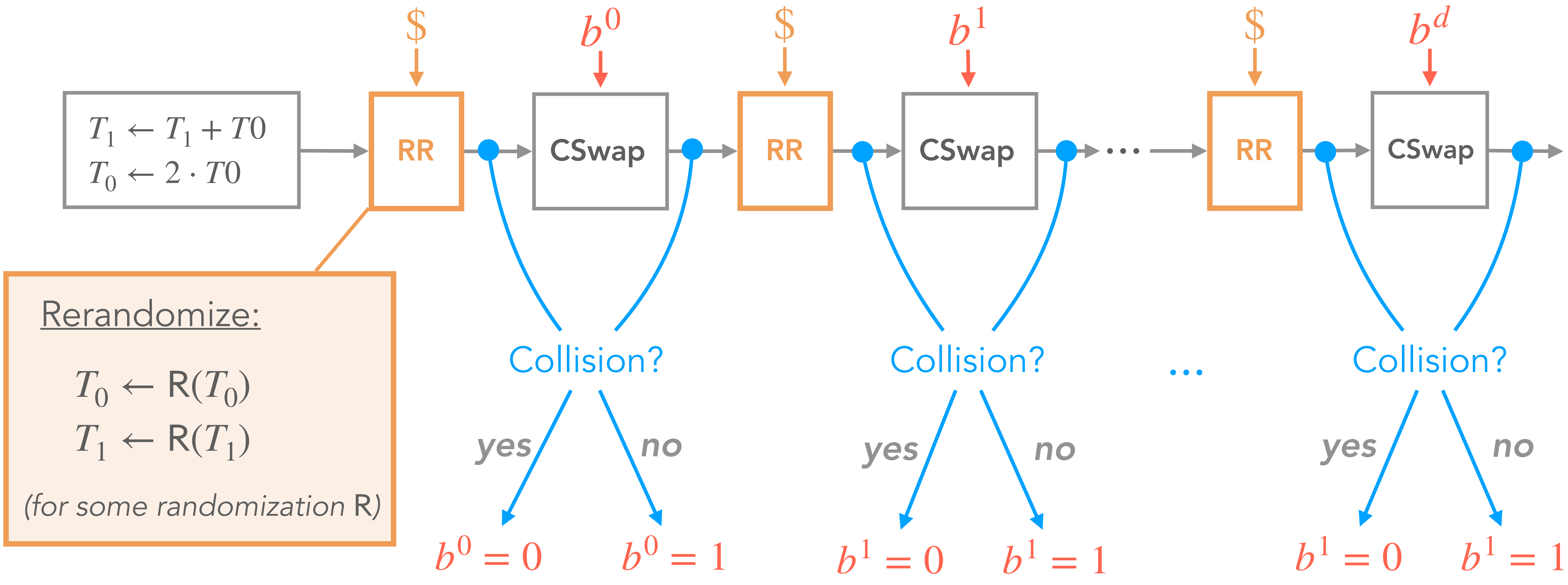

😈 Requires collision attack of order $d + 1$

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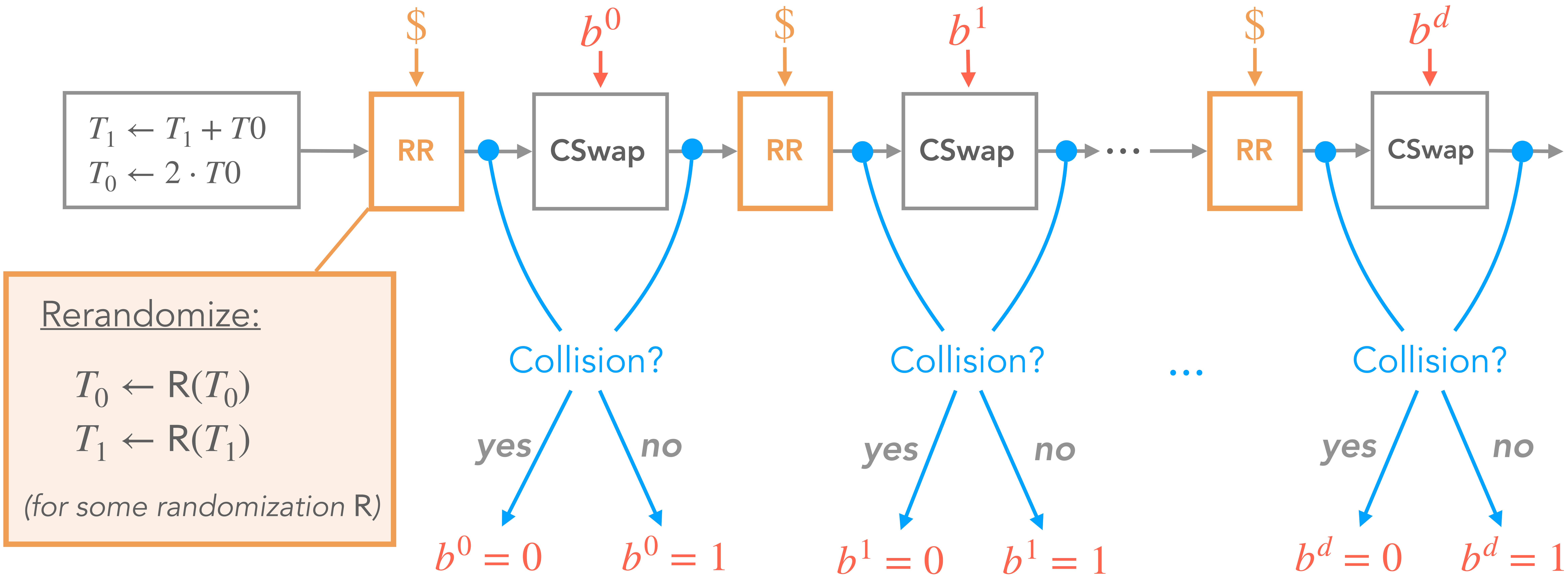

👿 Requires collision attack of order $d + 1$

🤔 How to formally prove this high order security?

Formal model

- Computation model: "Randomized Regular Algebraic Program" (RRAP)
  - Two types of variables
    - Algebraic variables $X_1, \ldots X_{\ell_X} \in \mathbb{A}$
    - Index variables $k_1, \ldots, k_{\ell_k} \in \mathbb{Z}$
  - Three types of operations
    - $k_{i_1} \leftarrow \mathsf{op}(k_{i_2}, k_{i_3})$
    - $X_{j_1} \leftarrow \mathsf{Op}(X_{j_2}, X_{j_3})$
    - $X_{j_1} \leftarrow \mathsf{R}(X_{j_1})$

    with $j_1, j_2, j_3 \in \{k_1, \ldots k_{\ell_k}\} \cup \{1, \ldots, \ell_X\}$
- Capture regular algorithms for ECC / RSA / pairings

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    → *Leaks* $f(k_{i_2}, k_{i_3})$

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    with f a *δ-noisy leakage function*:

$$\mathrm{SD}\big(U; (U \mid f(U)\big) \leq \delta$$

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  - Hiddenness assumption

💡 Capture that $x \mapsto f \circ \mathsf{R}(x)$ hides the information on x

Hiddenness assumption

📜 **Hiddenness assumption**
(simple version)

- Let f a (noisy) leakage function
- Let $\mathsf{R} : \mathbb{A} \rightarrow \mathbb{A}$ a rand. operation
- The pair (f, R) is ε-*hiding* if

