A Quest for Provable Security against Side-Channel Attacks

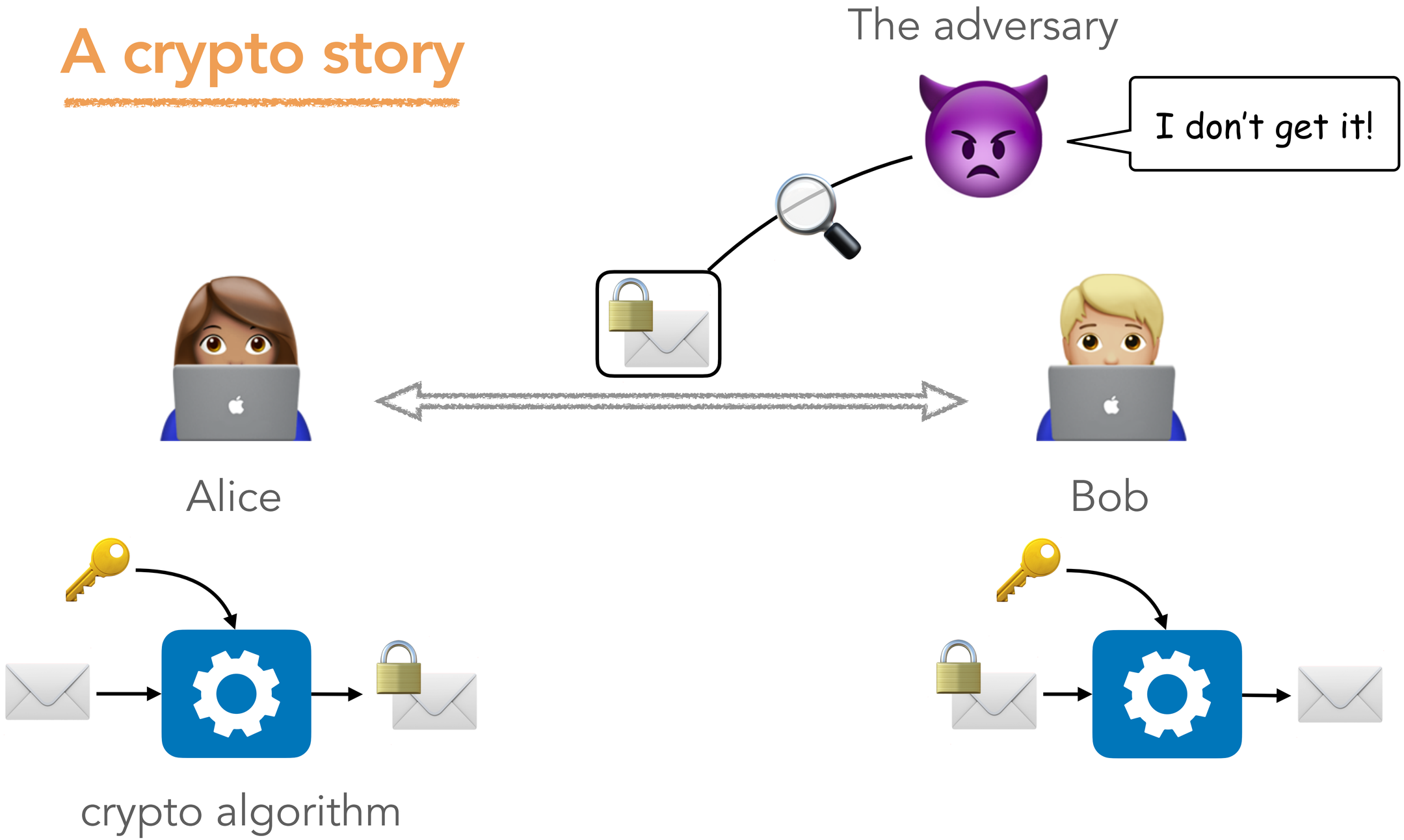
Matthieu Rivain

Cyber in Nancy

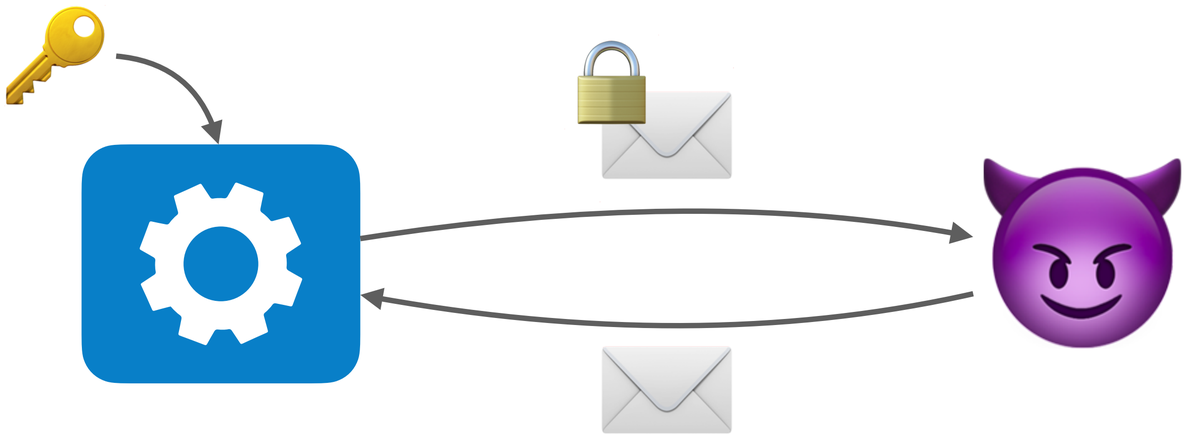
July 7, 2022, Loria, Nancy

Introduction

A crypto story



Provable security



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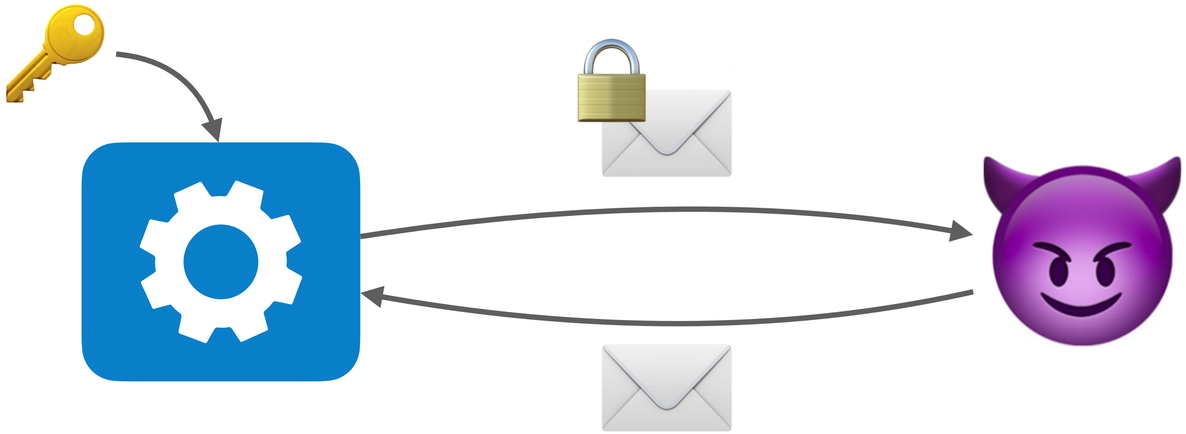
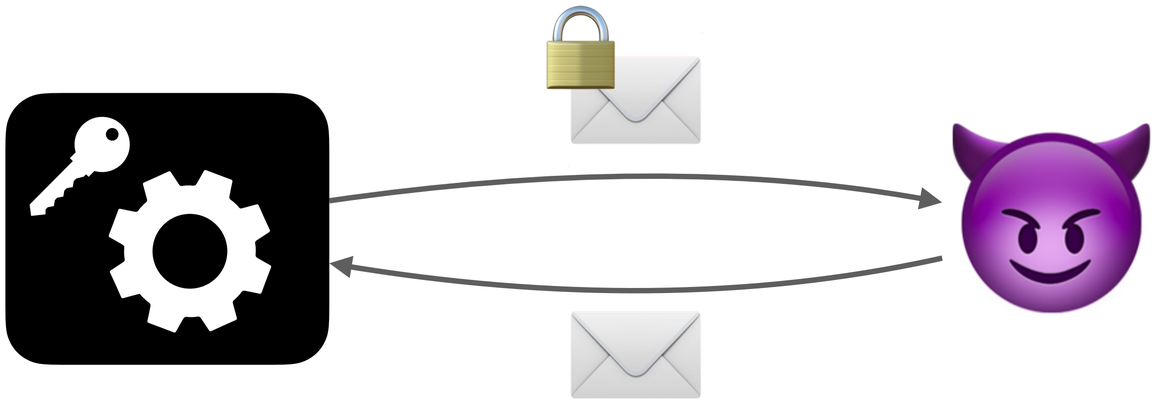


😈 needs unaffordable computing power to recover 🔑

security proof

Provable security

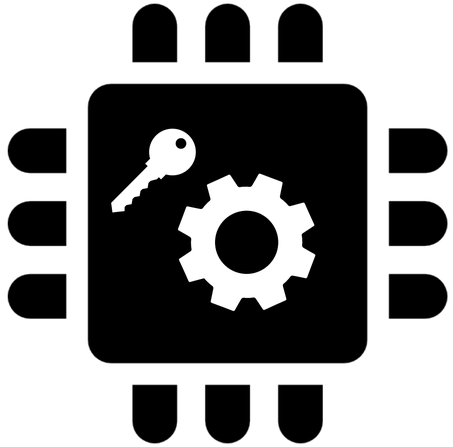


The “black-box model”

😈 needs unaffordable computing power to recover 🔑

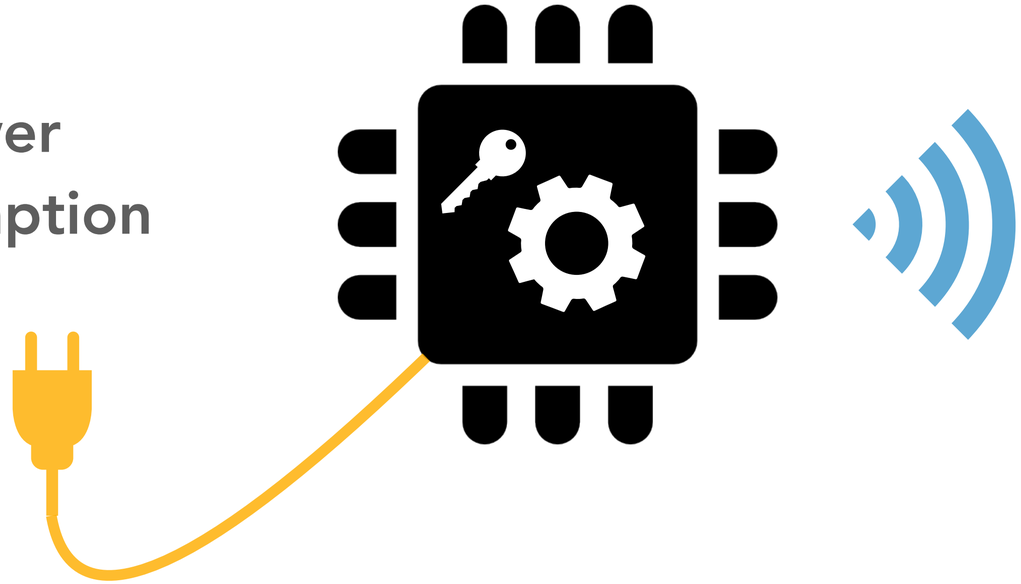
security proof

Side-channel attacks



Side-channel attacks

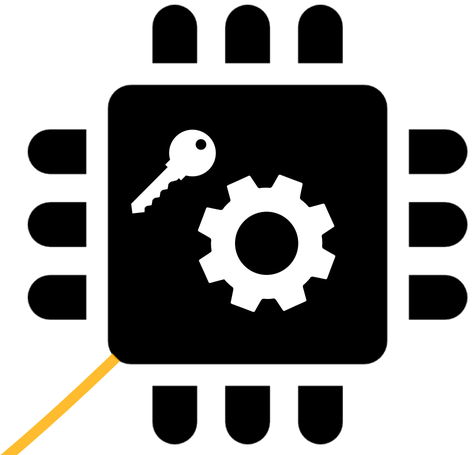
Power consumption

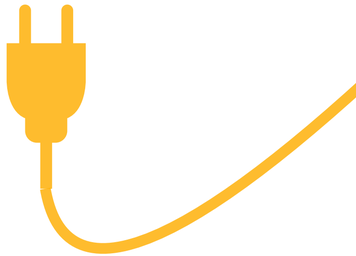


Electromagnetic emanations

Side-channel attacks

Power consumption





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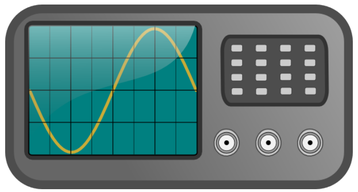
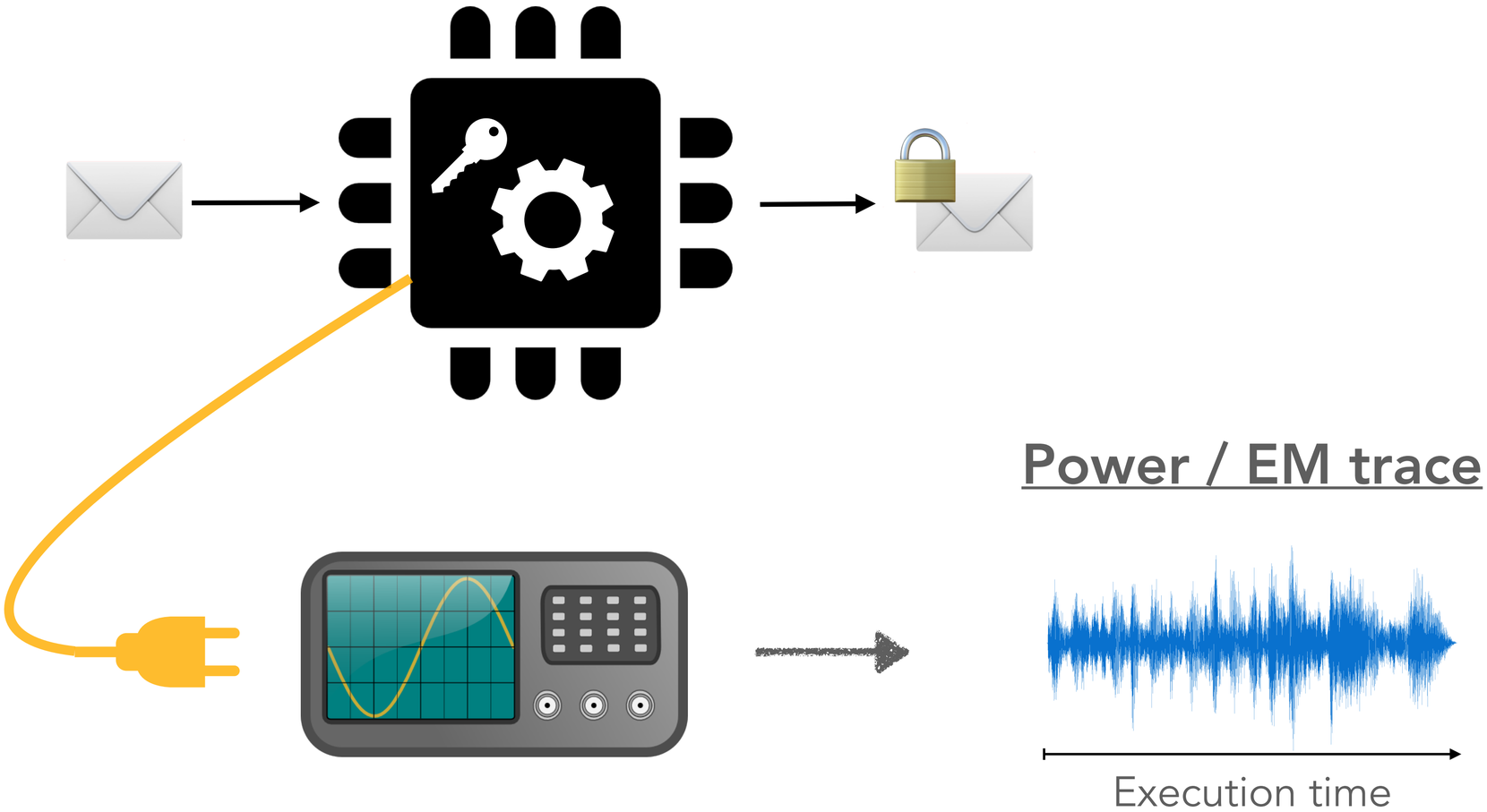


Differential power analysis



Differential power analysis

Key guess

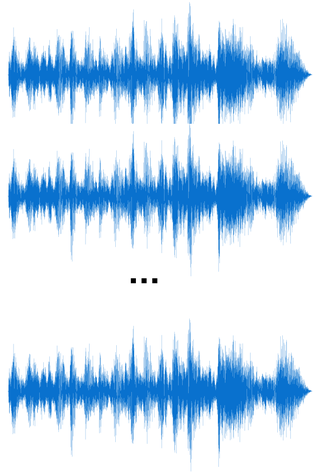
$f($ ✉️ , 🔑 $)$

$f($ ✉️ , 🔑 $)$

…

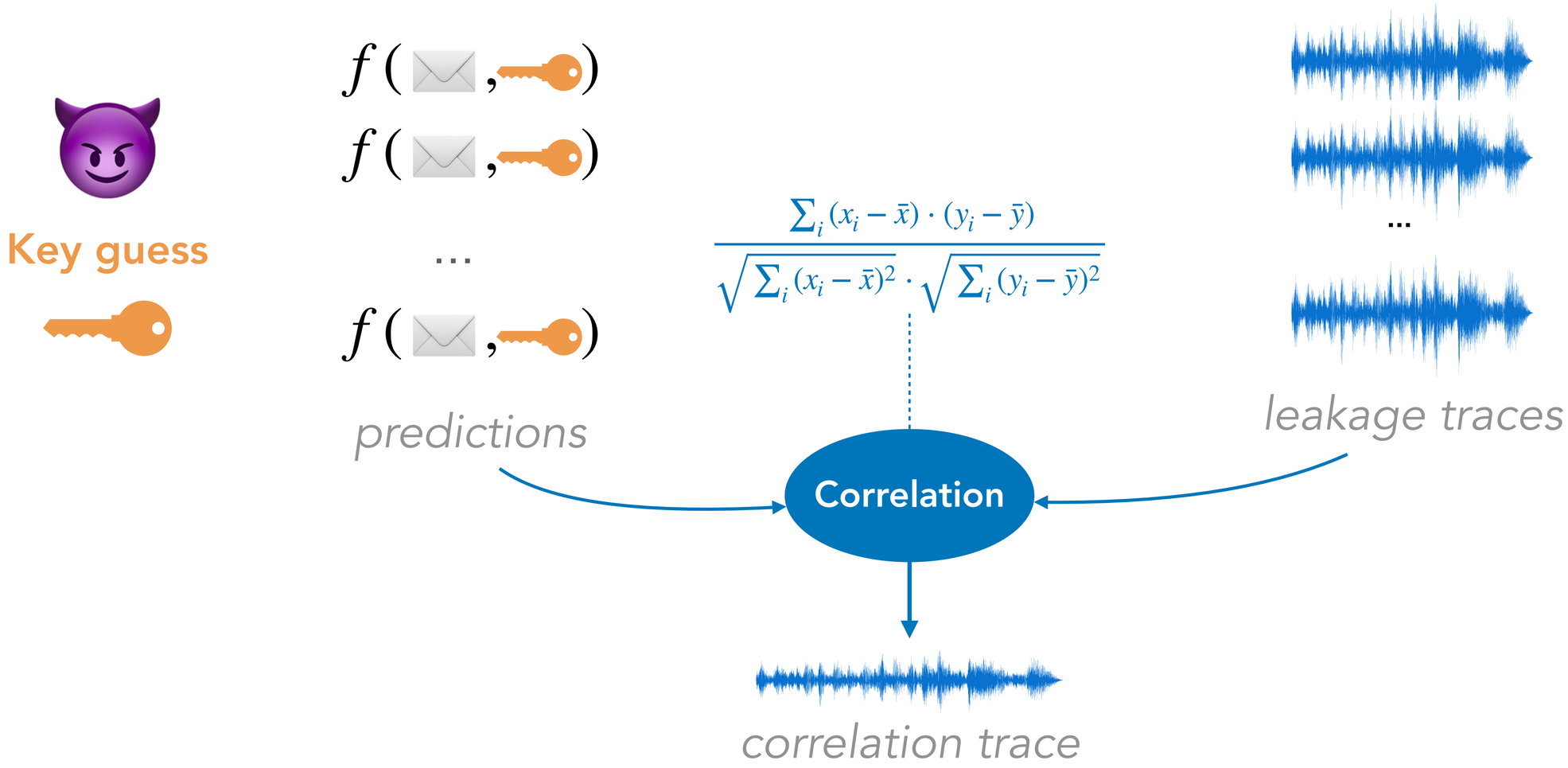
$f($ ✉️ , 🔑 $)$

predictions



leakage traces

Differential power analysis



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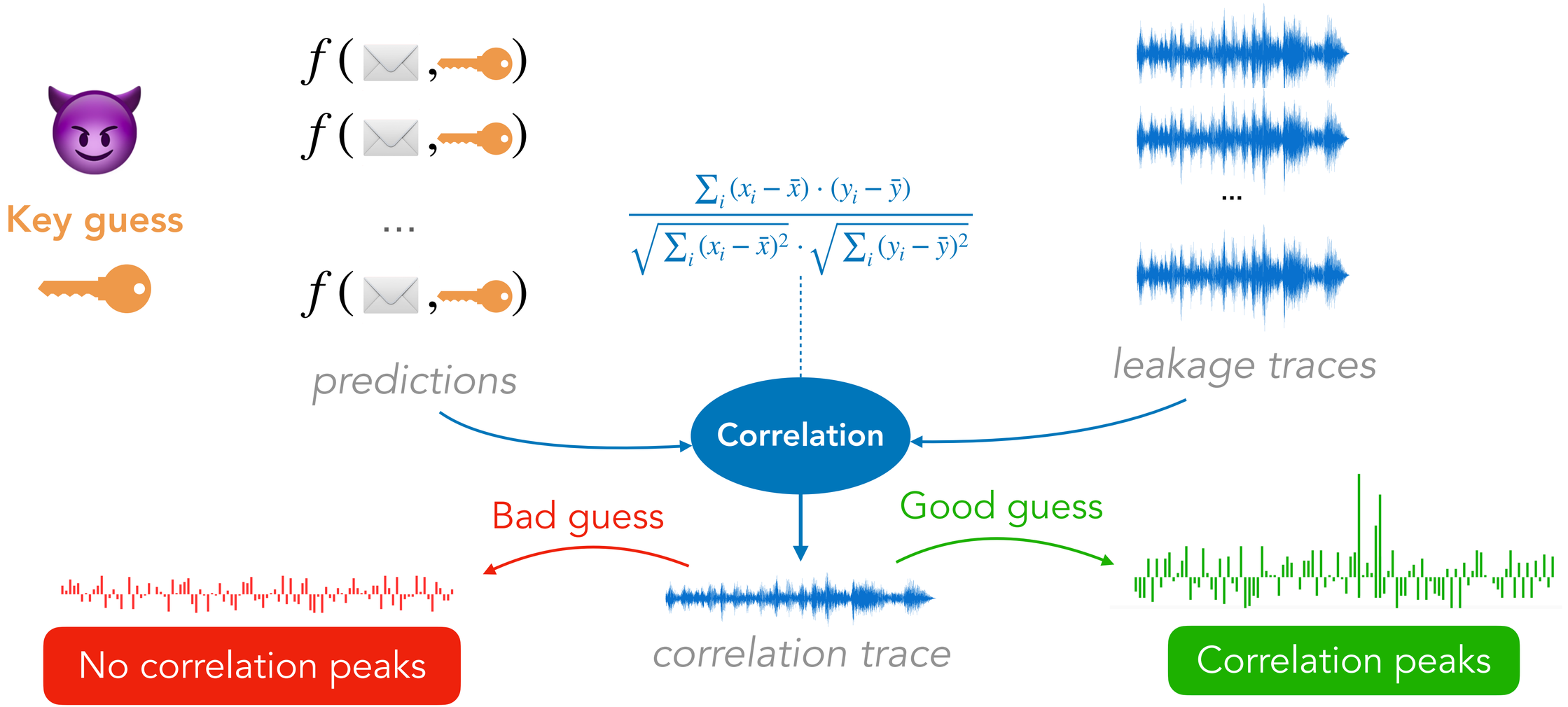
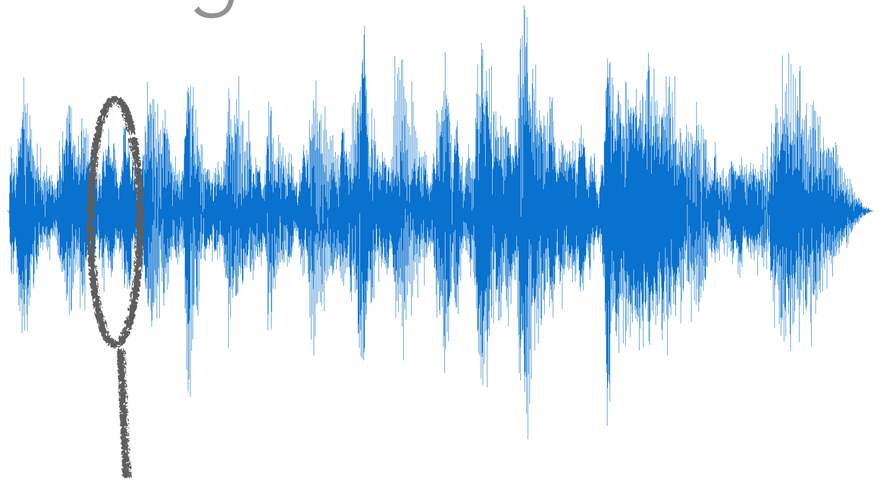