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KL divergence between $f(\mathsf{R}(x))$ and $f(U)$
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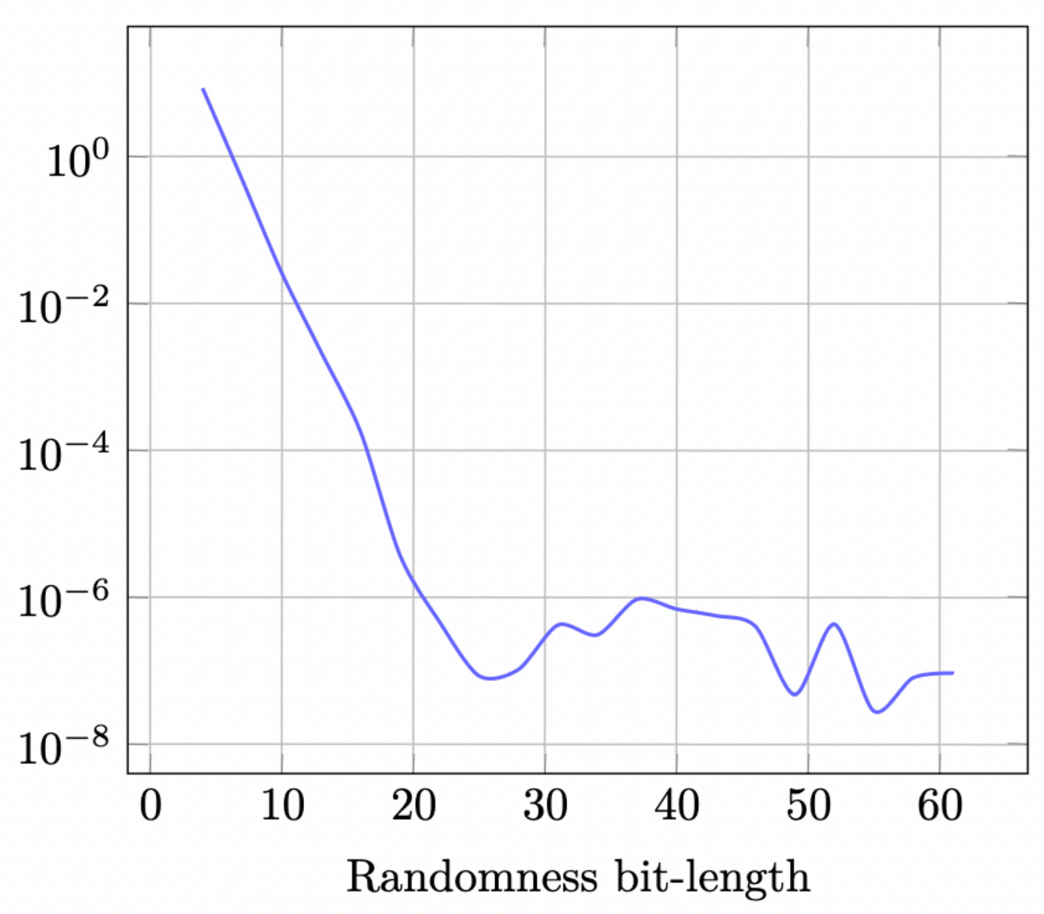
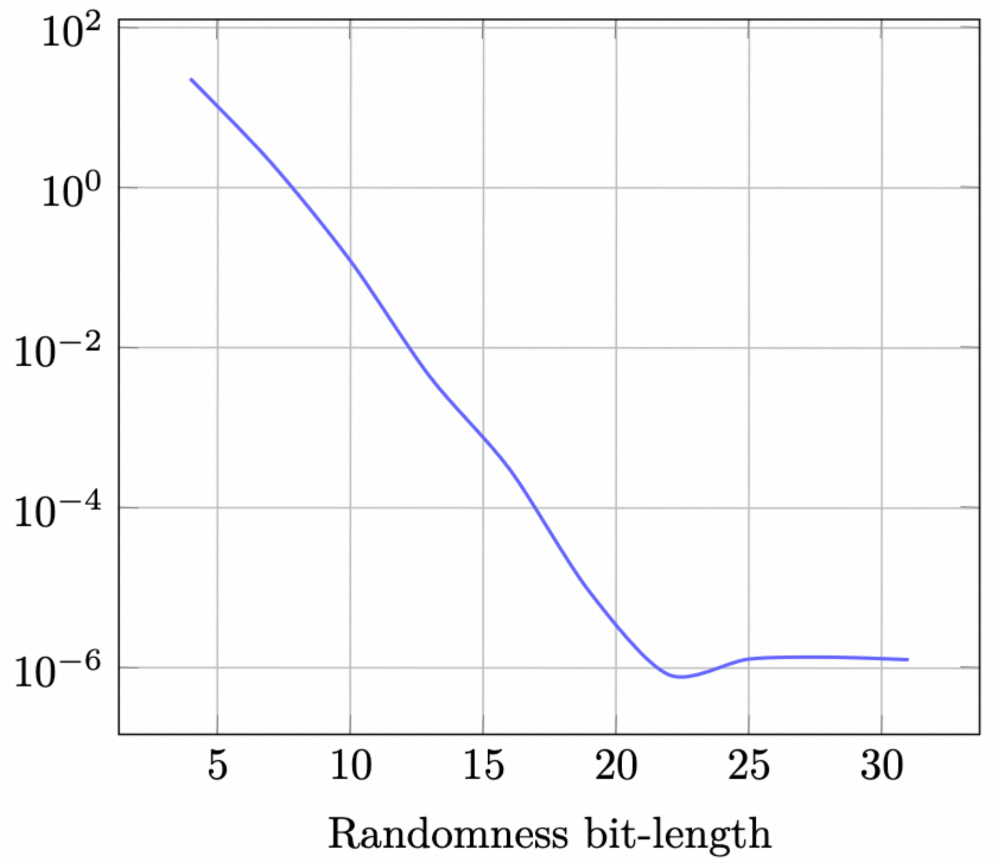


R = field element randomization



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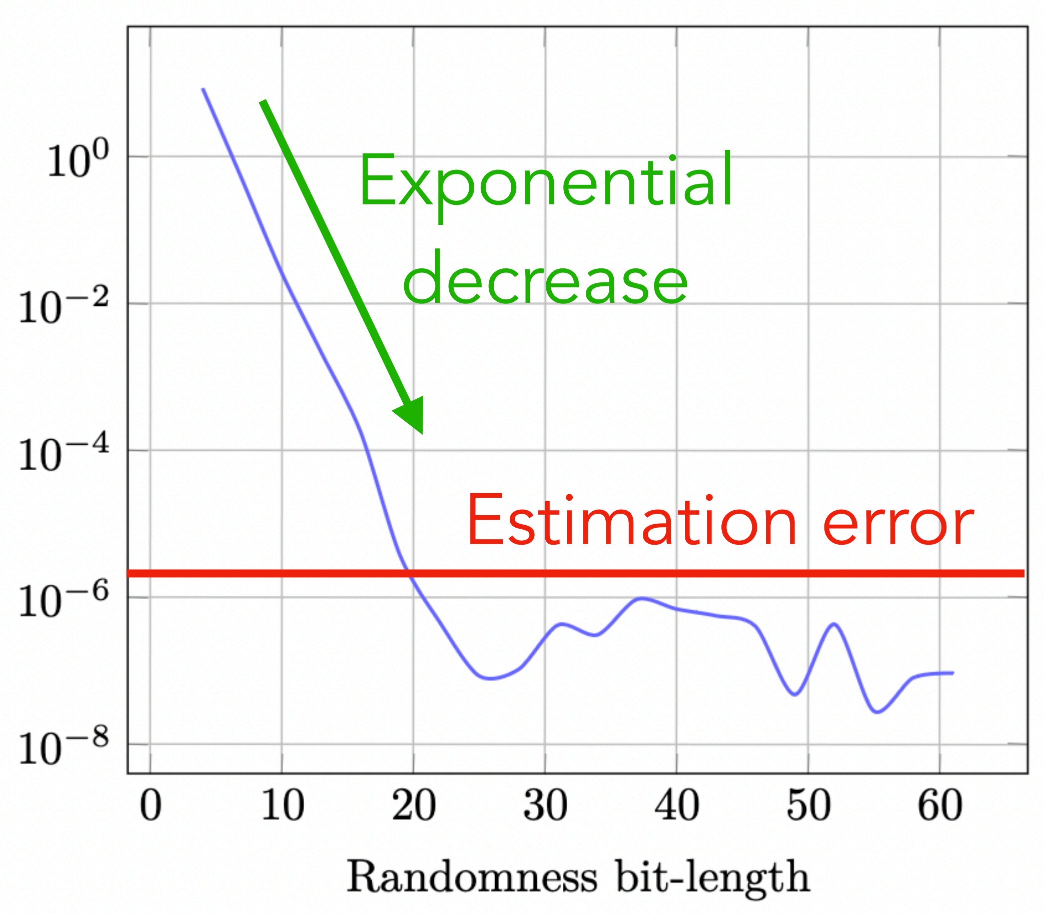