Example: DPA on AES

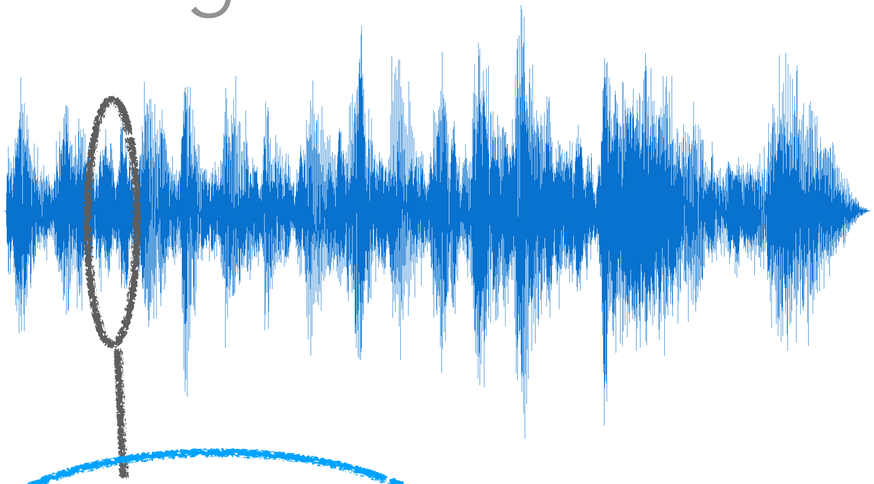
Leakage model



$\sim \mathrm{Hw}(\mathrm{S}(x \oplus k)) + \mathcal{N}(\mu, \sigma)$

Example: DPA on AES

Leakage model



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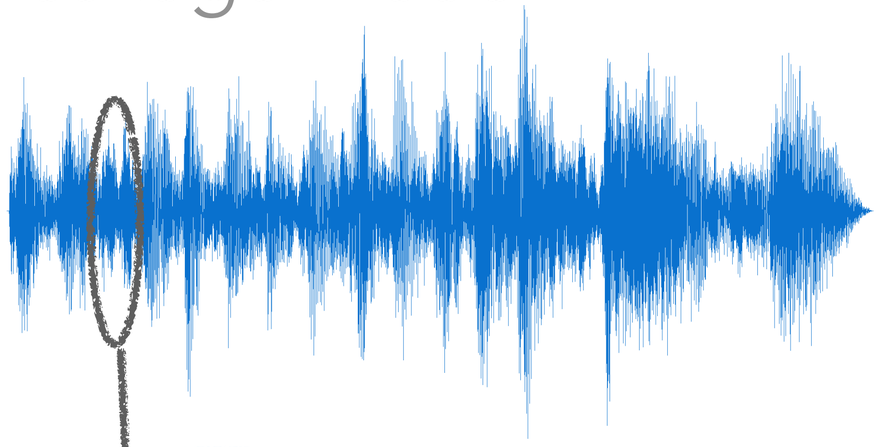
Hamming weight of s-box output

Example: DPA on AES

Leakage model



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Gaussian noise

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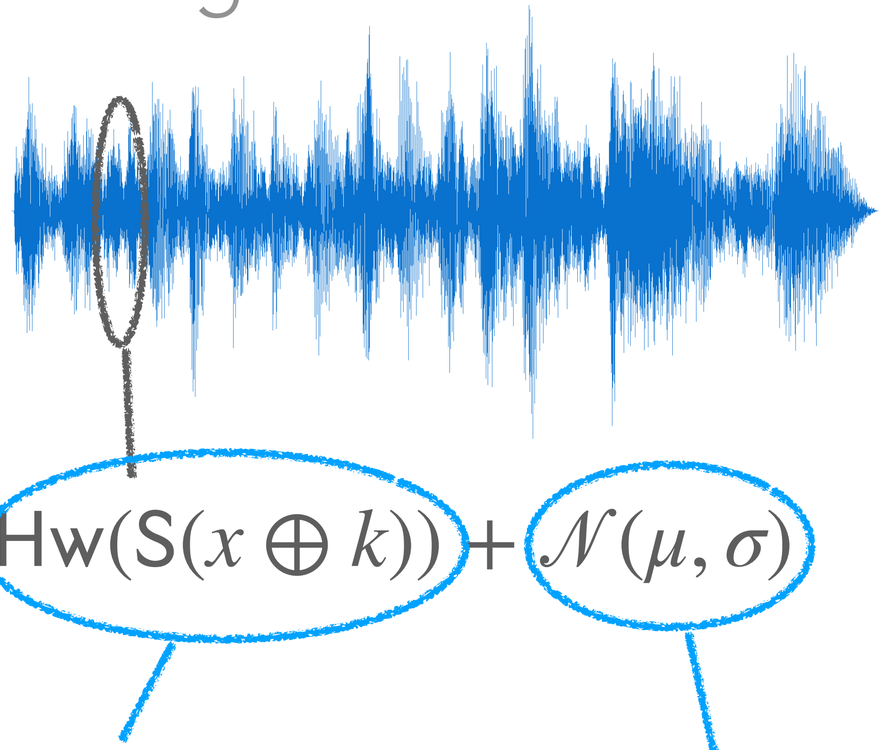

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Hamming weight of s-box output

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Prediction

$$f(\text{✉}, \text{🔑}) = \mathrm{Hw}(\mathrm{S}(x \oplus \hat{k}))$$

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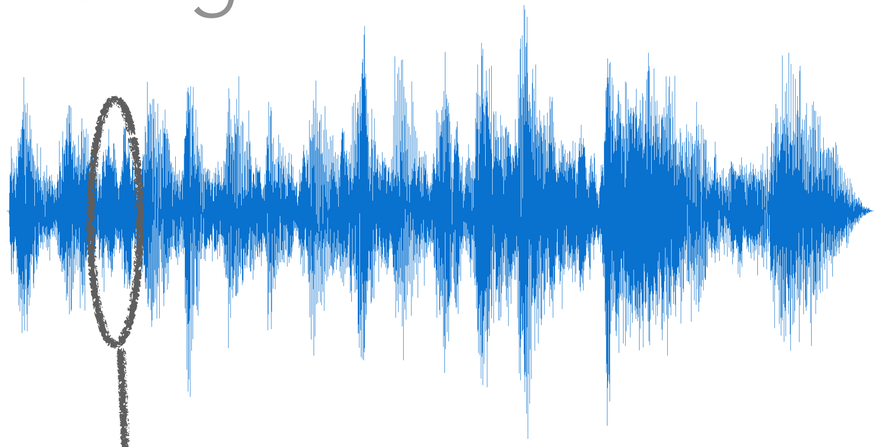
Leakage model



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(known) byte of the plaintext

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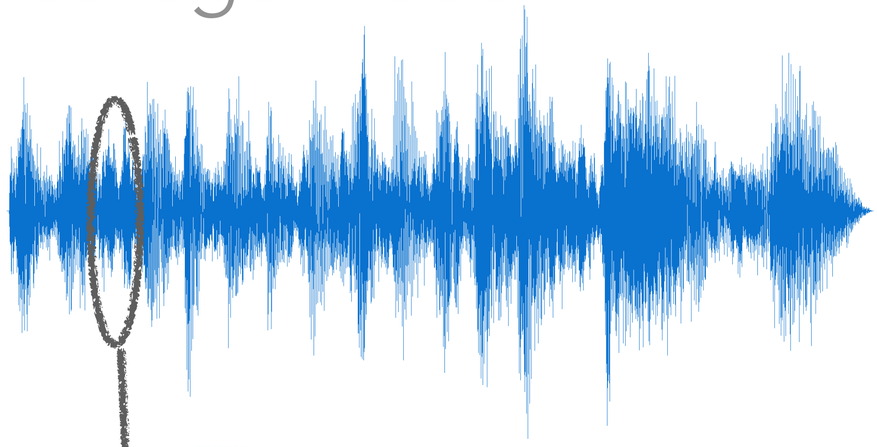
Leakage model



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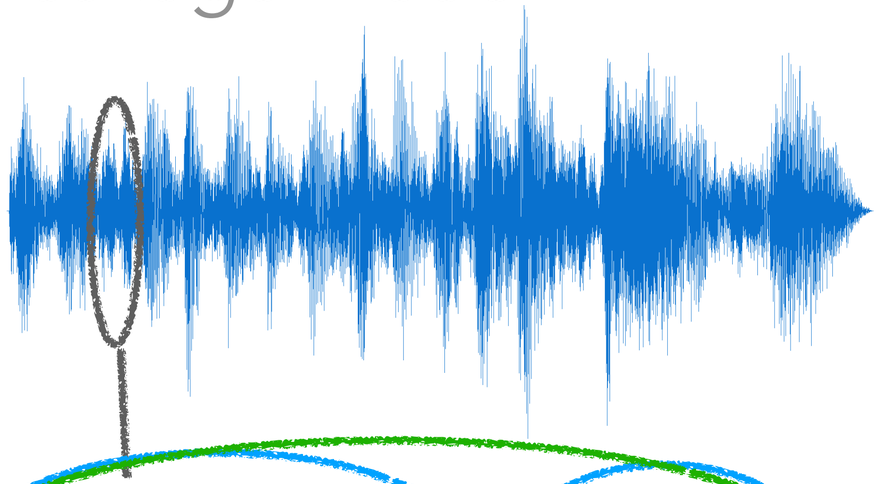
Example: DPA on AES

Leakage model



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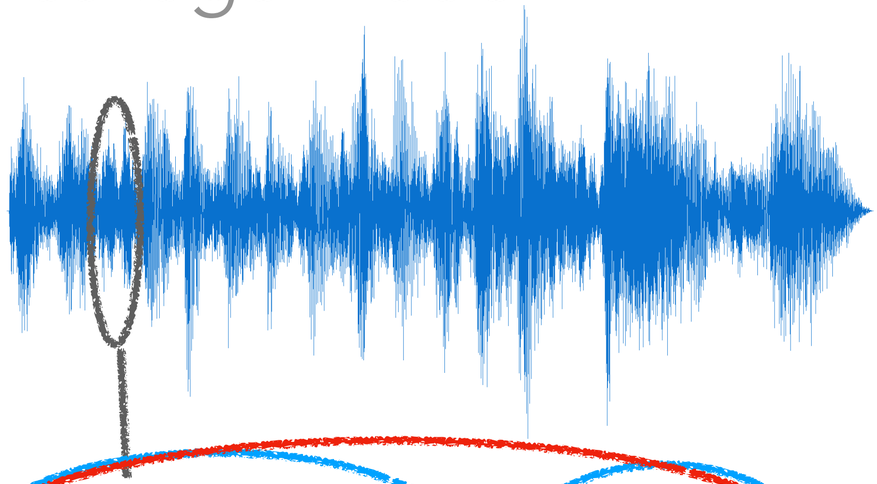
(guess) byte of the key

high correlation for the good guess

Example: DPA on AES

Leakage model



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Gaussian noise

Prediction

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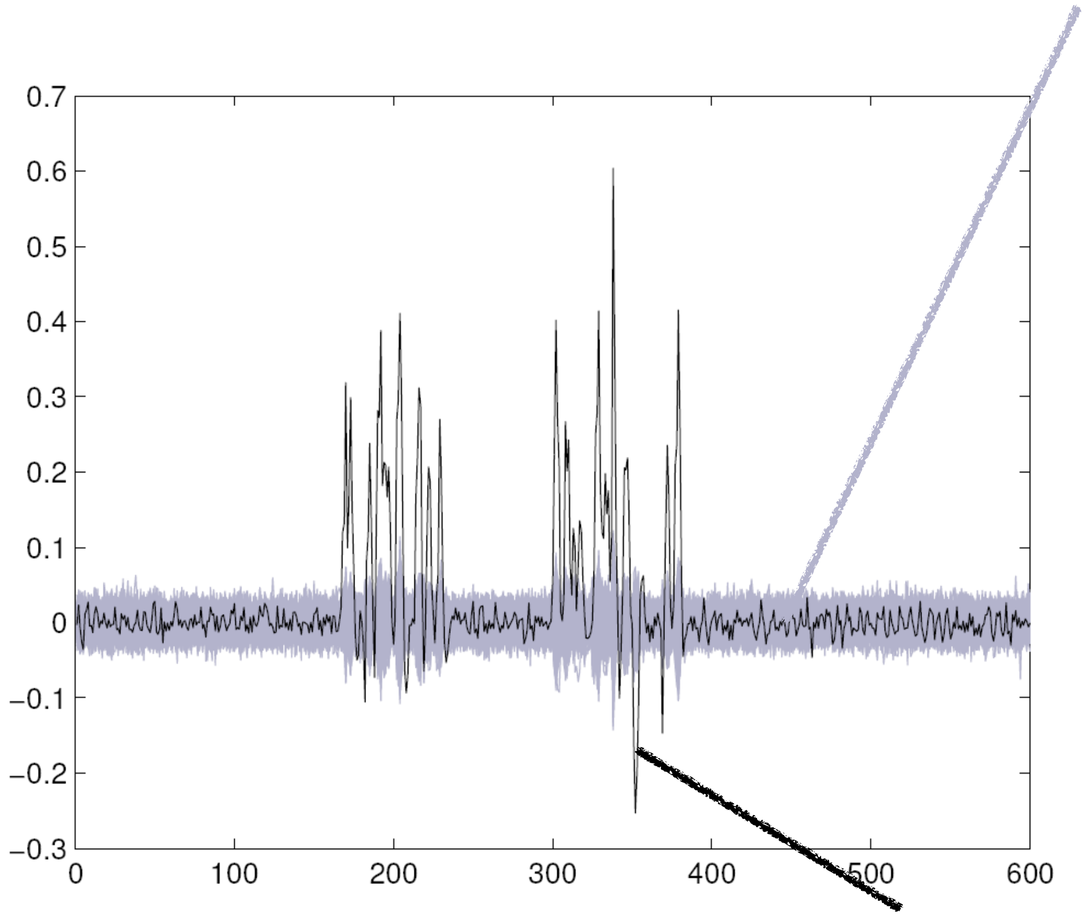
(guess) byte of the key

low correlation for the wrong guess

Example: DPA on AES

correlation traces for wrong guesses



correlation trace for good guess

Masking

Apply secret sharing at the computation level

$$x = x_1 \oplus \cdots \oplus x_n$$

Masking

Apply secret sharing at the computation level

$$x = x_1 \oplus \cdots \oplus x_n$$

randomly generated

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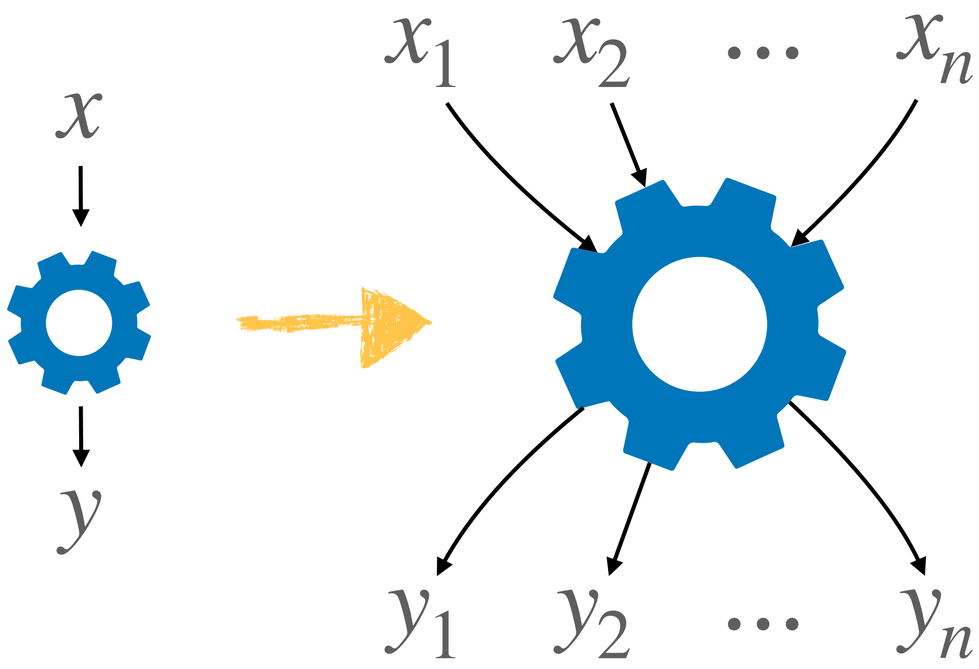
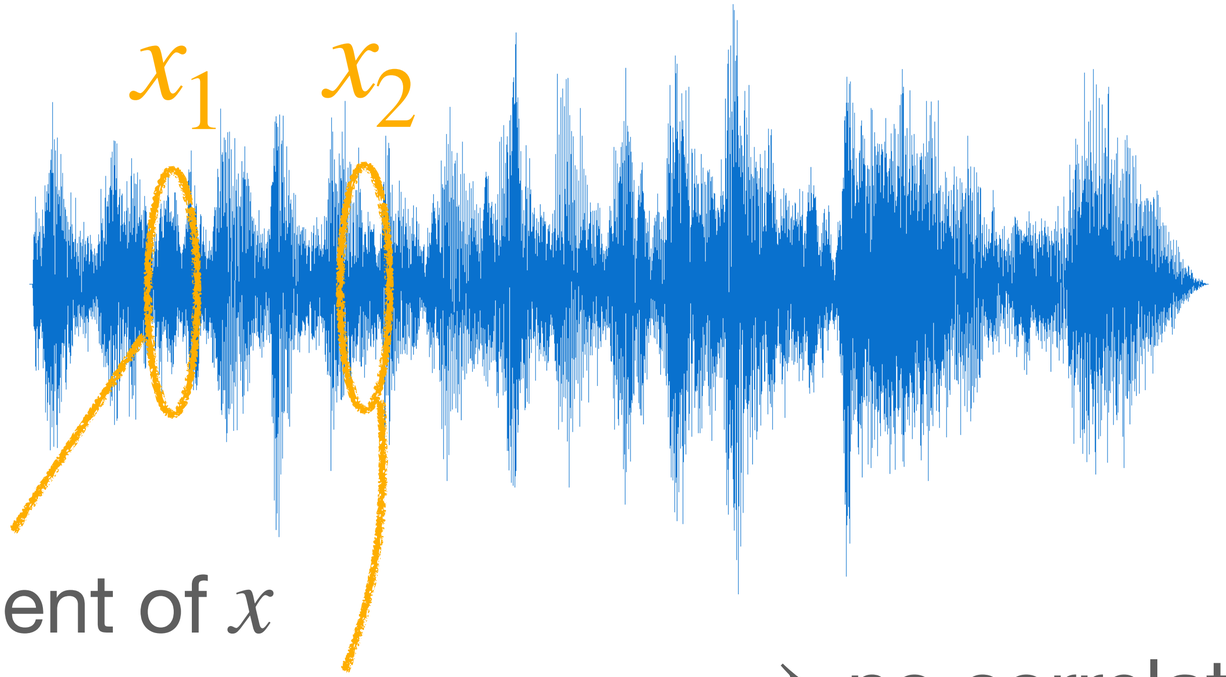


Masking

First order masking: $x = x_1 \oplus x_2$



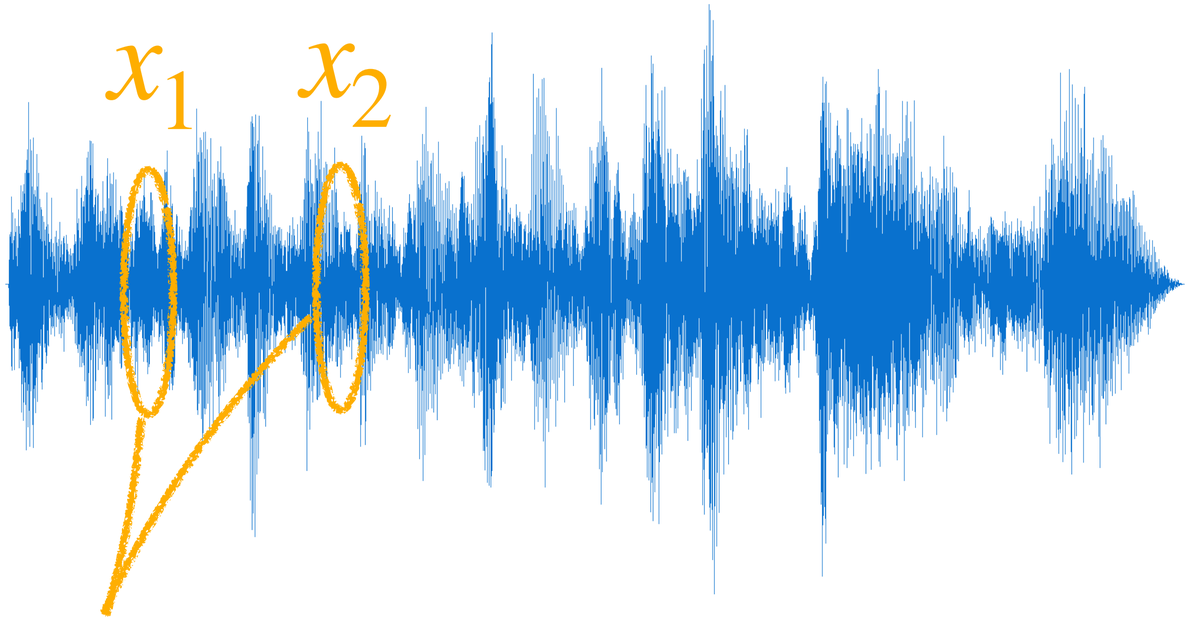
independent of x

independent of x

$\Rightarrow$ no correlation (DPA fails)

Masking

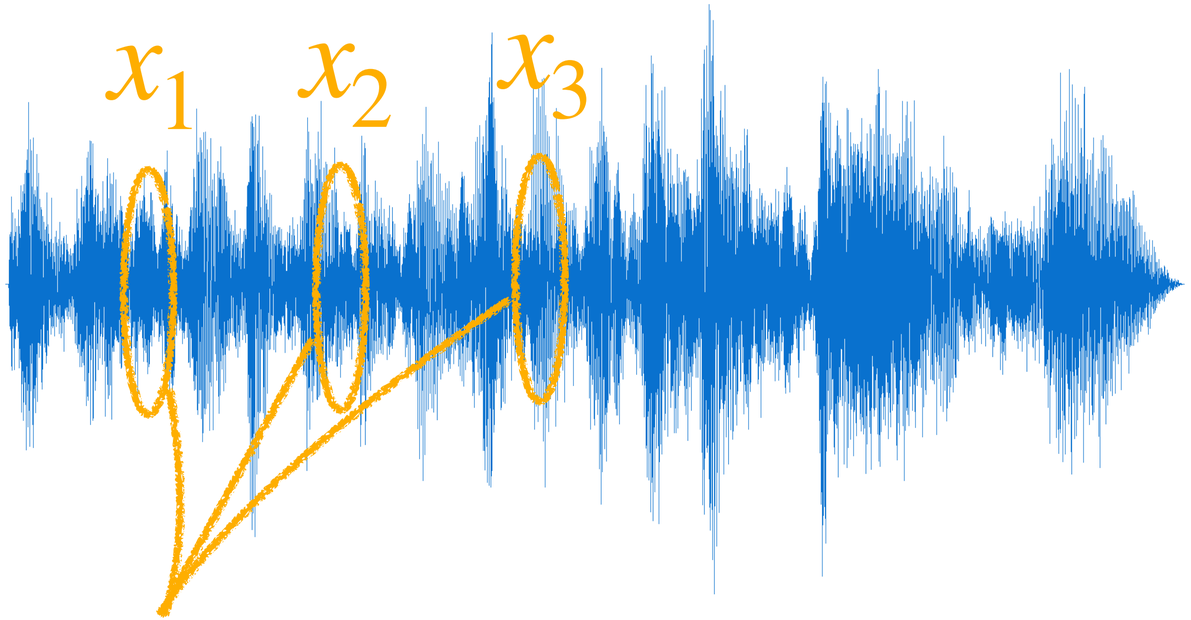
First order masking: $x = x_1 \oplus x_2$