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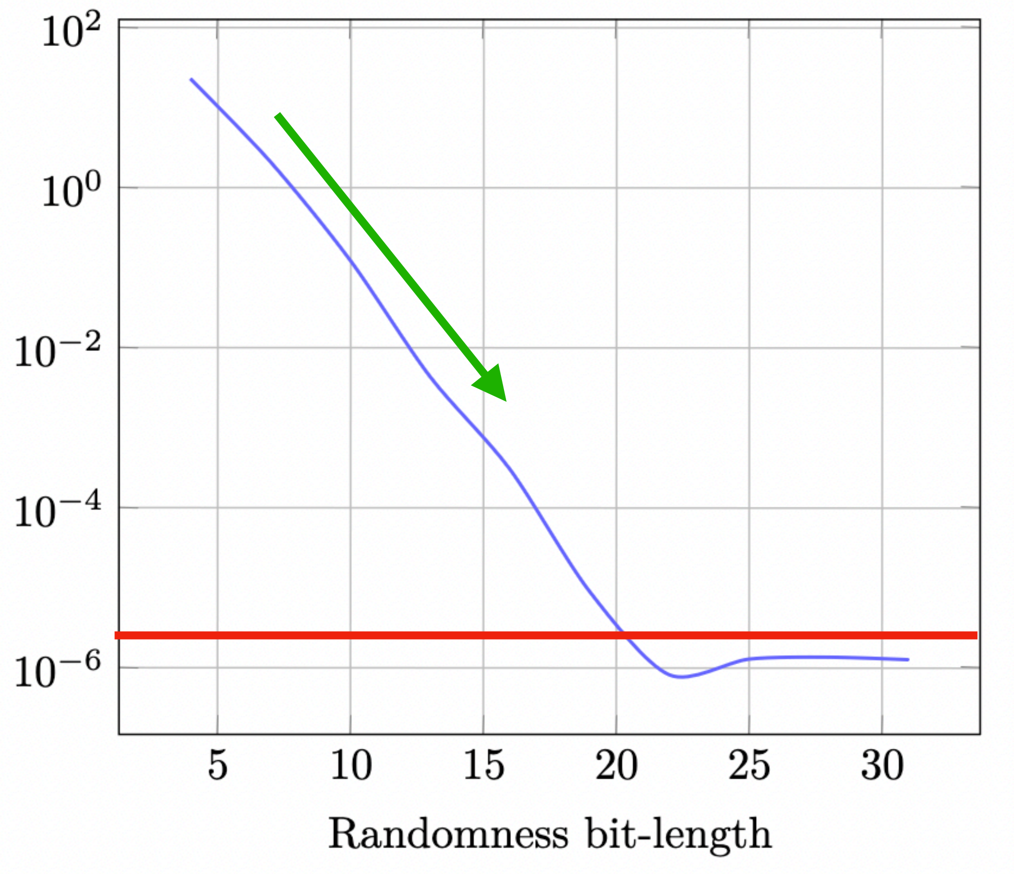