⚠ jointly depend on x ⇒ **2nd order** DPA

Masking

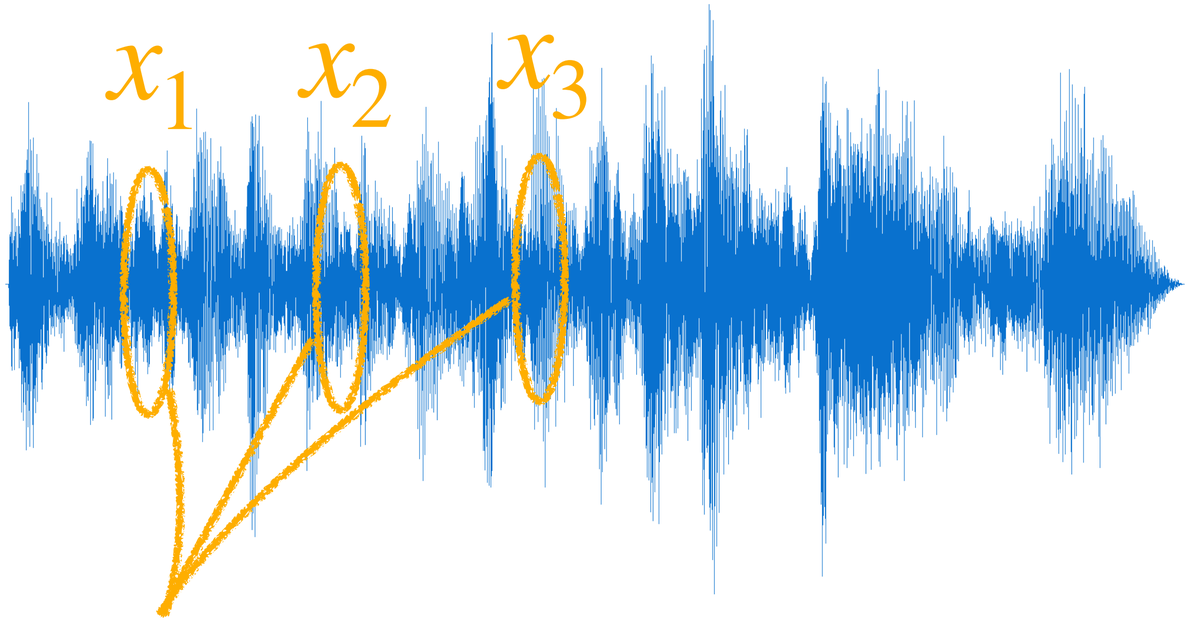
Second order masking: $x = x_1 \oplus x_2 \oplus x_3$



⚠ jointly depend on x $\Rightarrow$ **3rd order** DPA

Masking

Second order masking: $x = x_1 \oplus x_2 \oplus x_3$

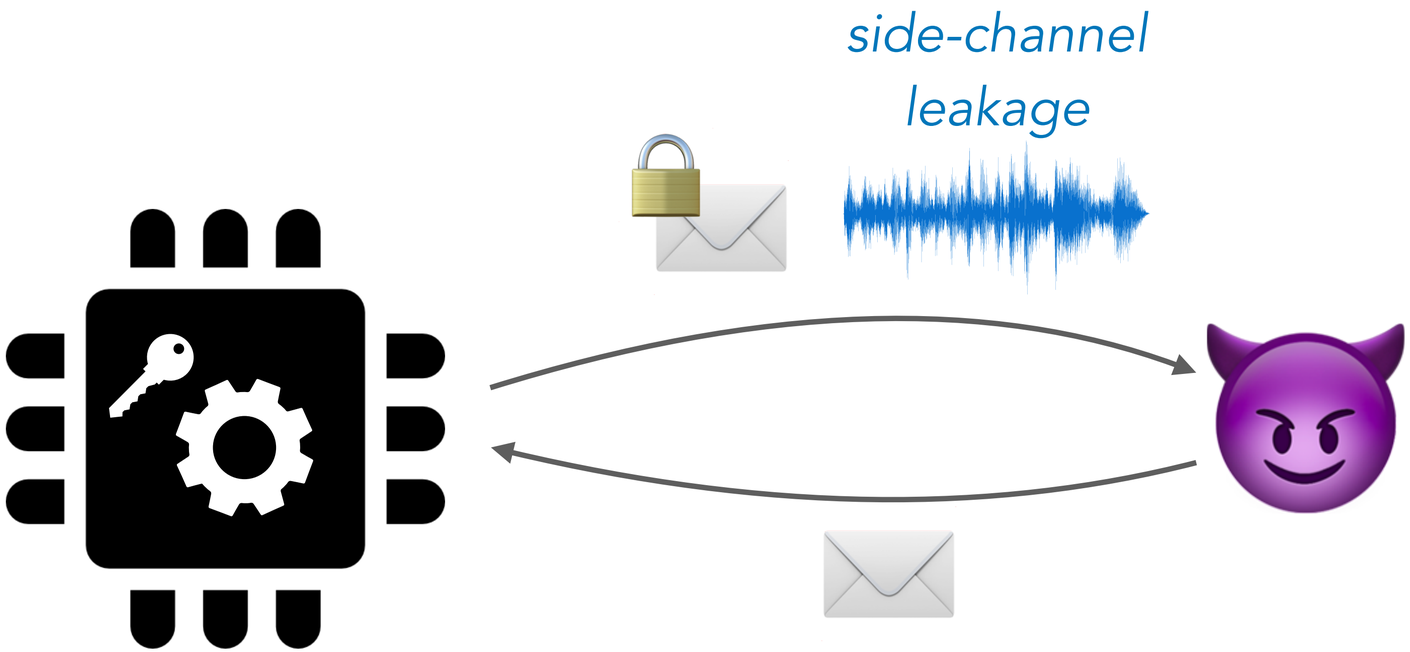



/


game!

⚠ jointly depend on x ⇒ **3rd order** DPA

Provable security in the presence of leakage

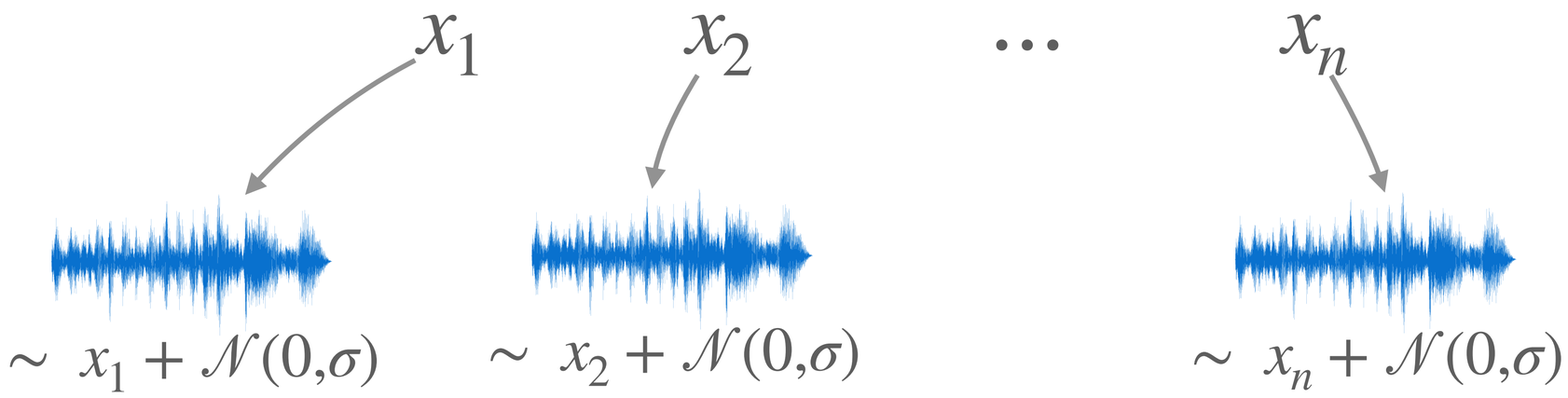


😈 needs unaffordable computing power to recover 🔑

security proof

Modelling noisy leakage

Motivation: soundness of masking with noise



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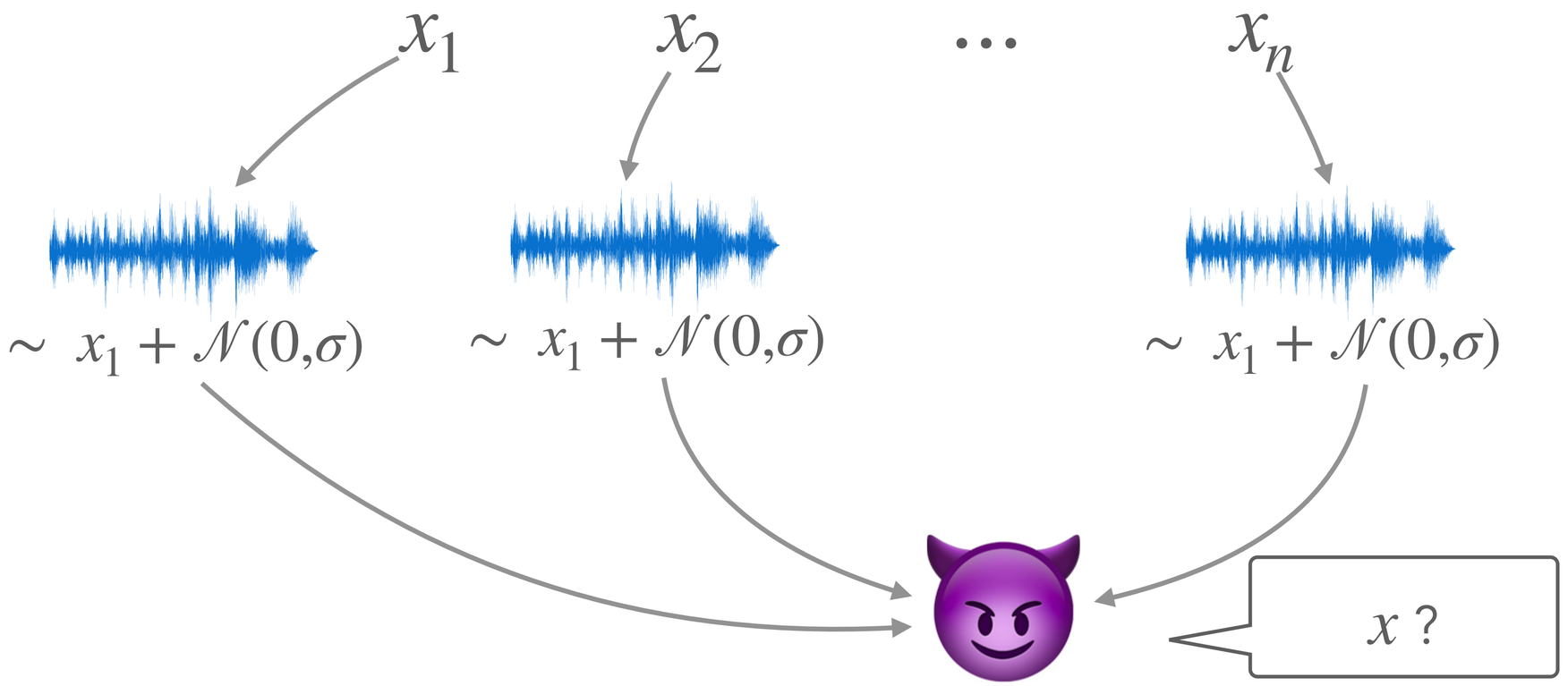

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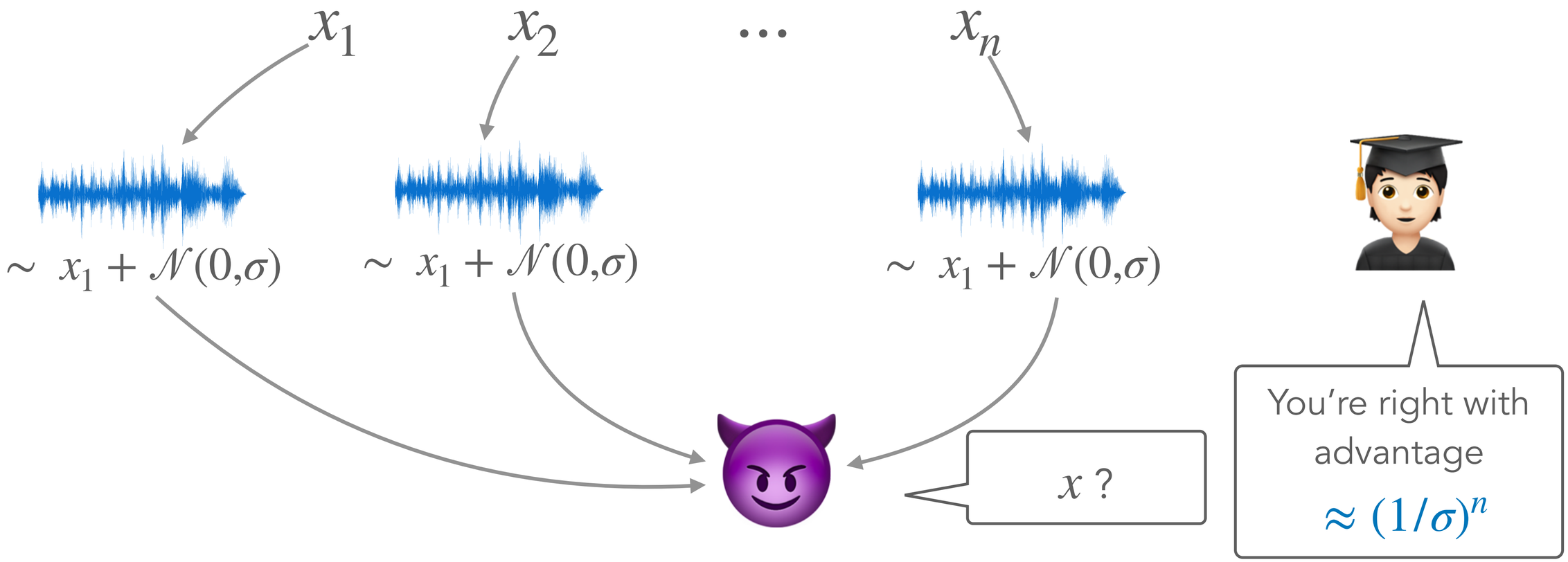

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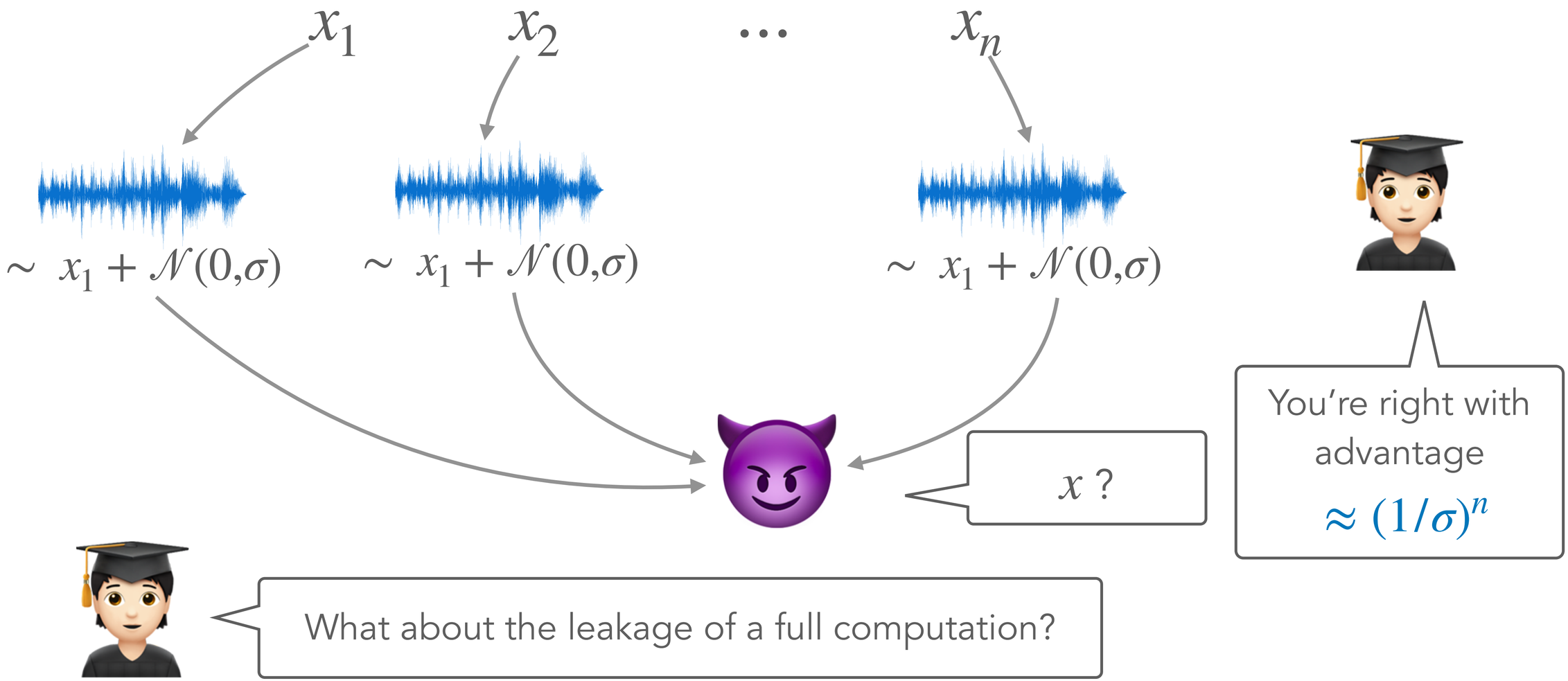
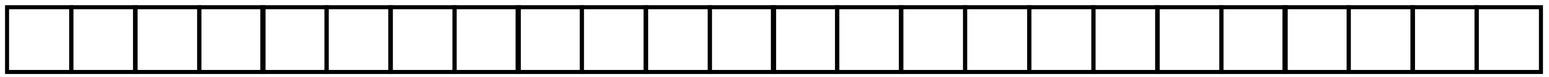


The noisy leakage model

Assumption: “only computation leaks”

Memory



Computation

The noisy leakage model

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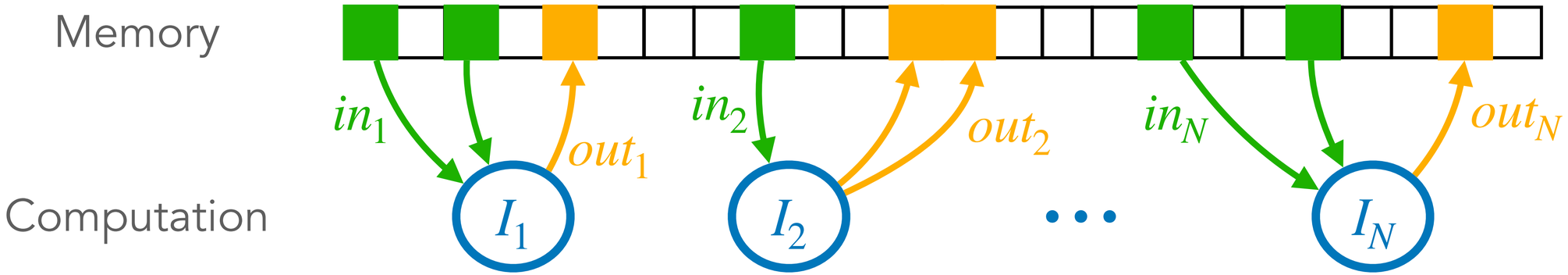

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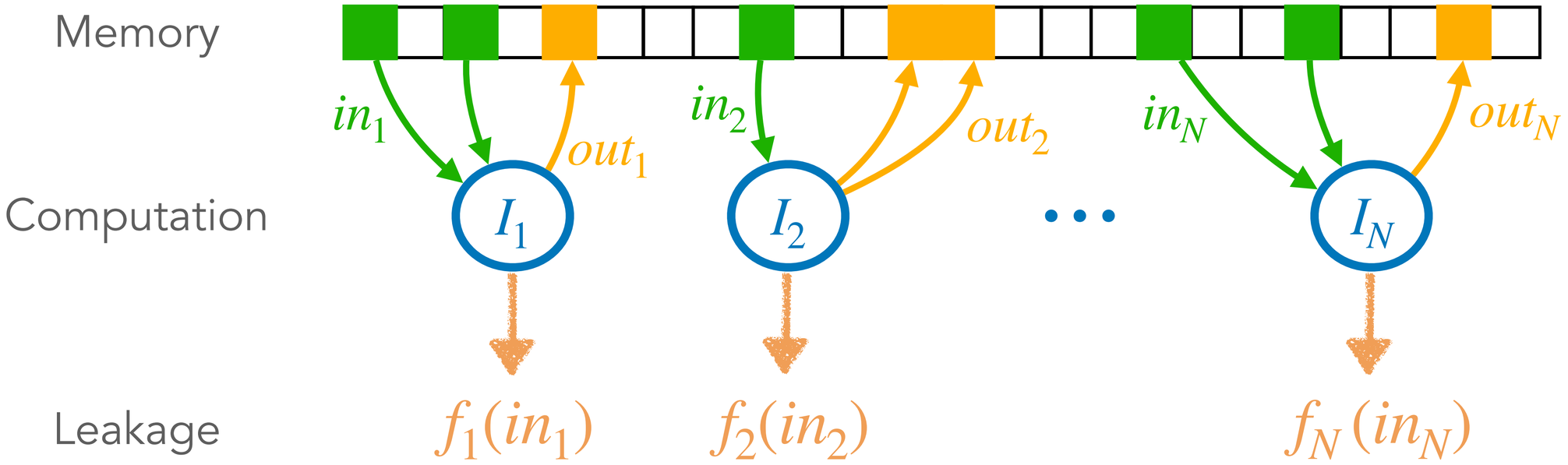

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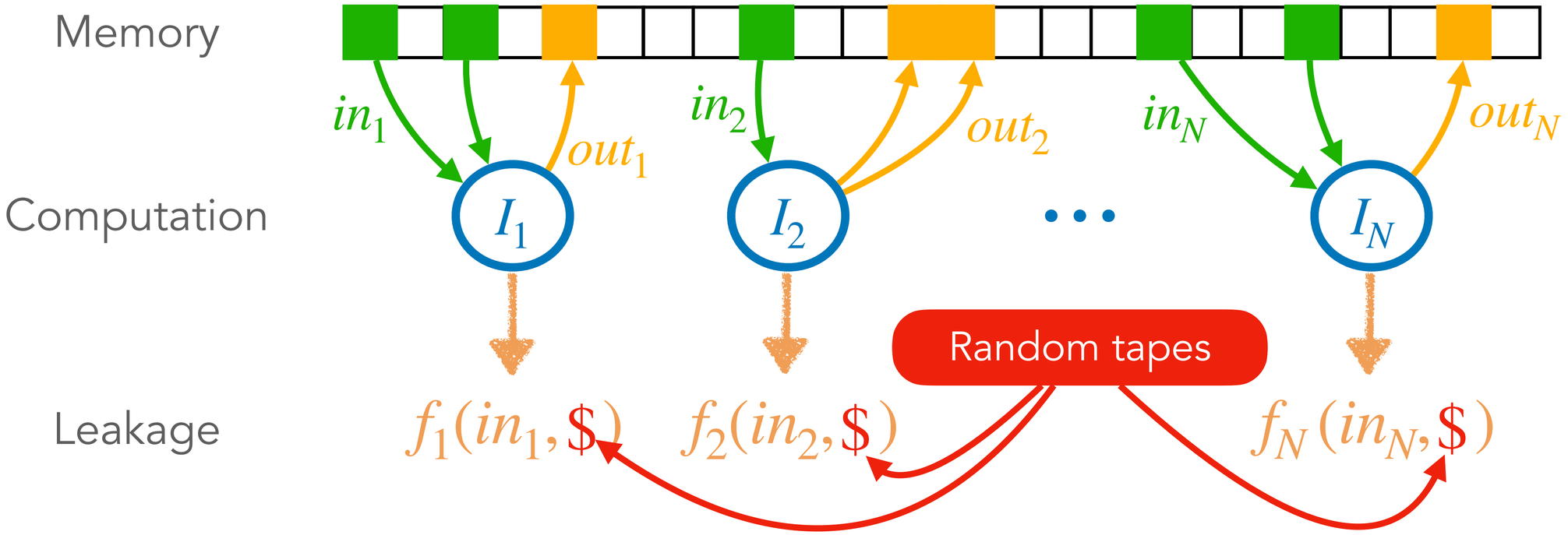

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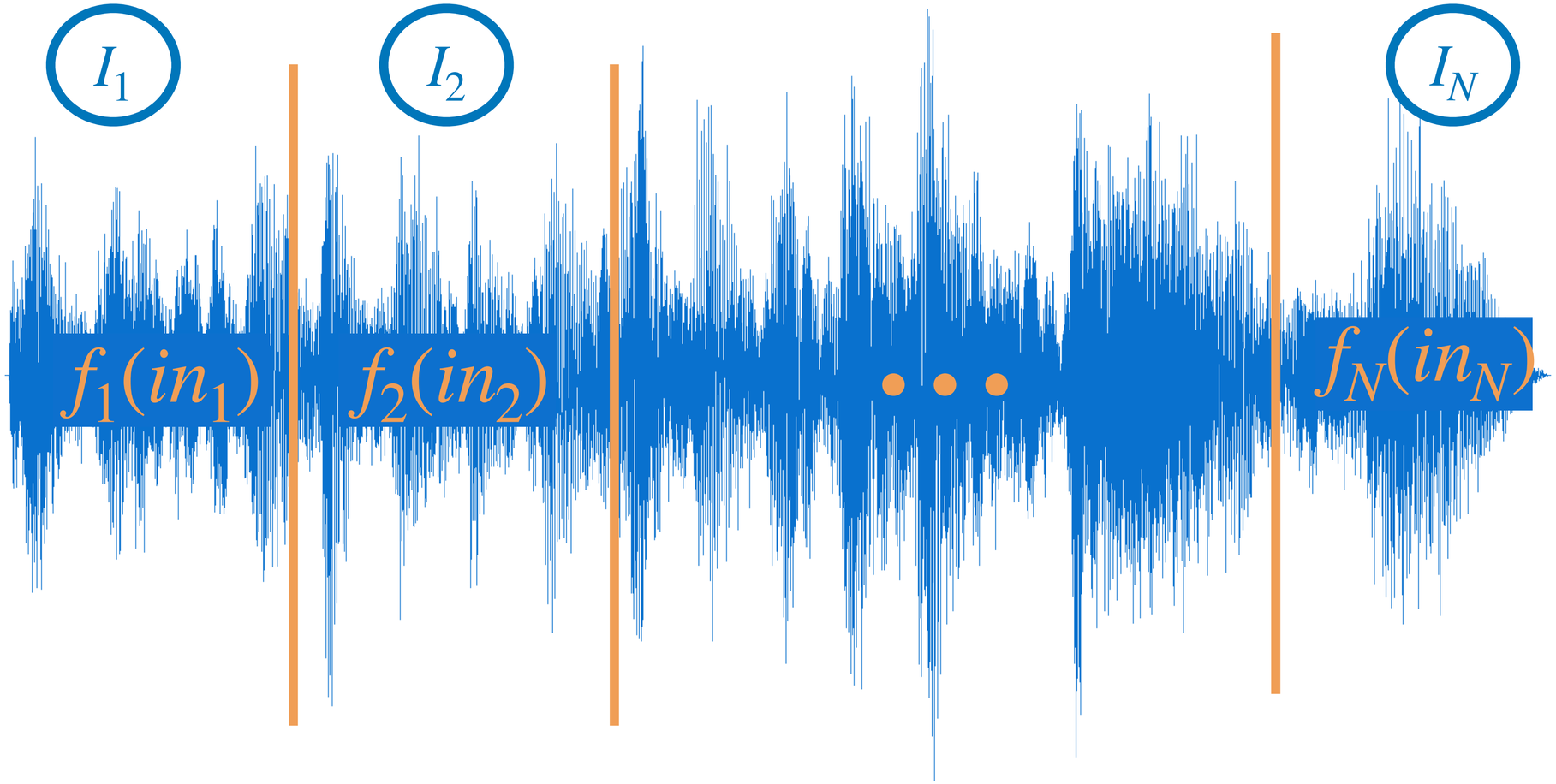

$f_i(in_i) \Rightarrow$ *multivariate noisy leakage*

Noisy leakage functions

A function is δ-**noisy** if (for $X \sim \mathcal{U}$):

$$\mathbb{E}_y\left[\Delta\left(X;\ \left(X \mid f(X) = y\right)\right)\right] \ \leq \ \delta$$

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statistical distance between X and X and given $f(X) = y$

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expectation on the possible leakage values

statistical distance between X and X and given $f(X) = y$

Noisy leakage functions

more noise
⇒ smaller δ

A function is **δ-noisy** if (for $X \sim \mathcal{U}$):

$$\mathbb{E}_y\left[\Delta\left(X;\ \left(X \mid f(X) = y\right)\right)\right] \leq \delta$$

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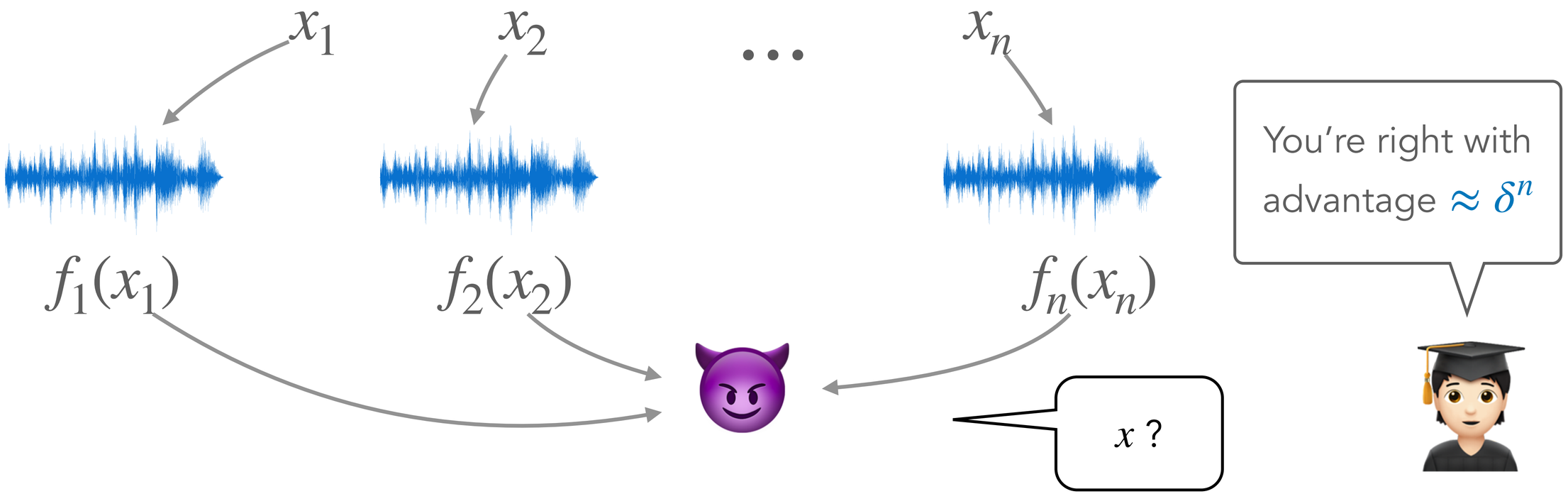
Soundness of masking with noise (generalised)



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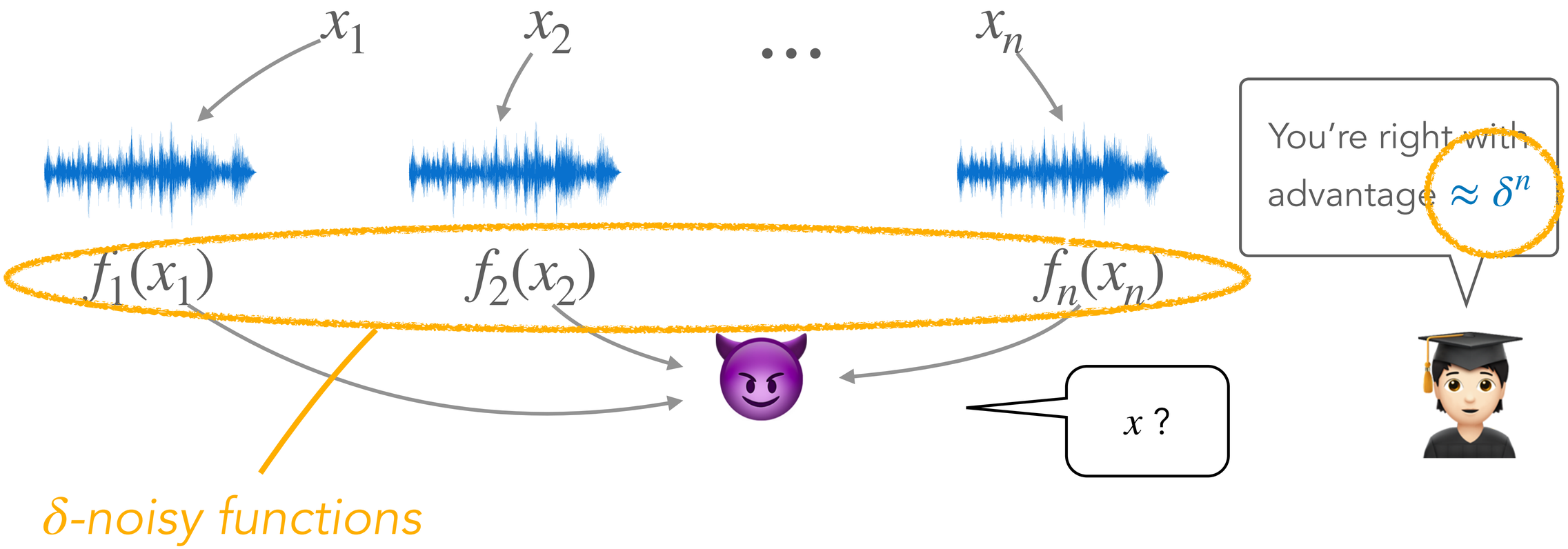
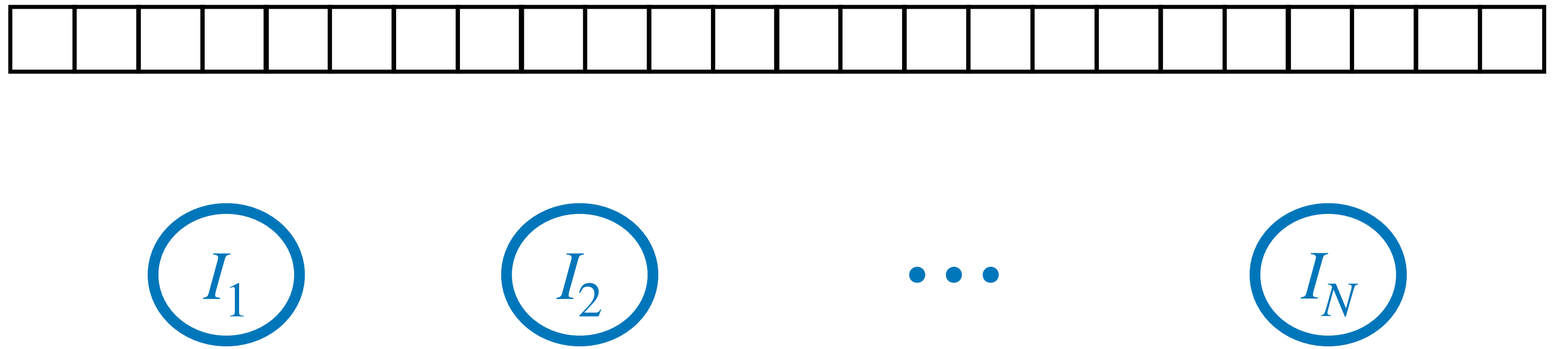


Simulation security

Secret input



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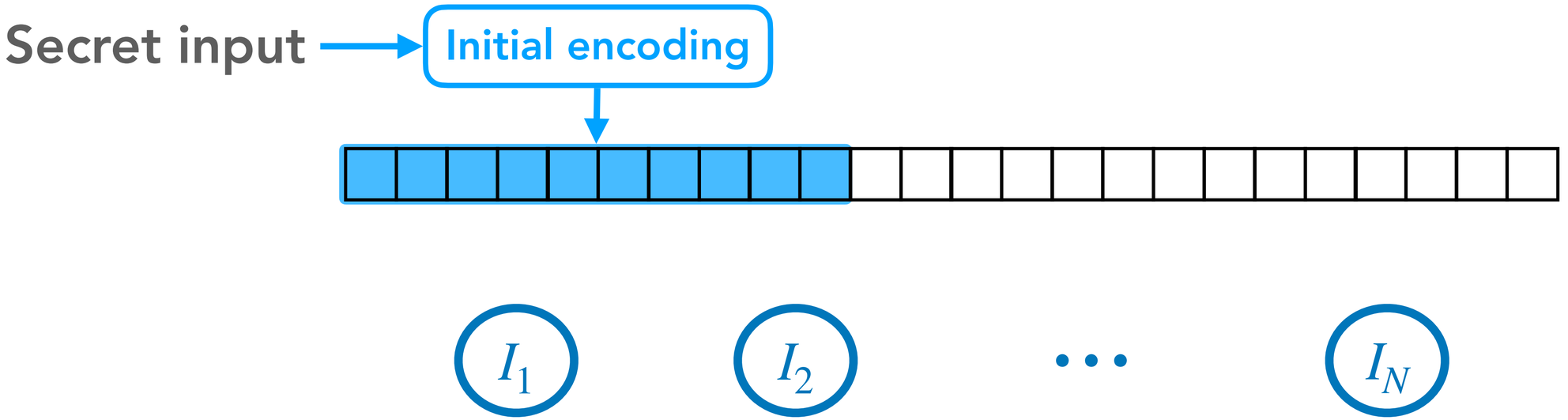

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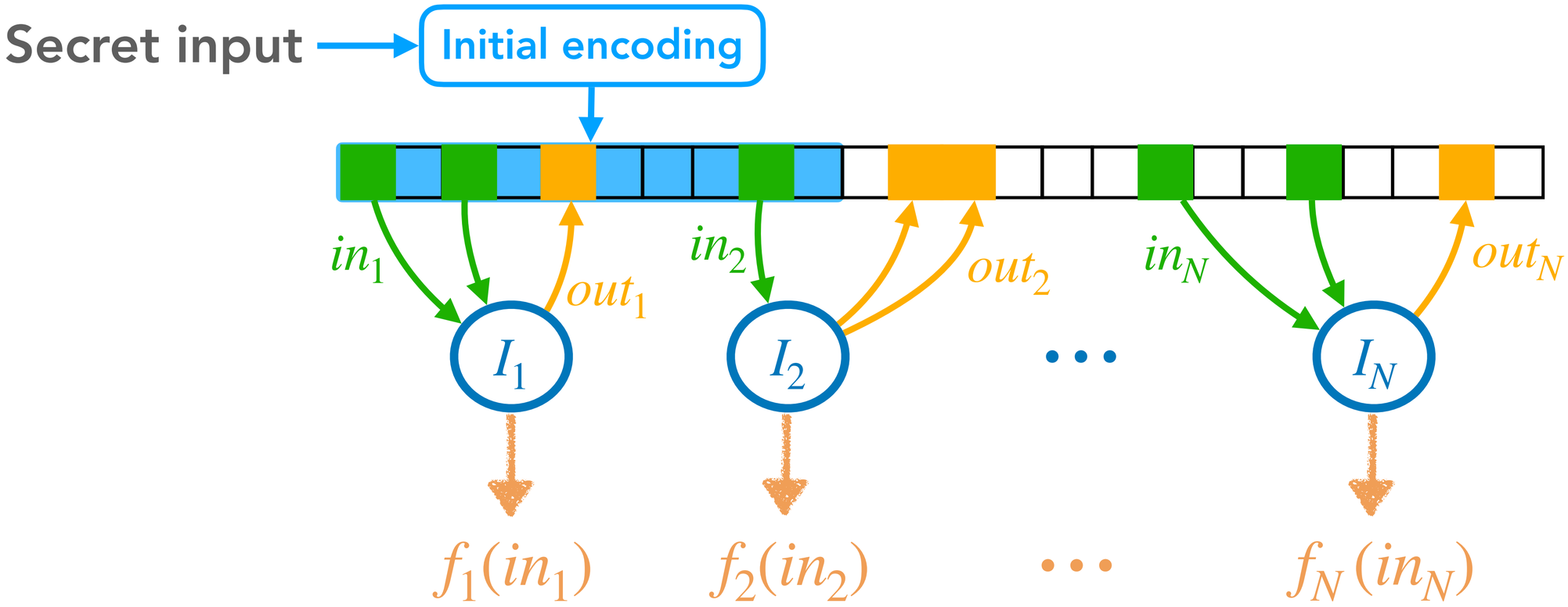

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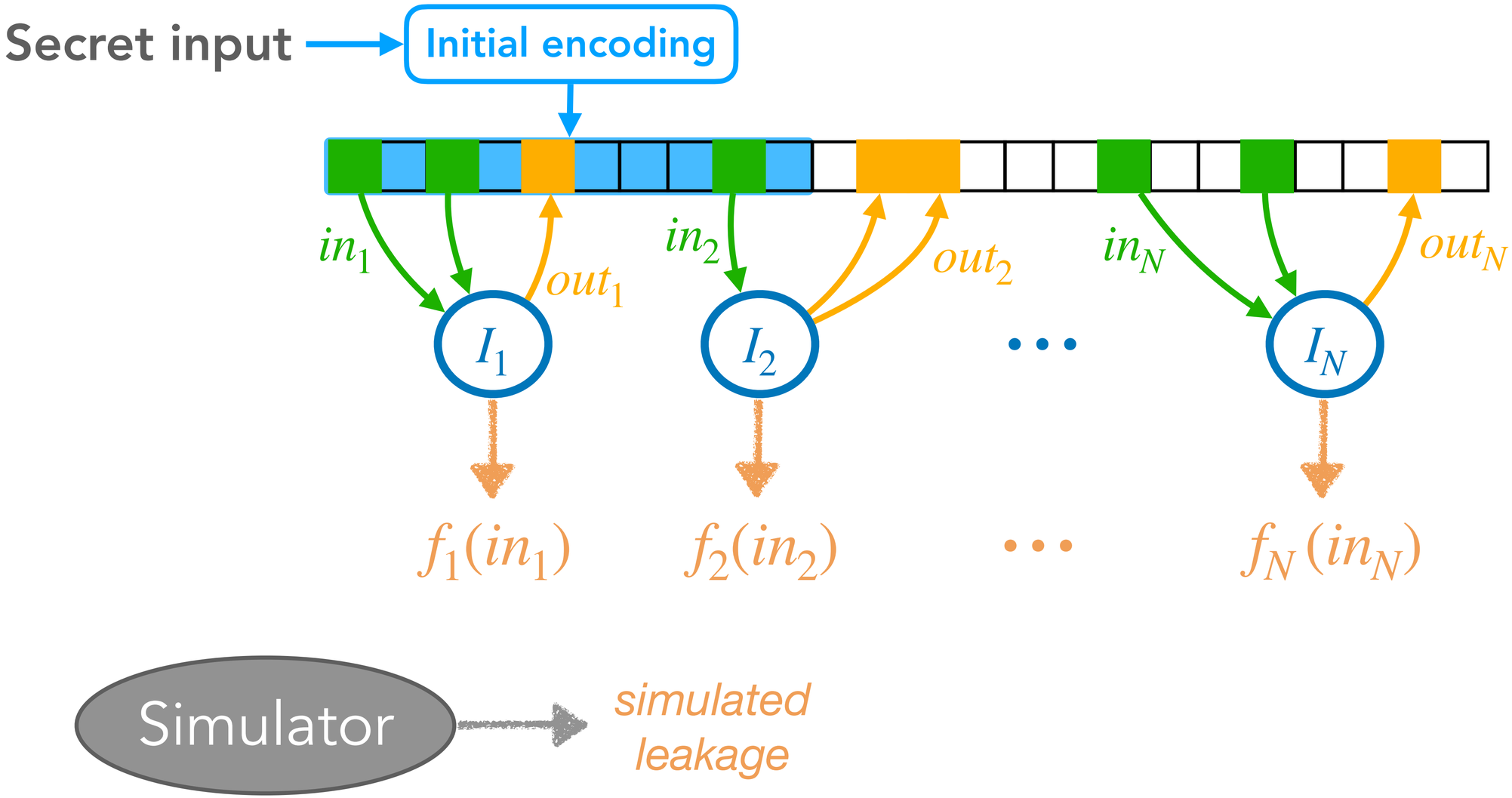

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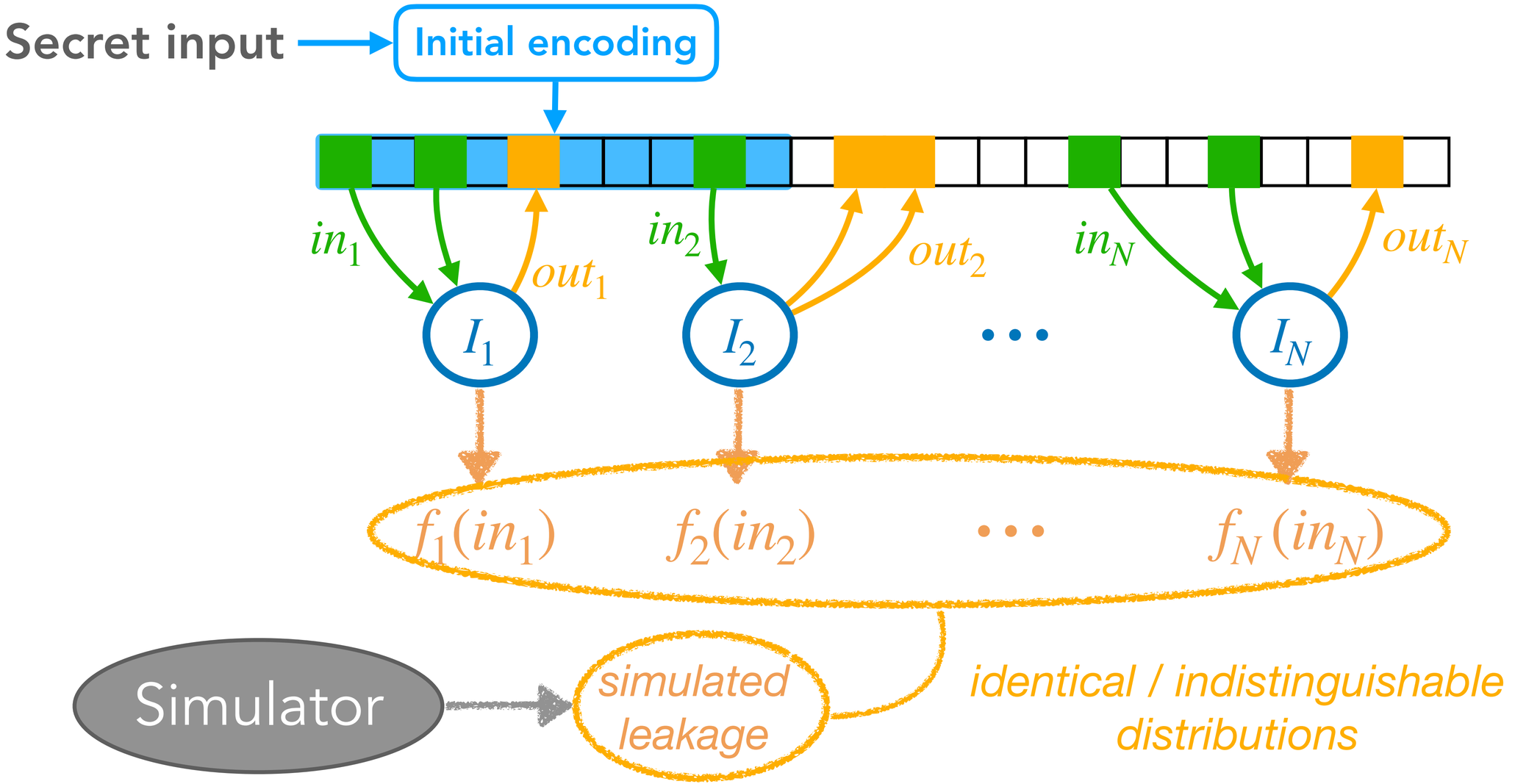