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Proof sketch:

1. Apply ε-hiddenness to replace re-randomized variables by new uniform variables

   $\rightarrow cst_2 \cdot \varepsilon$ gap

2. Replace noisy leakage by random probing leakage

   $\rightarrow$ no gap

   $\rightarrow (cst_1 \cdot \delta)^{d+1}$ probability of simulation failure

Application

- Generic algorithm applicable to any RRAP
- Several ECC scalar mult. algorithms expressed in our framework:
  - Montgomery ladder (point level & coordinate level)
  - Joye ladder
  - Signed binary ladder
  - Fixed-window scalar multiplication
- PoC smart card implementation
  - (signed binary ladder with XY-only co-Z coordinates)

Performance estimations

	order 1	order 2	order 4	order 8
Our countermeasure (overhead)				
R_1 - $h = 32$	1,35	1,39	1,47	1,64
R_1 - $h = 64$	1,73	1,81	1,98	2,31
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* Assume 12 multiplications per loop iteration

** Neglect add / sub vs. multiplications

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Jacobian coordinate randomization

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R_1 - $h = 128$	2,58	2,75	3,08	3,75
R_2	3	4	6	10
R_1 & R_2 - $h = 32$	5,48	7,59	11,81	20,25
R_1 & R_2 - $h = 64$	6,77	9,38	14,58	25
R_1 & R_2 - $h = 128$	9,75	13,5	21	36
Other countermeasures (overhead)				
scalar splitting	2	3	5	9
naive ISW	4	9	25	81

Field element randomization

Jacobian coordinate randomization

Double randomization

* Assume 12 multiplications per loop iteration

** Neglect add / sub vs. multiplications

Performance estimations

	order 1	order 2	order 4	order 8
Our countermeasure (overhead)				
R_1 - $h = 32$	1,35	1,39	1,47	1,64
R_1 - $h = 64$	1,73	1,81	1,98	2,31
R_1 - $h = 128$	2,58	2,75	3,08	3,75
R_2	3	4	6	10
R_1 & R_2 - $h = 32$	5,48	7,59	11,81	20,25
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Other countermeasures (overhead)				
scalar splitting	2	3	5	9
naive ISW	4	9	25	81

Field element randomization

Jacobian coordinate randomization

Double randomization

$[k]P = [k_0]P + \cdots + [k_d]P$

* Assume 12 multiplications per loop iteration

** Neglect add / sub vs. multiplications

Performance estimations

	order 1	order 2	order 4	order 8
Our countermeasure (overhead)				
R_1 - $h = 32$	1,35	1,39	1,47	1,64
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Field element randomization

Jacobian coordinate randomization

Double randomization

$[k]P = [k_0]P + \cdots + [k_d]P$

ISW applied to all mult.

* Assume 12 multiplications per loop iteration

** Neglect add / sub vs. multiplications

Performance estimations

	order 1	order 2	order 4	order 8
Our countermeasure (overhead)				
R_1 - $h = 32$	1,35	1,39	1,47	1,64
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Other countermeasures (overhead)				
scalar splitting	2	3	5	9
naive ISW	4	9	25	81

Field element randomization

Jacobian coordinate randomization

Double randomization

$[k]P = [k_0]P + \cdots + [k_d]P$

ISW applied to all mult.

Less secure than our solution (provided that hiddenness holds)

* Assume 12 multiplications per loop iteration

** Neglect add / sub vs. multiplications

Conclusion

- Formal model for regular exponentiation-like algorithms (with randomization)
- Formalisation of the hiddenness assumption
- Generic provably secure countermeasure
- Application to several ECC scalar mult. algorithms
- **Perspectives**:
    - Challenge the hiddenness assumption in practice
    - Applications to other algorithms / randomization techniques
    - Practical implementations and attacks