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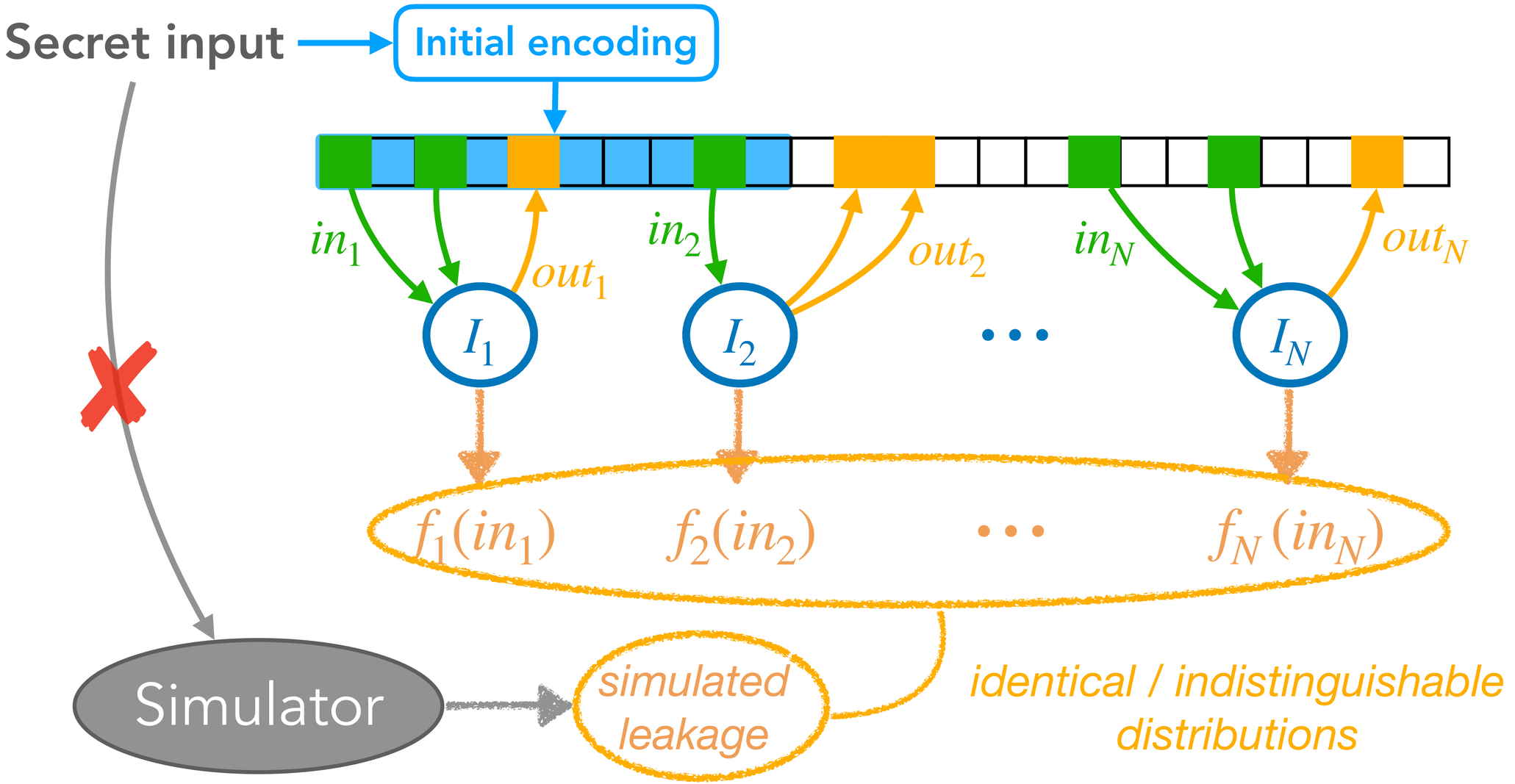
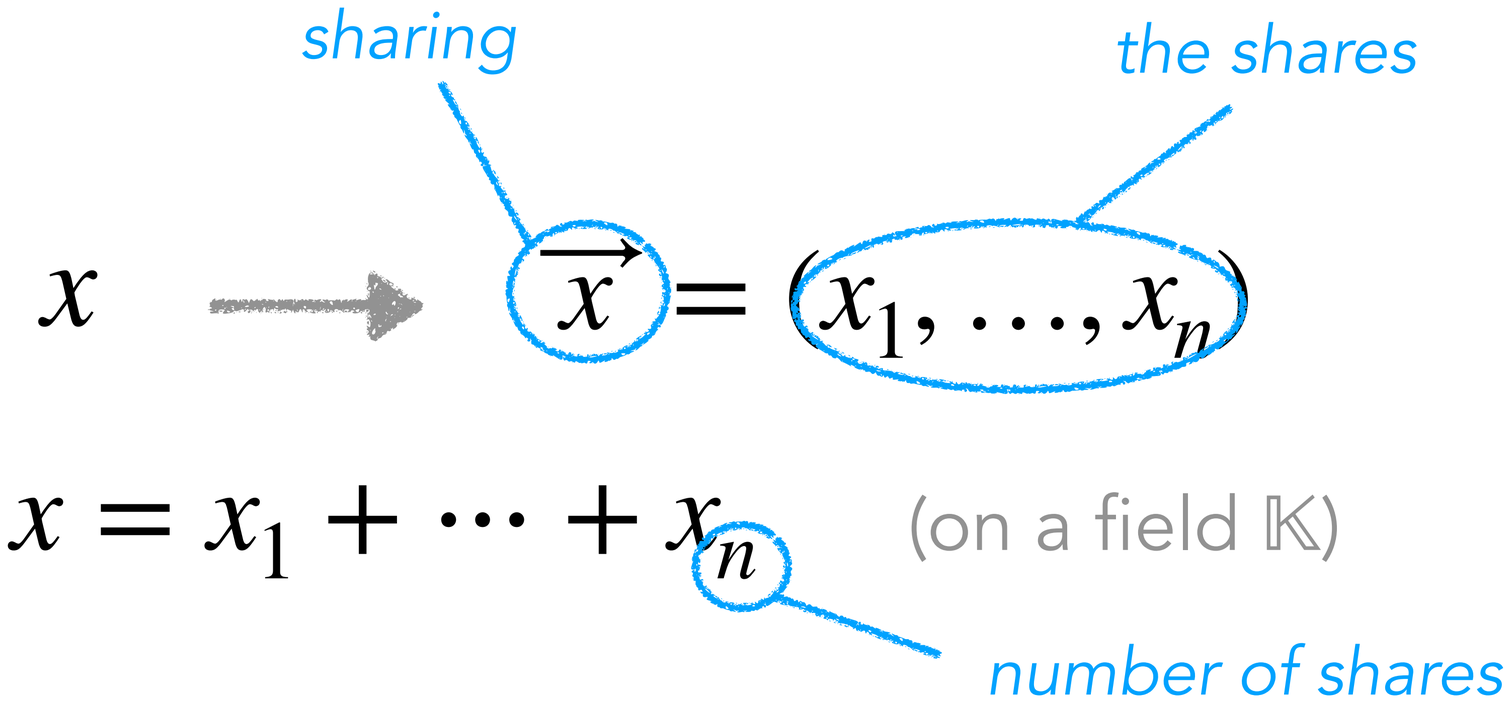
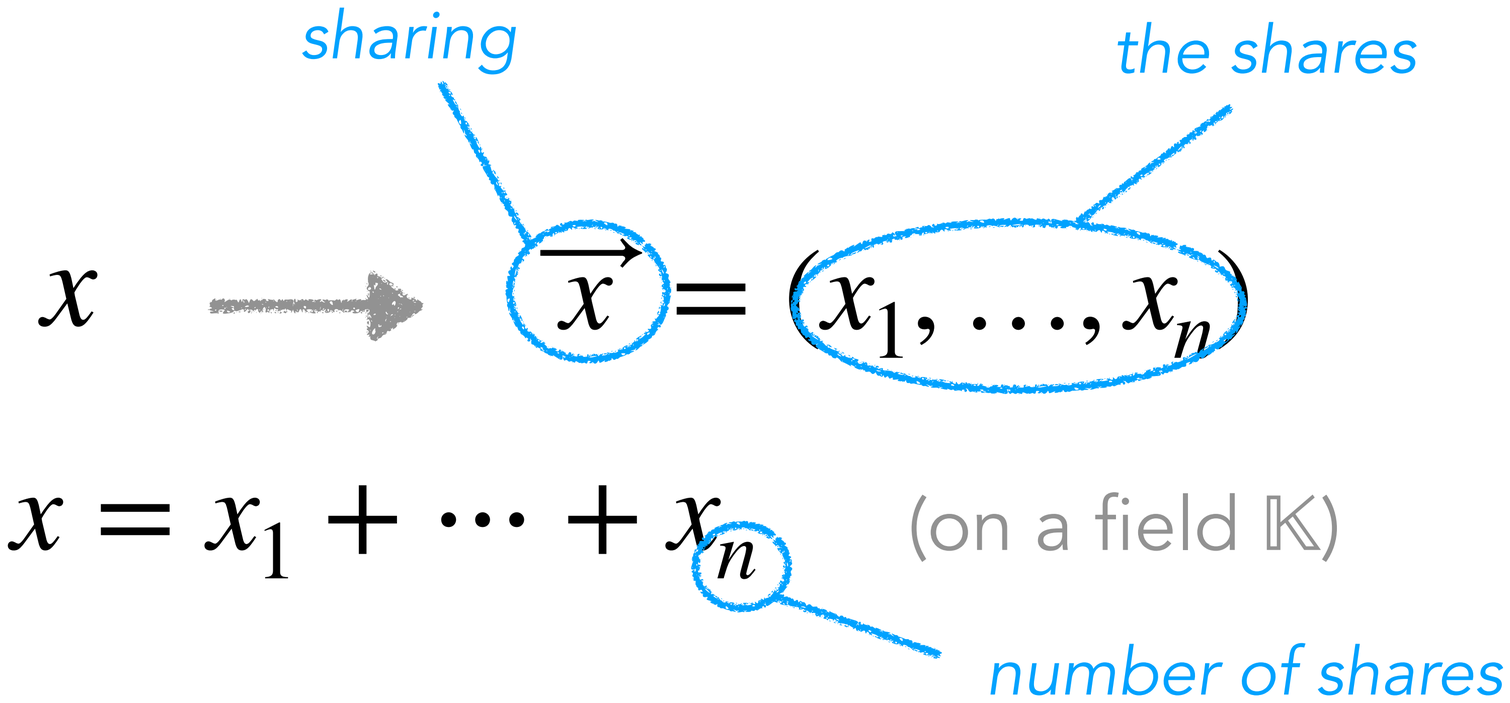


Masked computation

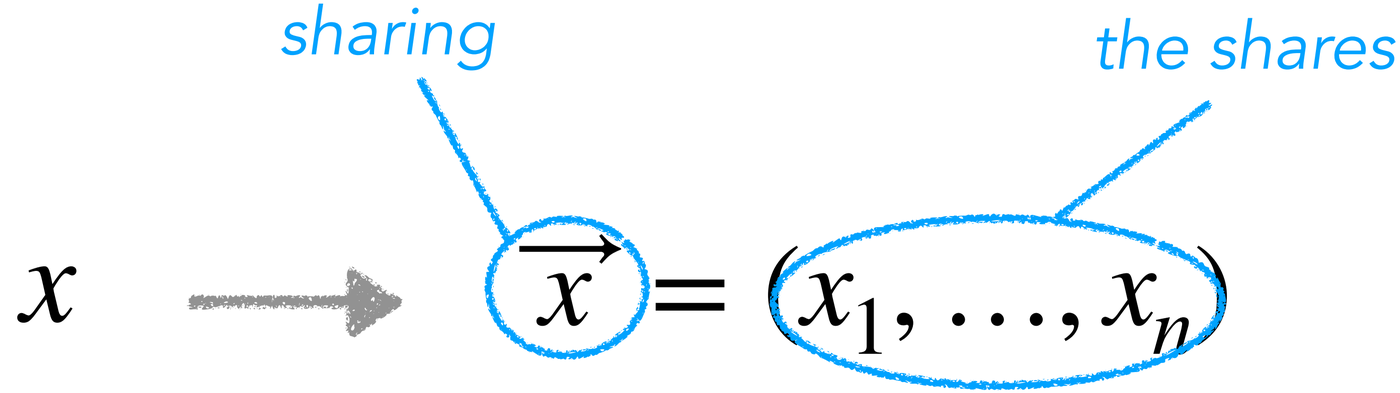
Masking


sharing
the shares
$x \longrightarrow \vec{x} = (x_1, \ldots, x_n)$
$x = x_1 + \cdots + x_n$
(on a field $\mathbb{K}$)
number of shares

Masking


sharing
the shares
$x \longrightarrow \vec{x} = (x_1, \ldots, x_n)$
$x = x_1 + \cdots + x_n$
(on a field $\mathbb{K}$)
number of shares

Masking



$$x = x_1 + \cdots + x_n$$ (on a field $\mathbb{K}$)

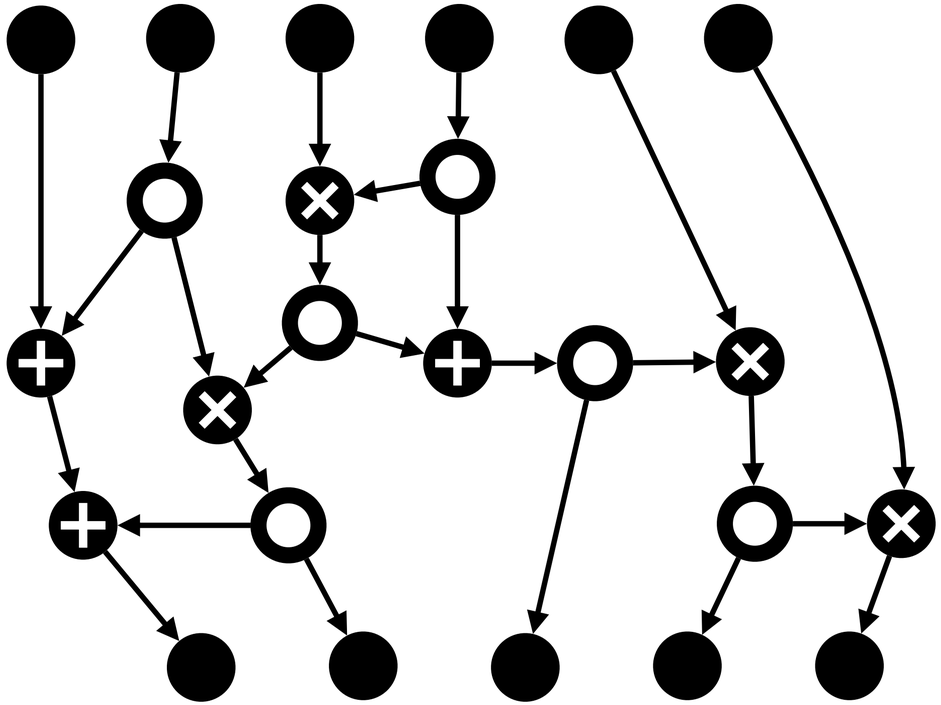
number of shares

⚠️ all the shares are necessary to recover x

🎲 any $n - 1$ shares are completely random

Circuit model

Crypto computation modelled as an arithmetic circuit on $\mathbb{K}$



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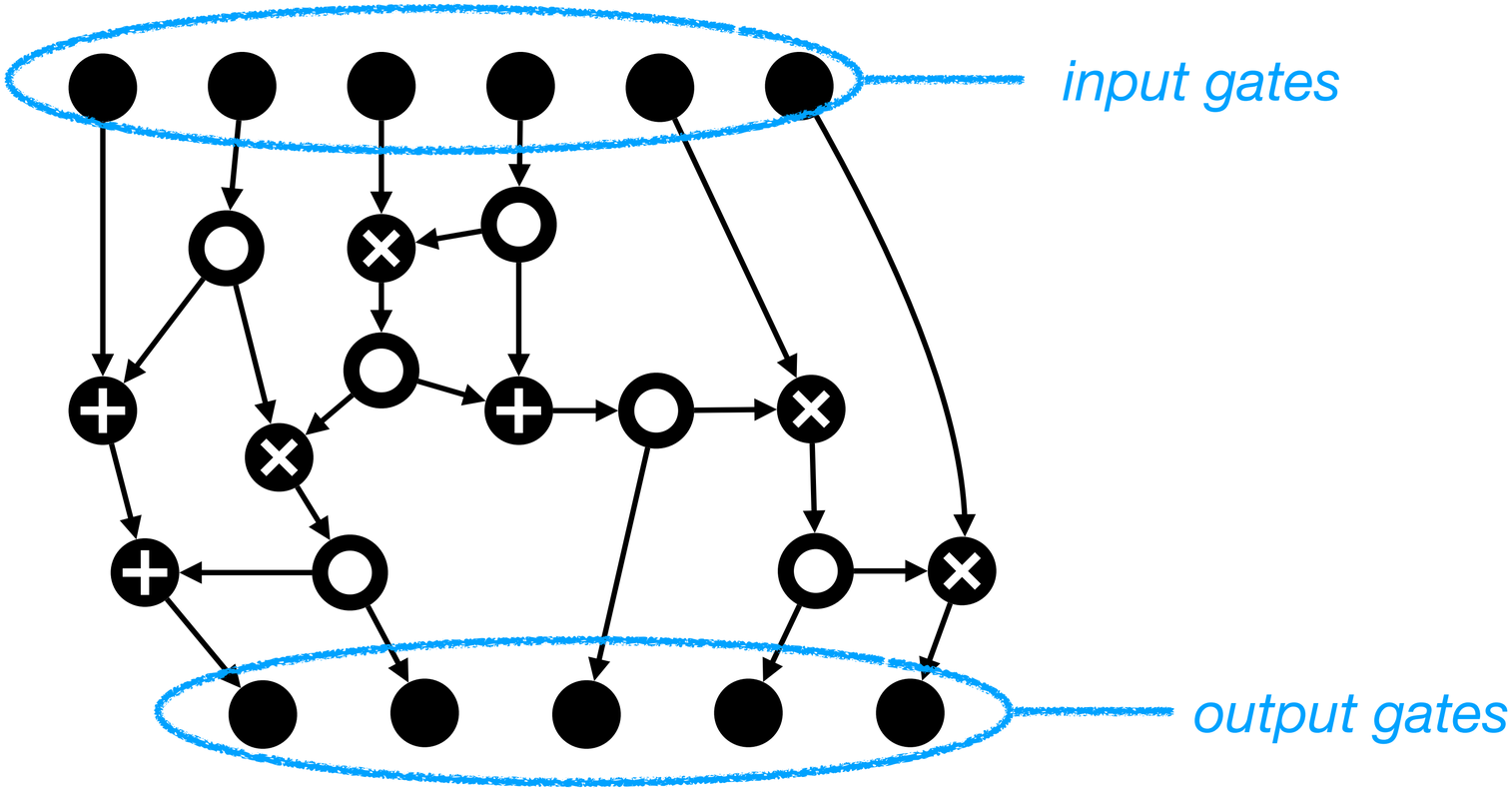

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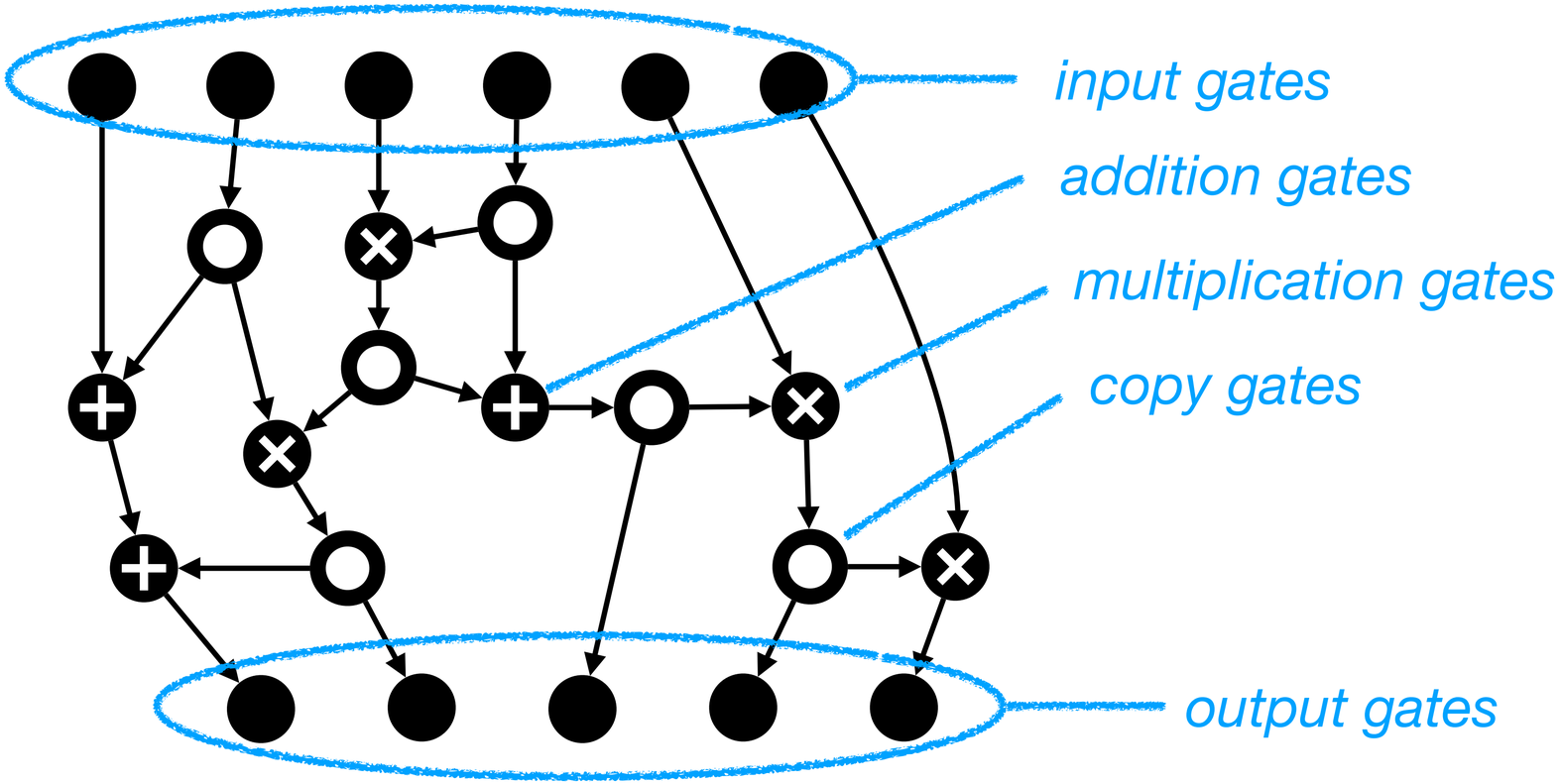

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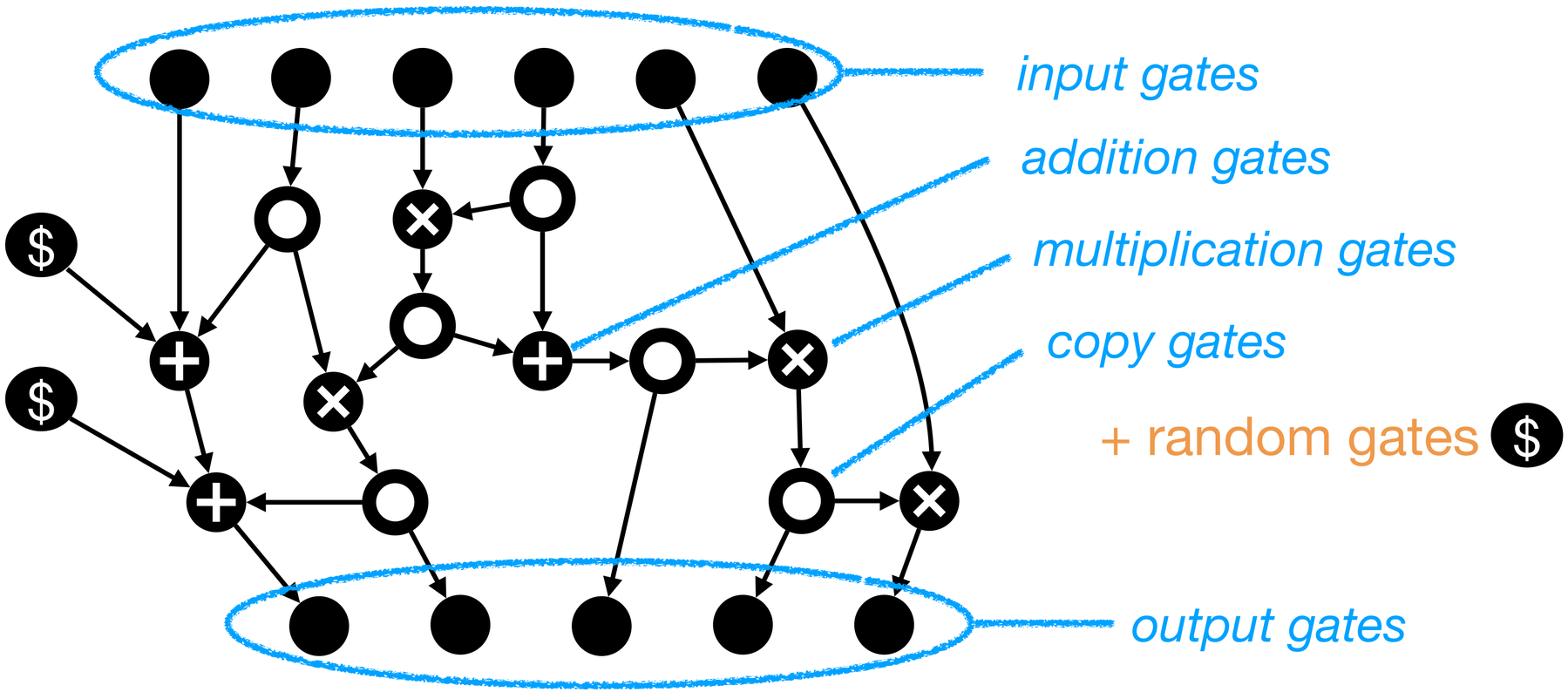

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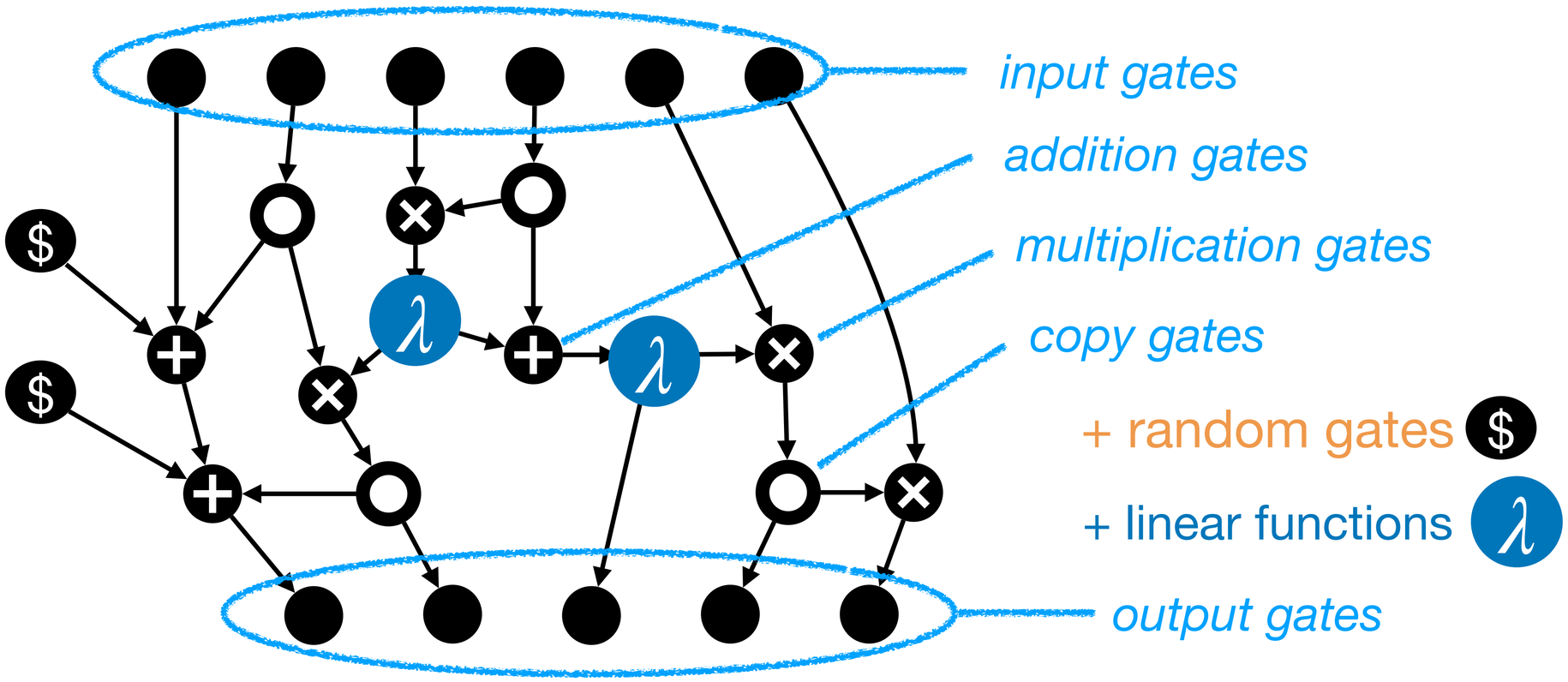
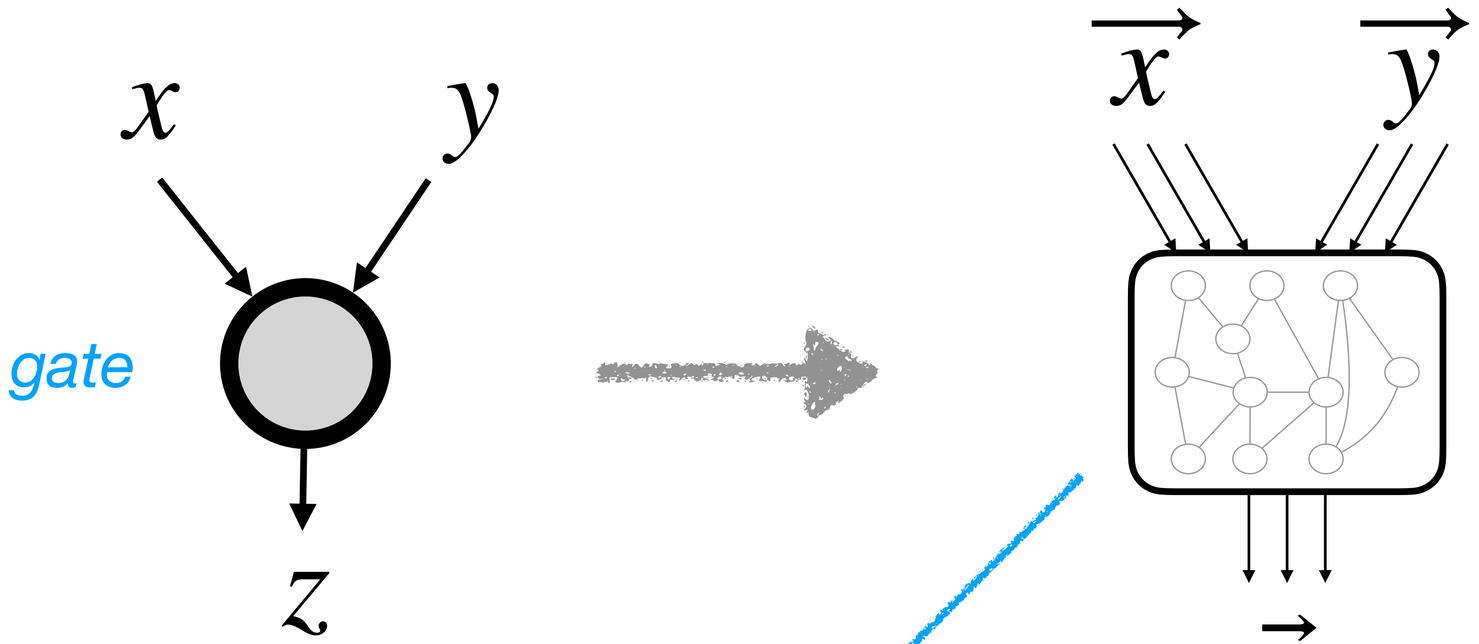
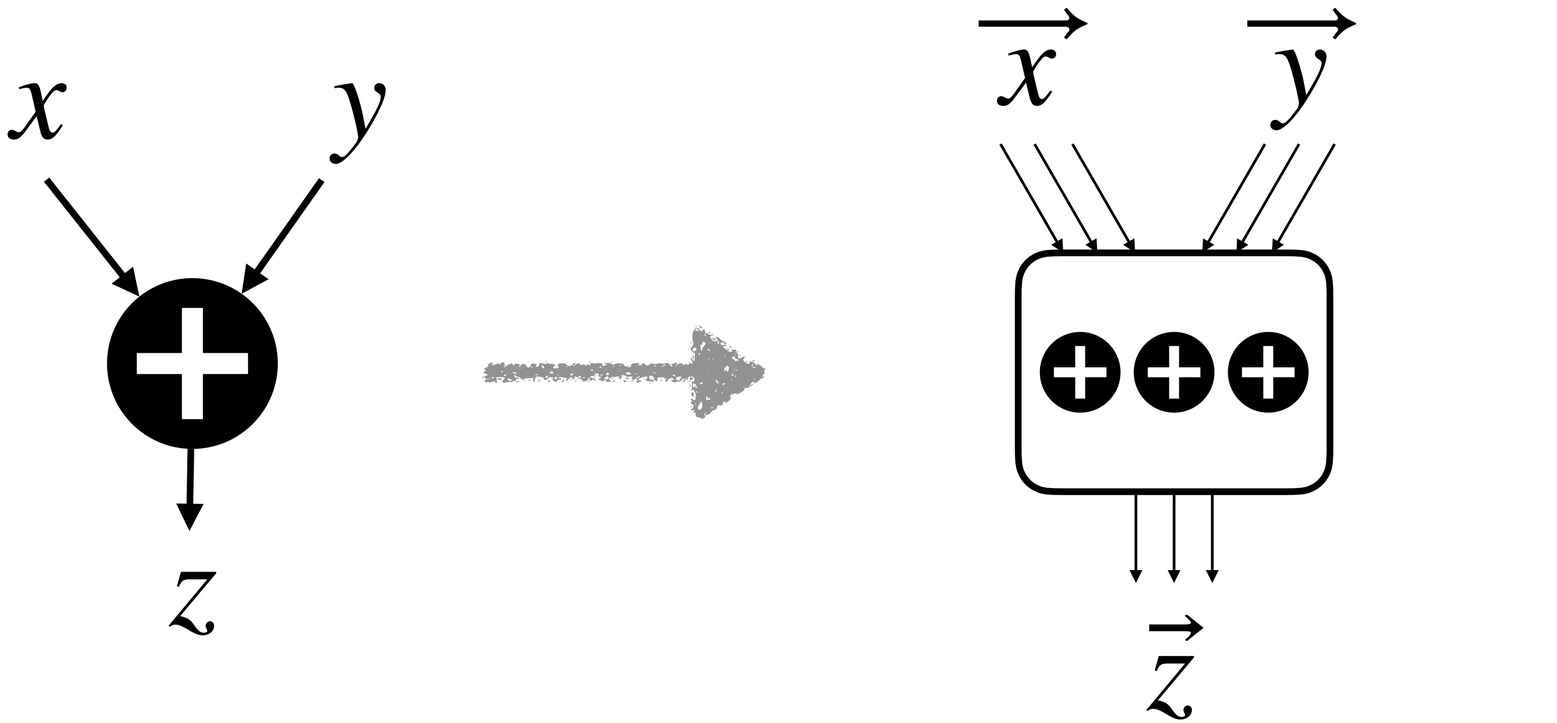


Gadgets



gadget : small circuit computing an operation on sharings

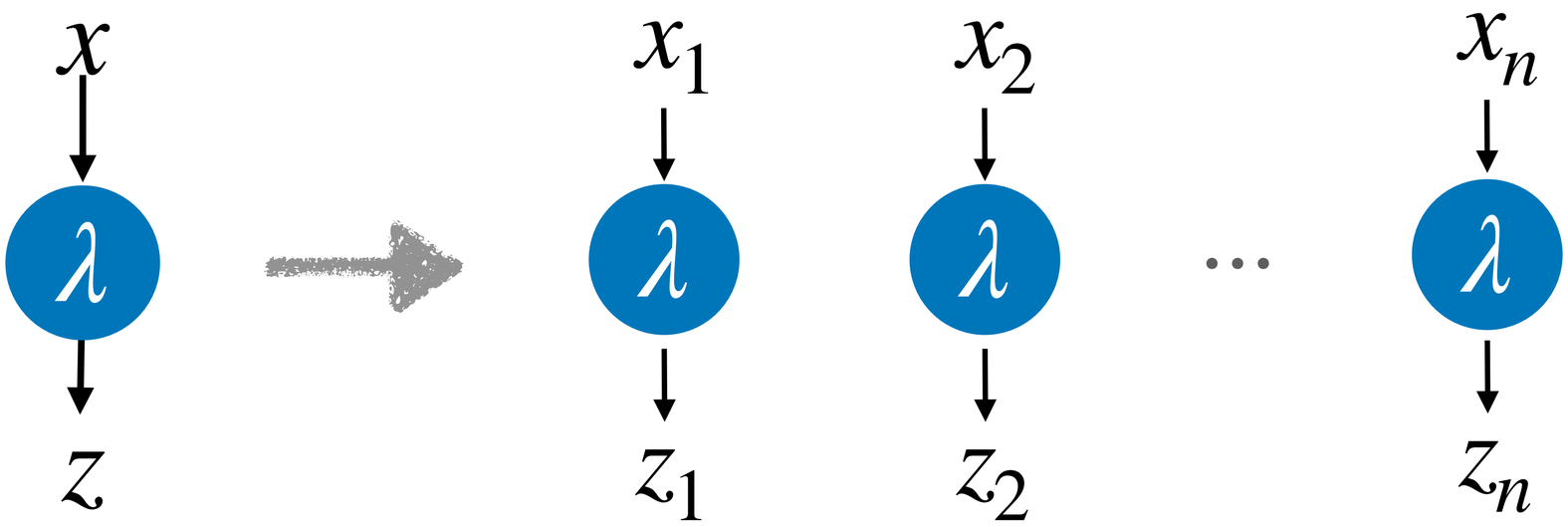
Adition gadget



sharewise computation

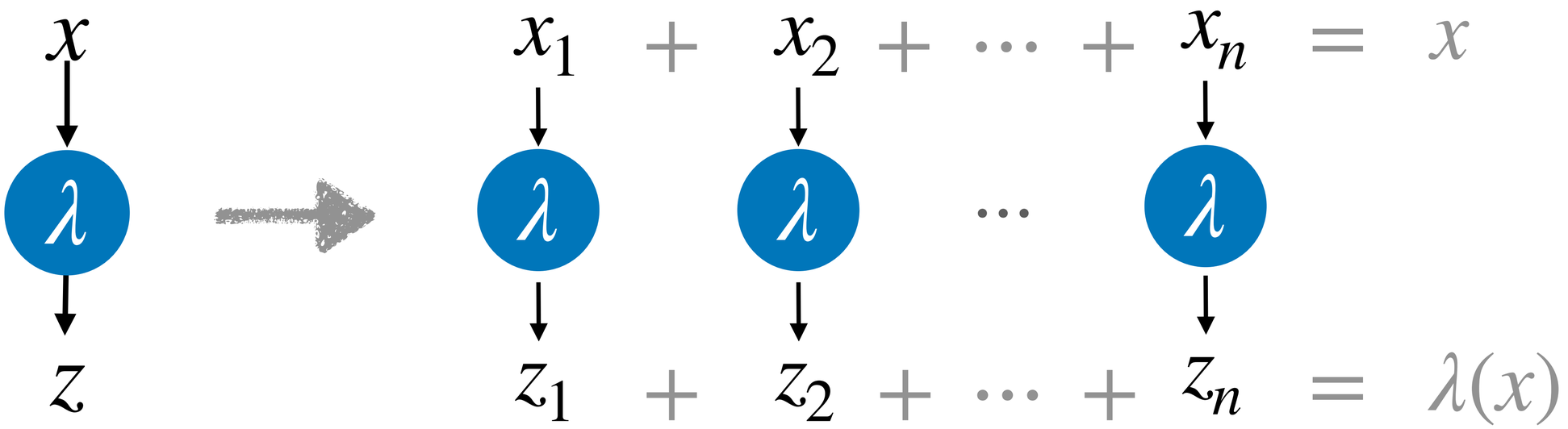
$\Rightarrow$ n addition gates

Linear gadget



sharewise computation ⇒ n evaluations of λ

Linear gadget



sharewise computation ⇒ n evaluations of λ

Multiplication gadget

$$z = x \cdot y = \left(\sum\nolimits_i x_i\right)\left(\sum\nolimits_i y_i\right) = \sum\nolimits_{i,j} x_i y_j$$

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$$z = x \cdot y = \left(\sum_i x_i\right)\left(\sum_i y_i\right) = \sum_{i,j} x_i y_j$$

split into n shares

$$z_1$$
$$z_2$$
$$\vdots$$
$$z_n$$

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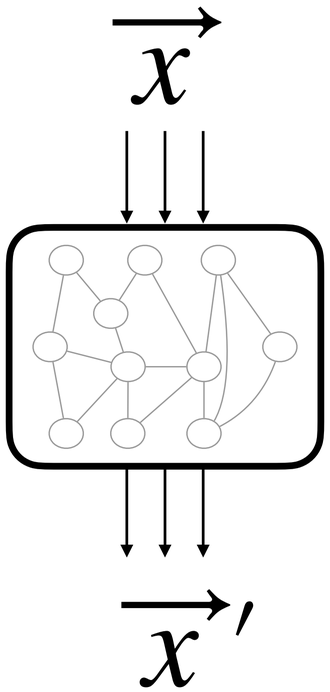
$$z = x \cdot y = \left(\sum_i x_i\right)\left(\sum_i y_i\right) = \sum_{i,j} x_i y_j$$

split into n shares

$$z_1$$
$$z_2$$
$$\vdots$$
$$z_n$$

+ fresh randomness

Refresh gadget



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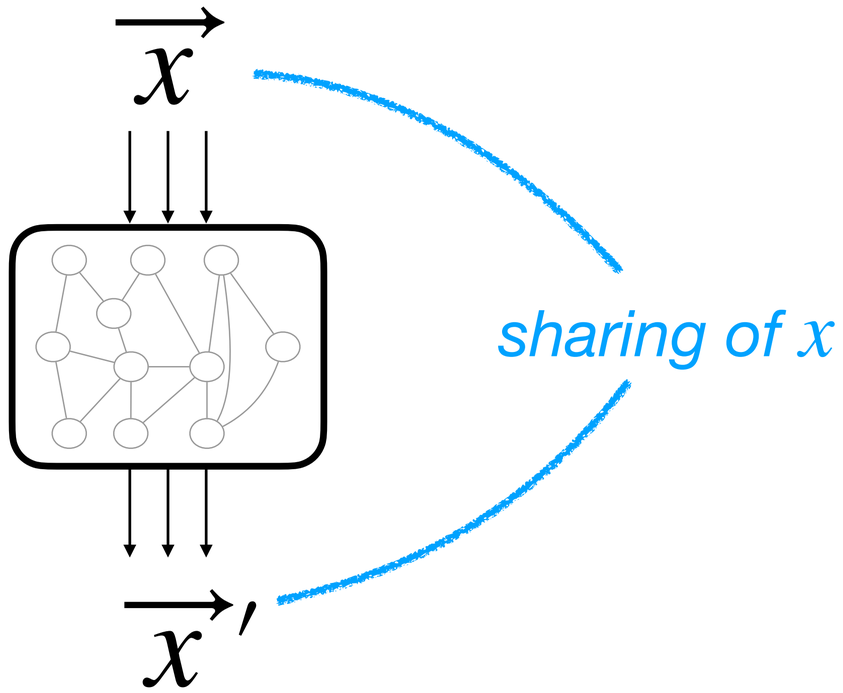

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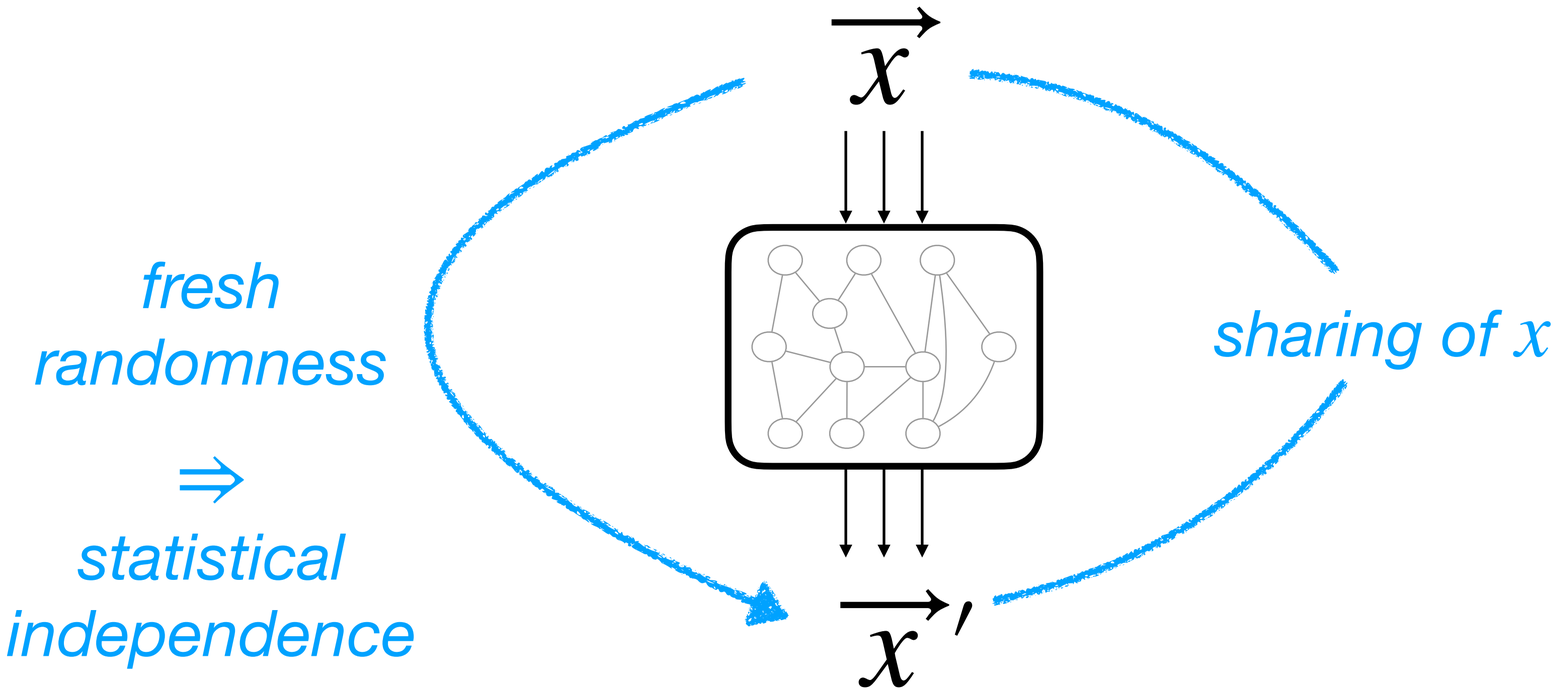
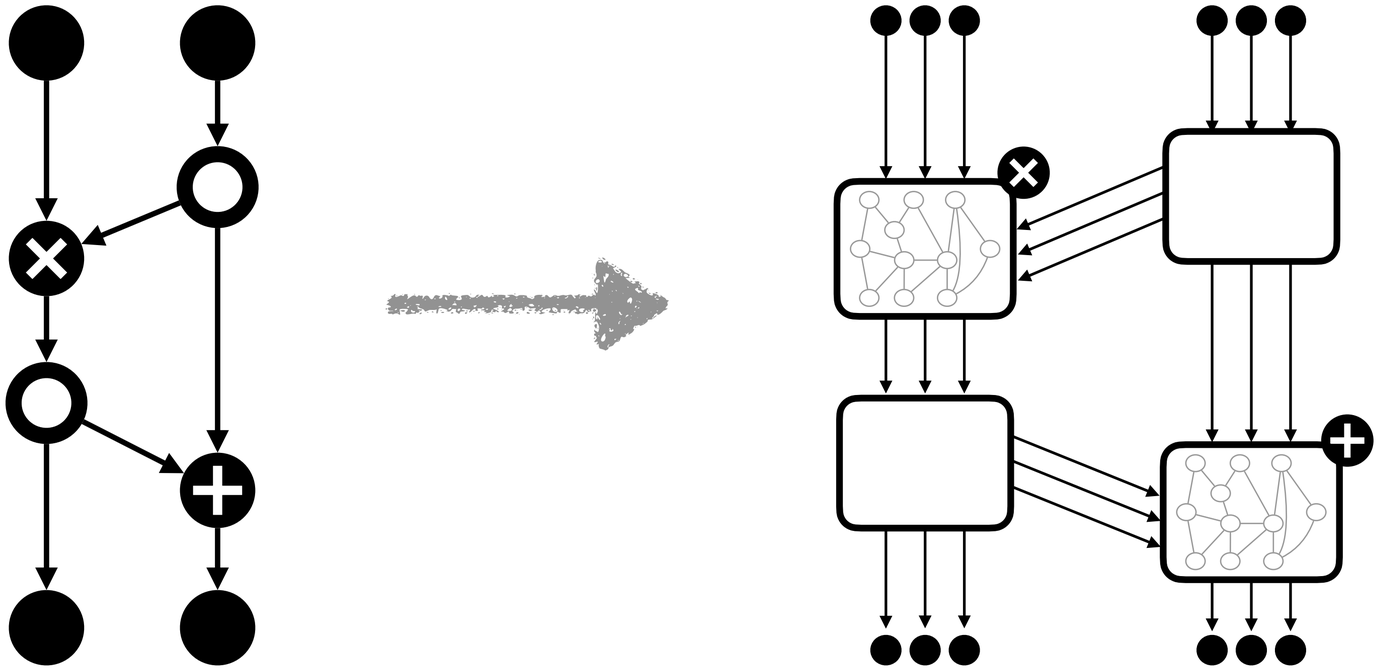


Standard circuit compiler

wire $\rightarrow$ n wires (sharing)

gate $\rightarrow$ gadget



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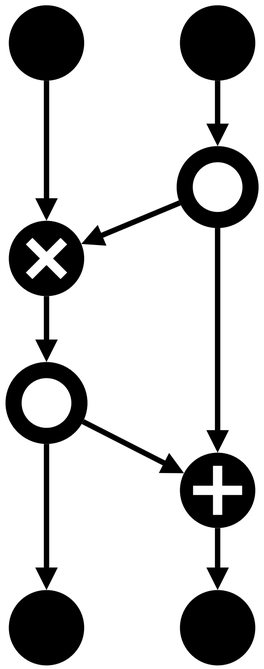
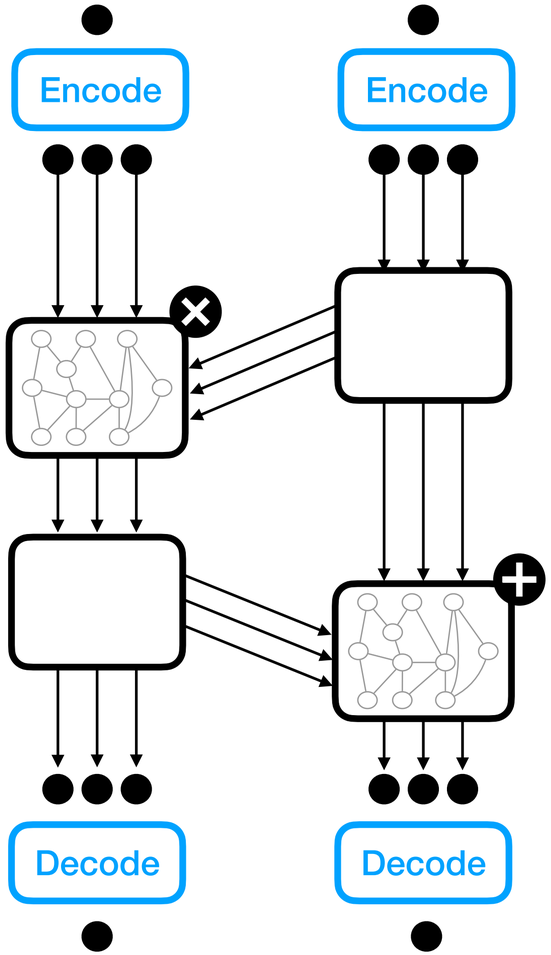


functional equivalence



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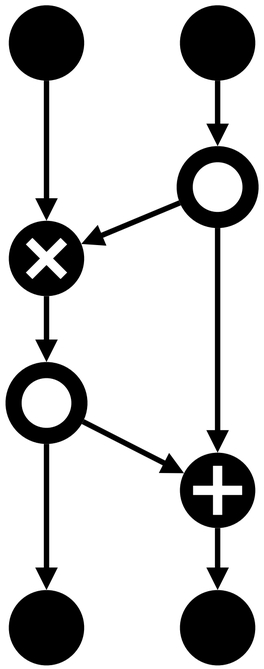

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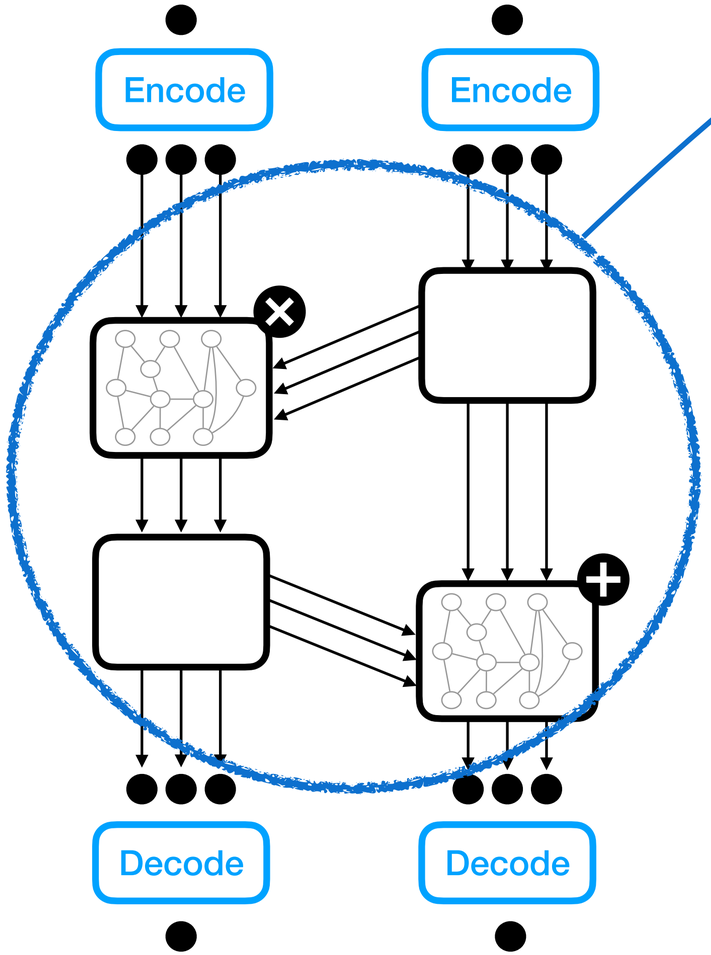
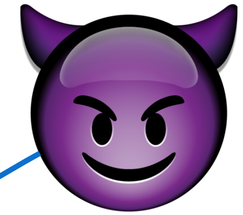
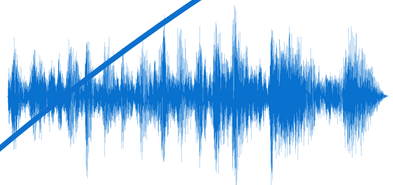




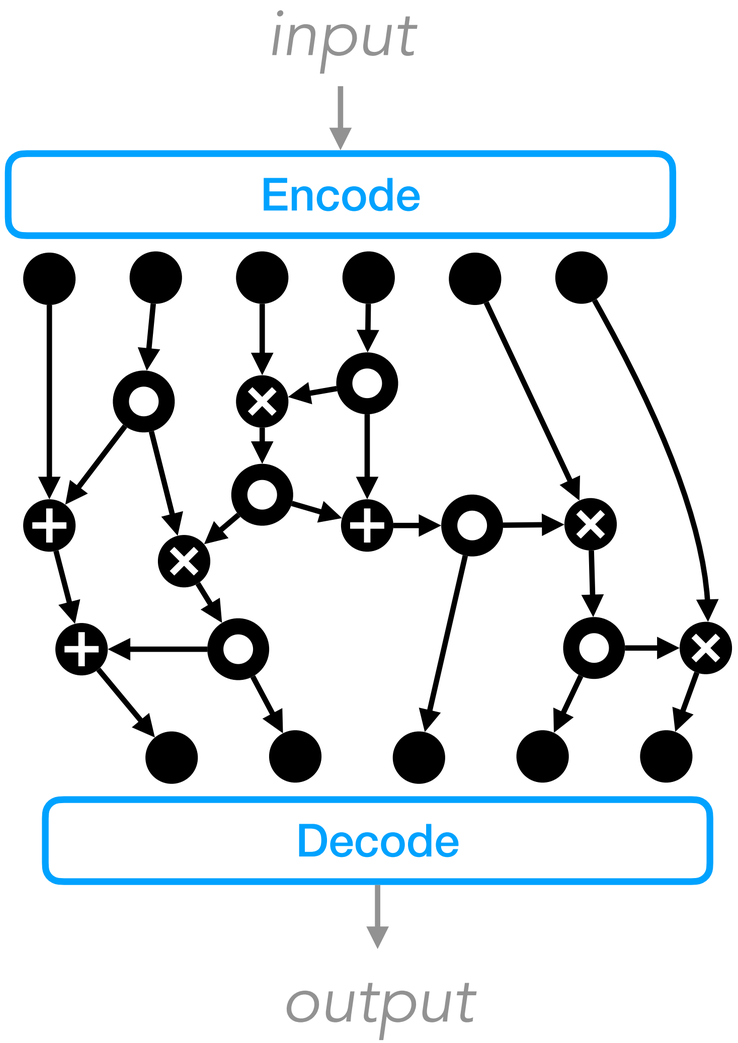


noisy leakage

How to prove the security vs. δ-noisy leakage?

Probing models

Probing model



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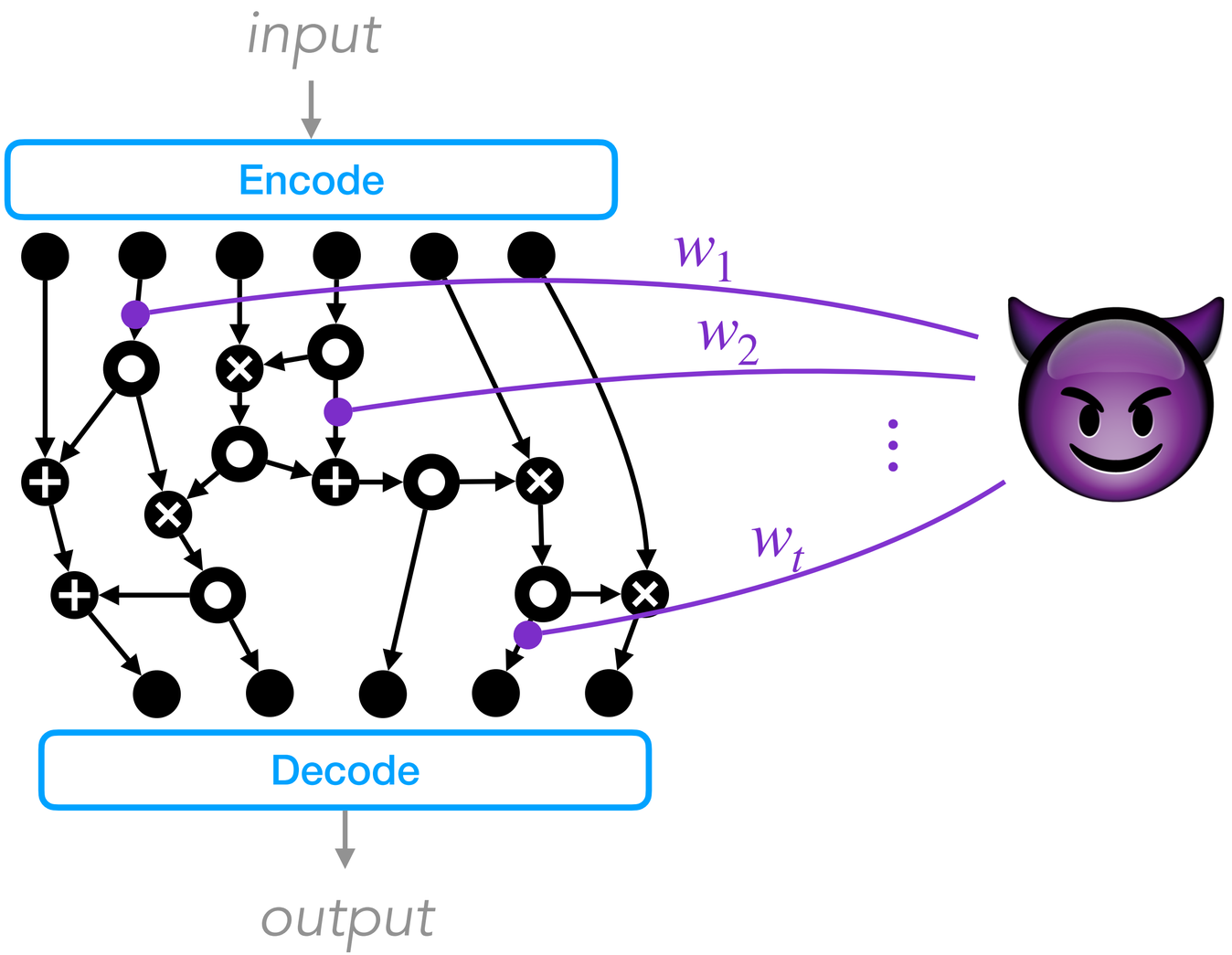
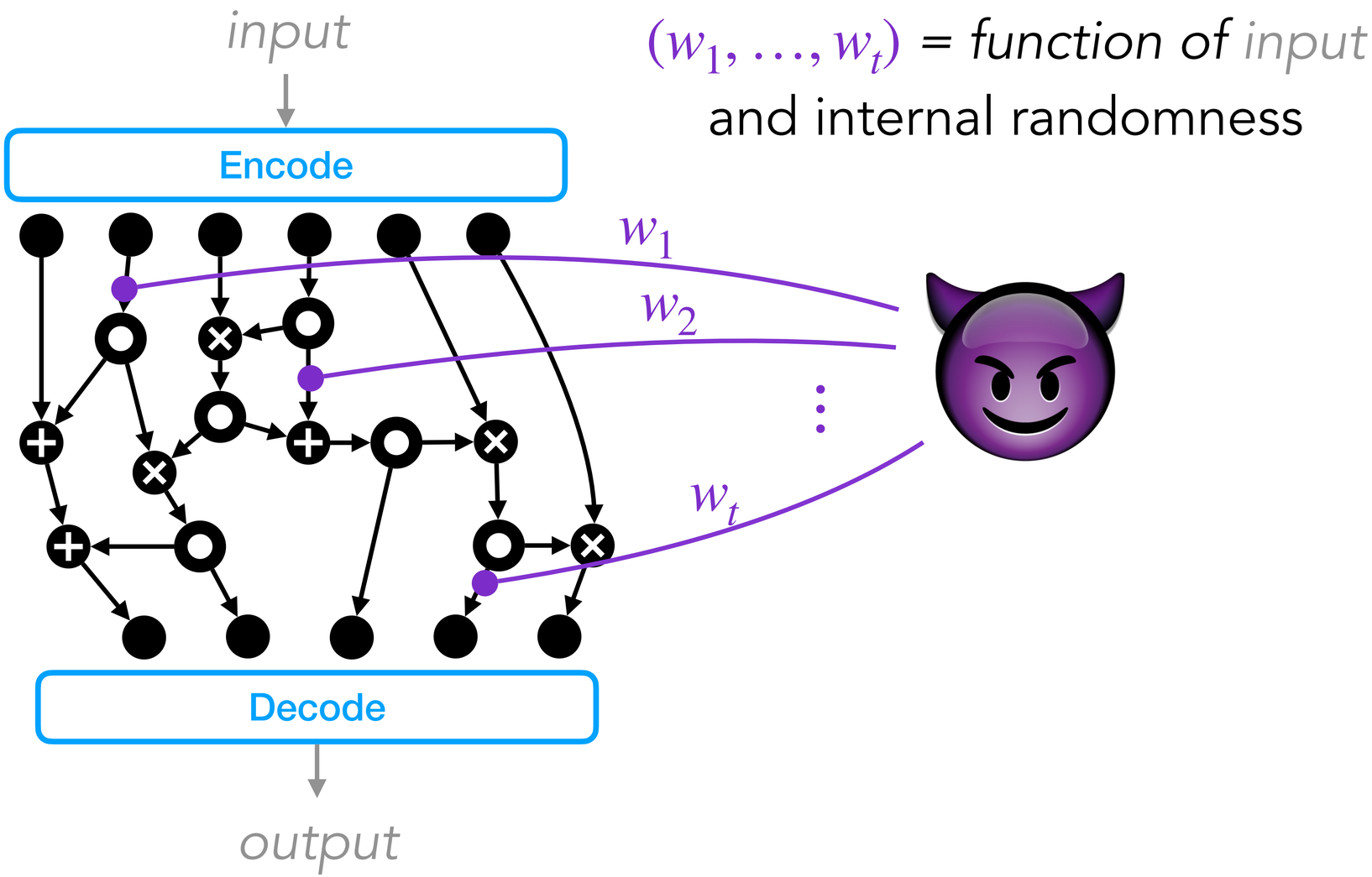
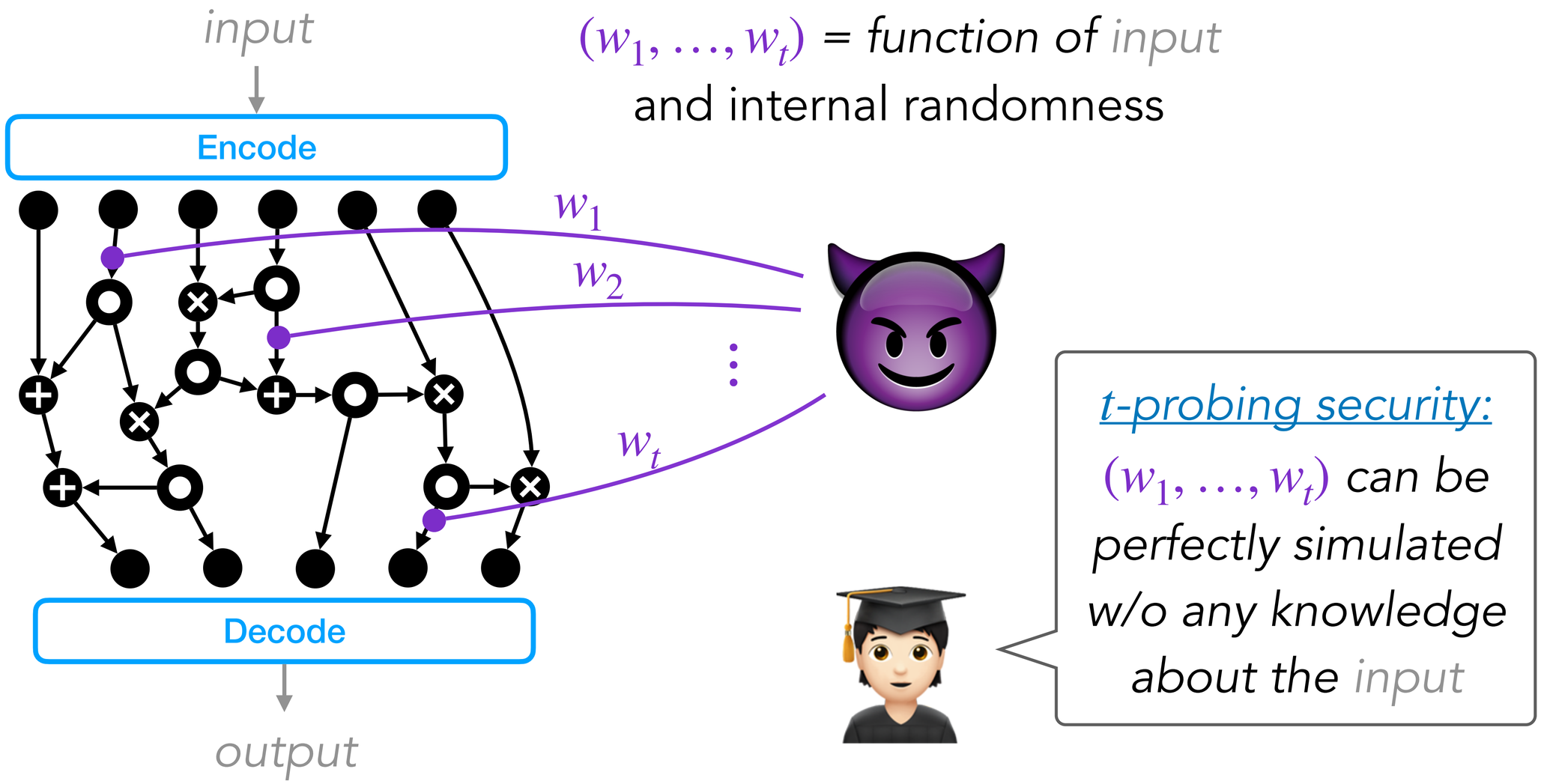


Probing model



$(w_1, \ldots, w_t)$ = *function of input*
and internal randomness

Probing model



$(w_1, \ldots, w_t)$ = *function of input* and internal randomness

t-probing security:

$(w_1, \ldots, w_t)$ *can be perfectly simulated w/o any knowledge about the input*

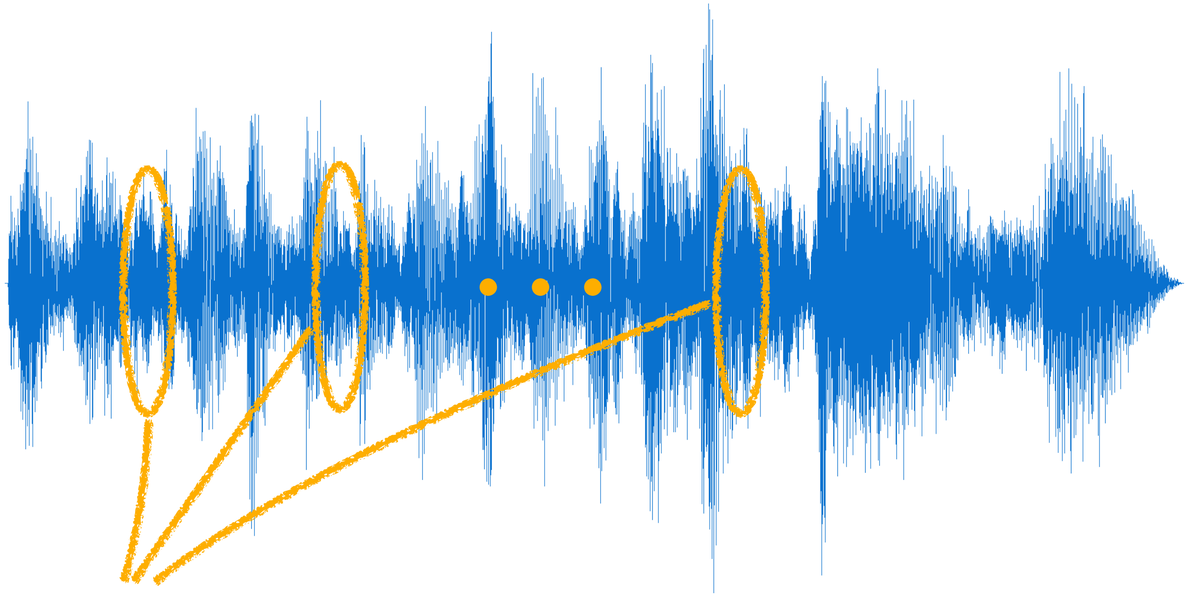
Probing model

t-probing security

⇔

security against

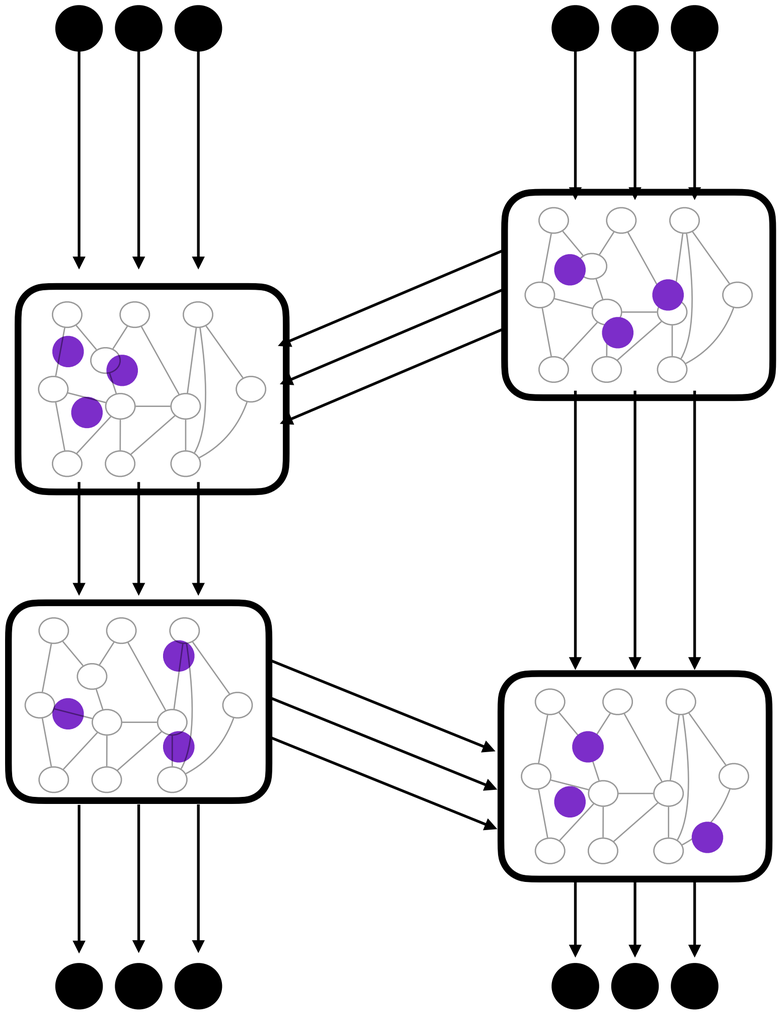
t-order DPA



any t leakage points

independent of the secrets

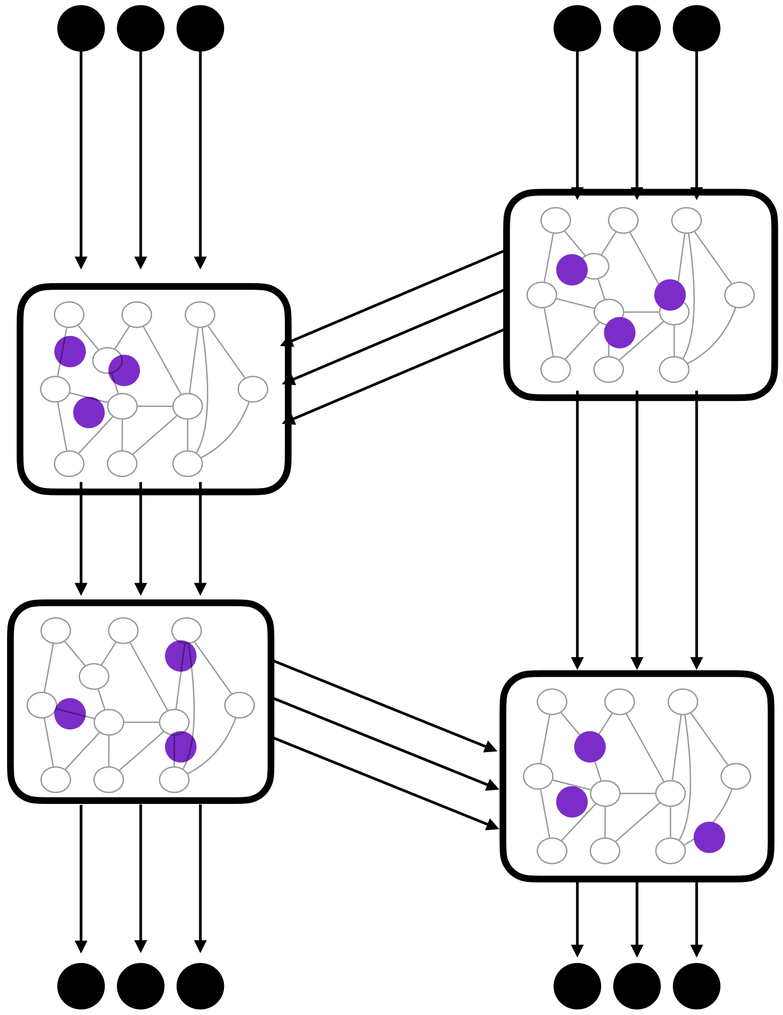
Region probing model



t probes per gadget (or region)

with $t = r \times |G|$

Region probing model



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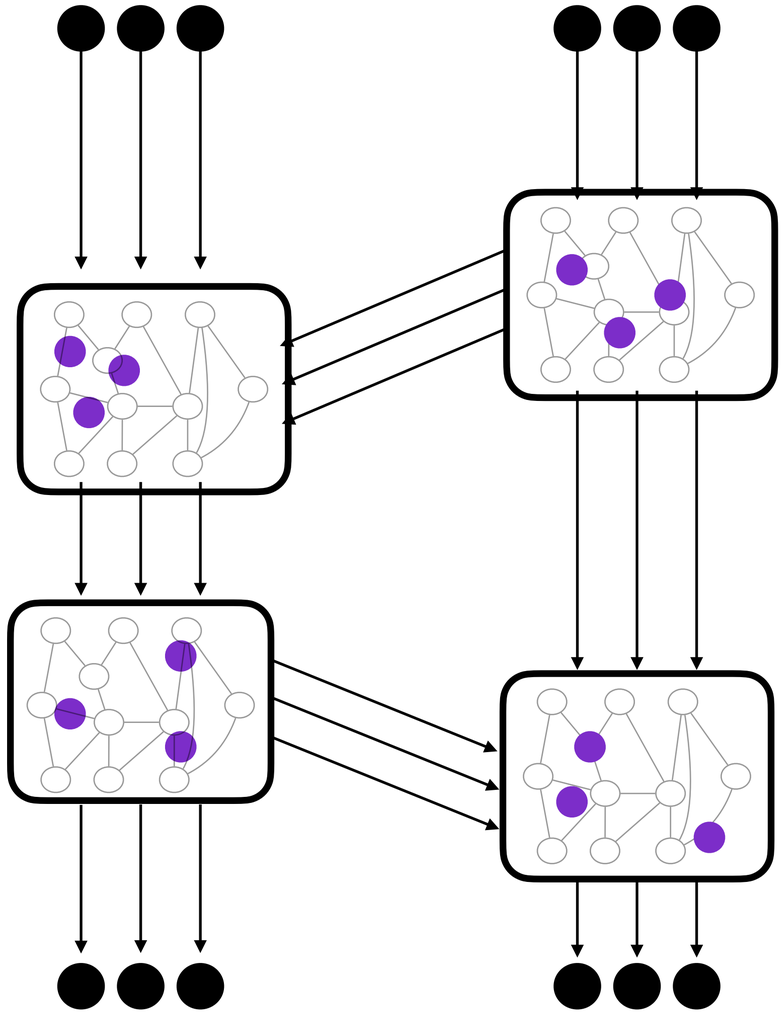
probing rate

number of wires in G

Region probing model



t probes per gadget (or region)

$$\text{with } t = r \times |G|$$

probing rate

number of wires in G

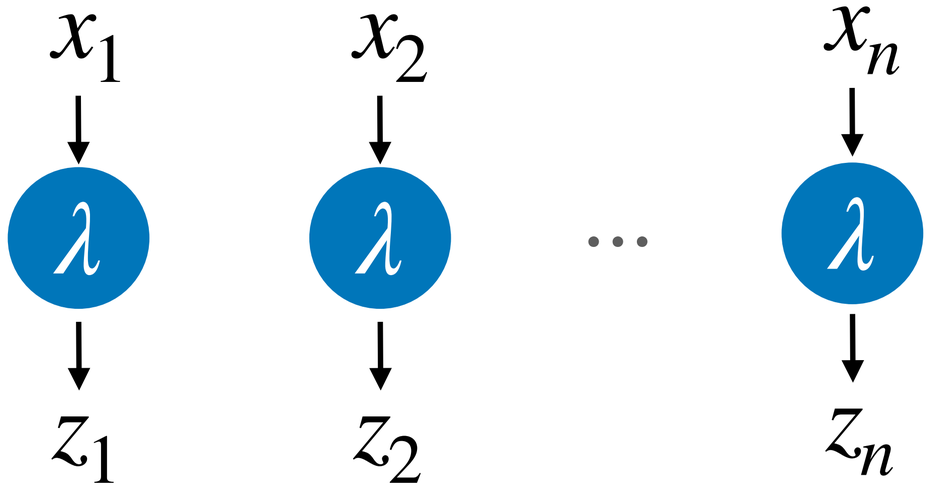
$\Rightarrow$ *r-region probing security*

Security of sharewise gadgets

sharewise gadget
⇒ inherent probing security

$n-1$ probes

= completely random 🎲



ISW multiplication gadget

$$\begin{pmatrix} x_1y_1 & x_1y_2 & x_1y_3 \\ & x_2y_2 & x_2y_3 \\ & & x_3y_3 \end{pmatrix} + \begin{pmatrix} & & \\ x_2y_1 & & \\ x_3y_1 & x_3y_2 & \end{pmatrix}^{\mathsf{T}}$$

ISW multiplication gadget

cross-products $\sum_{i,j} x_i y_j$

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fresh randomness (cancelling out)

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row sum →

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

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row sum → $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$

t probes ⇒ info on at most t shares

Composition security

⚠️ probing security for gadgets $\not\Rightarrow$ global (region) probing security

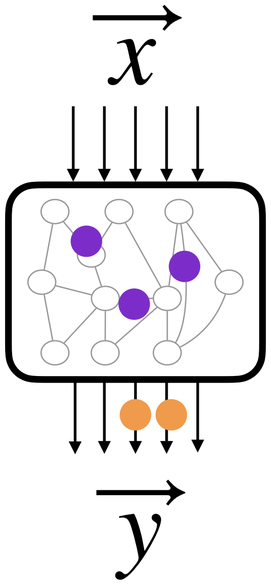
💡 composition security notions

Composition security

⚠️ probing security for gadgets $\not\Rightarrow$ global (region) probing security

💡 composition security notions

Example: strong non-interference (SNI)



t_1 *internal probes*

t_2 *output probes*

Composition security

⚠️ probing security for gadgets $\not\Rightarrow$ global (region) probing security

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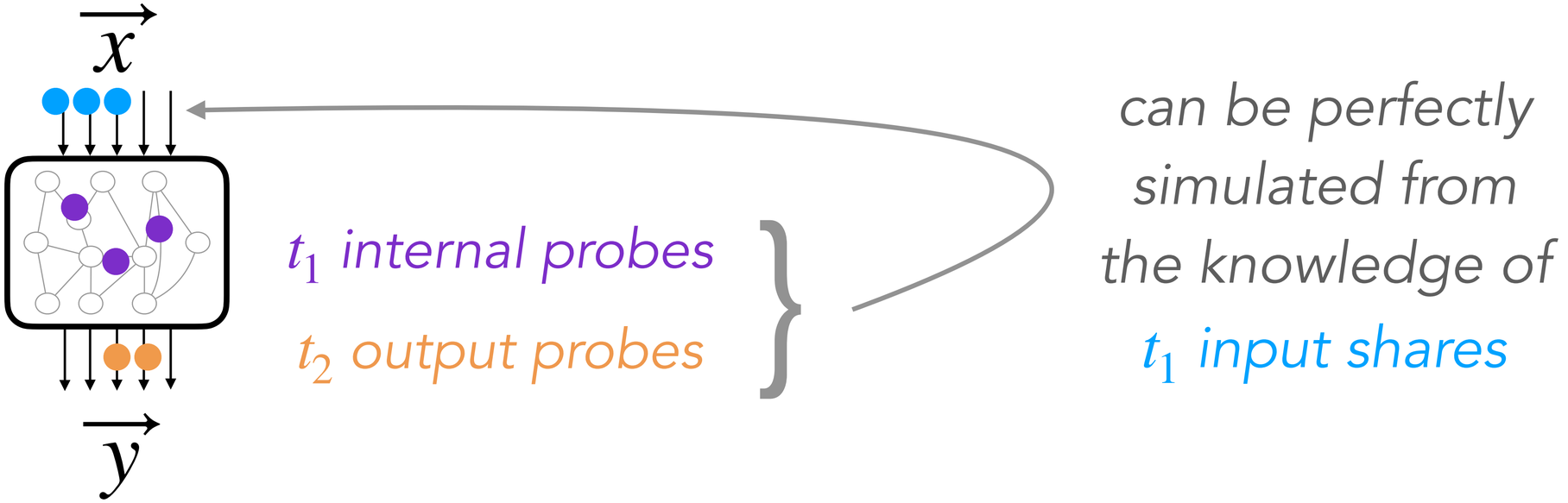
Example: strong non-interference (SNI)



Composition security

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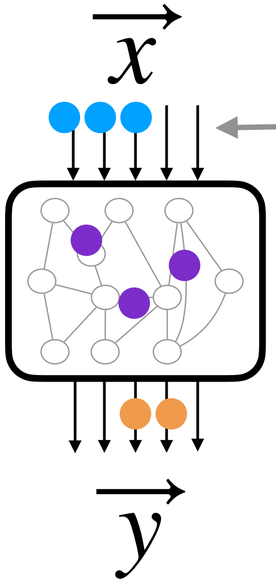


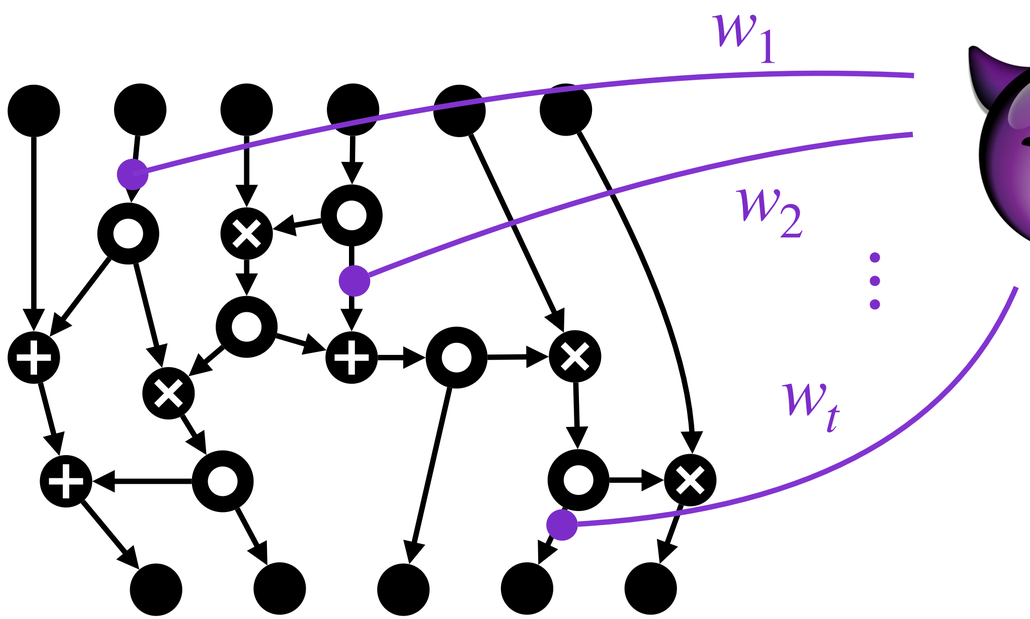
t_1 *internal probes*

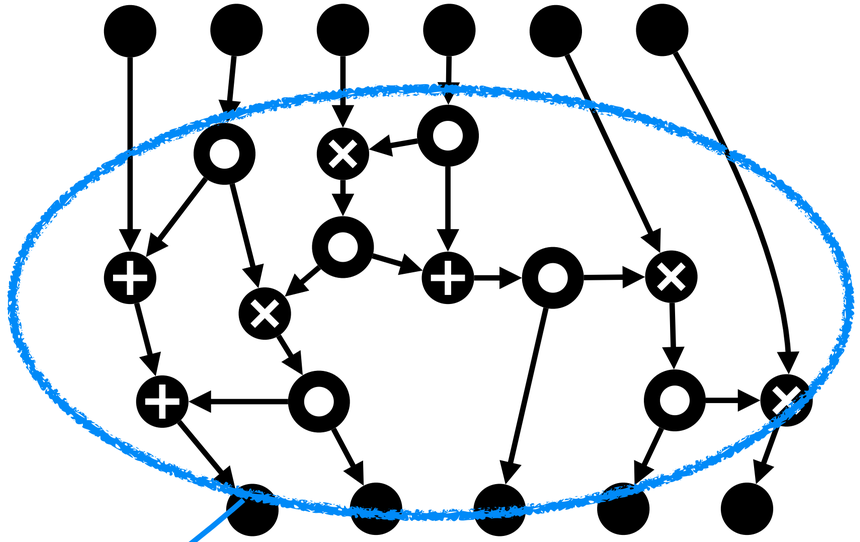
t_2 *output probes*

can be perfectly simulated from the knowledge of t_1 *input shares*

SNI gadgets ⇒ global region probing security

But… wait a minute!

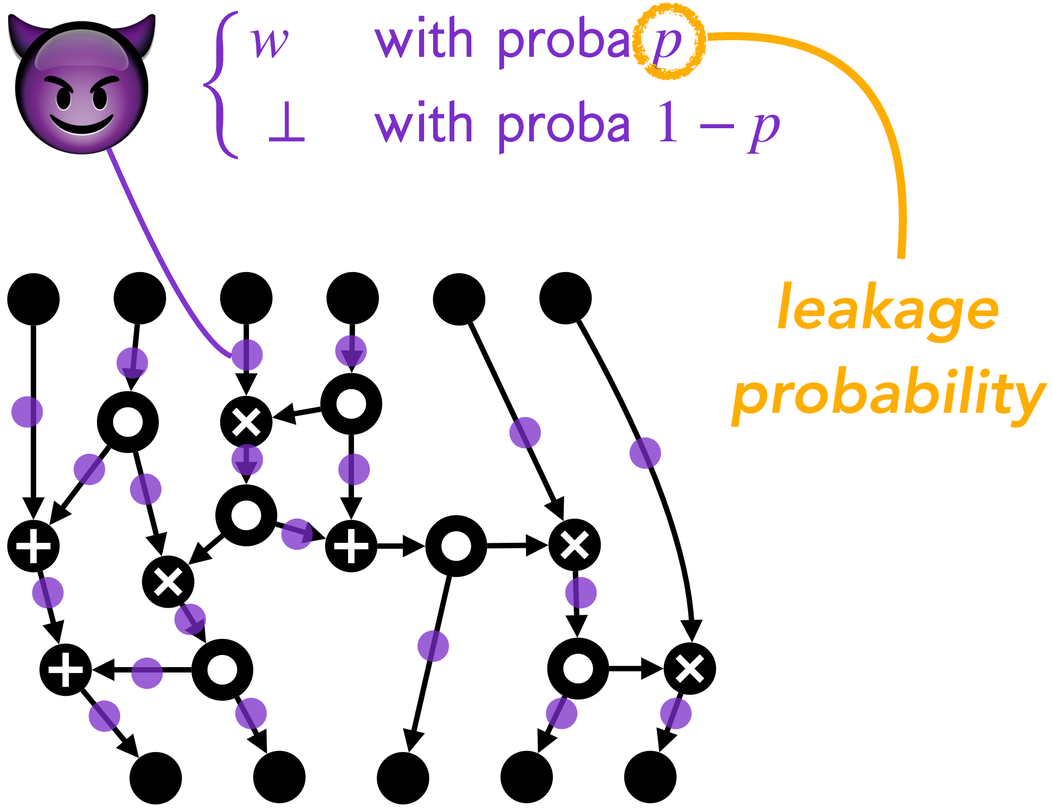




How do I get from probing security …

… to noisy leakage security?!

Random probing model



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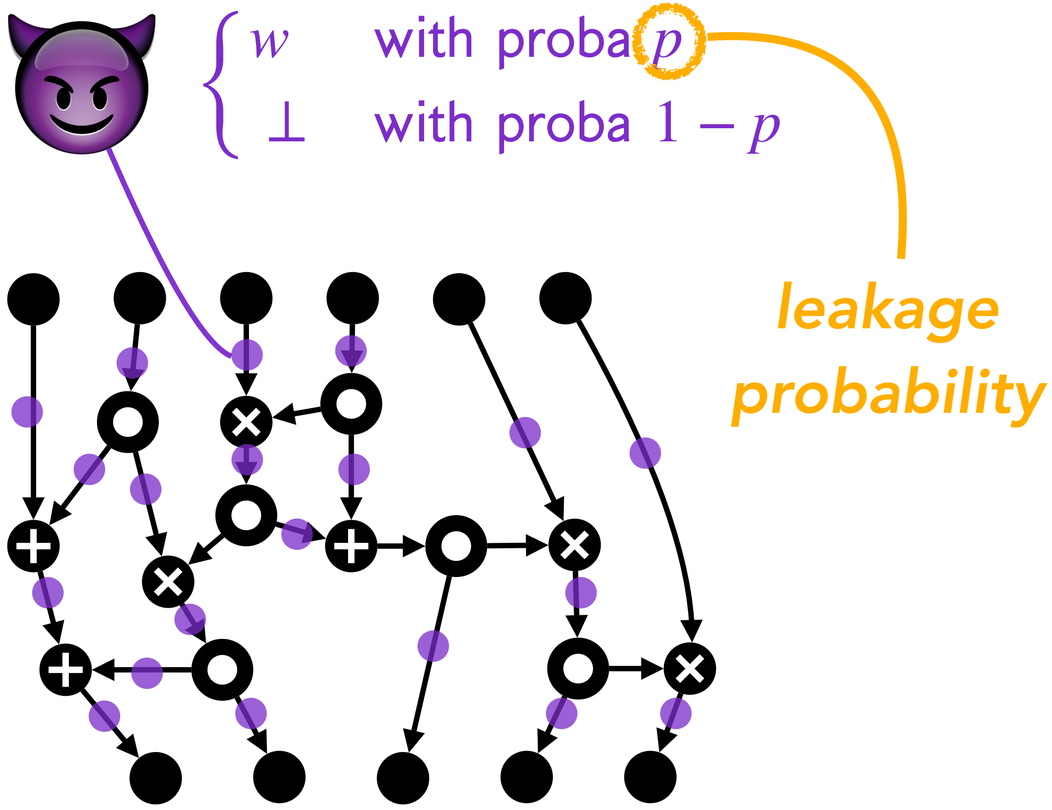
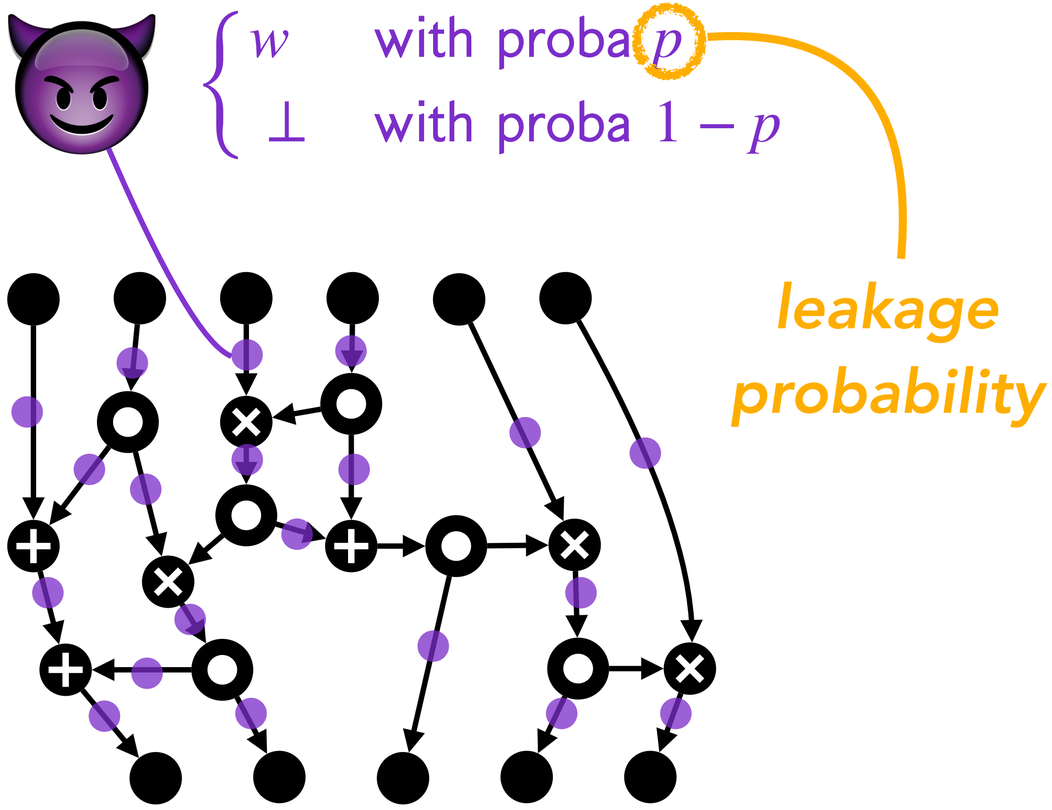


In average $p \cdot |G|$ leaking wires per gadget

Random probing model

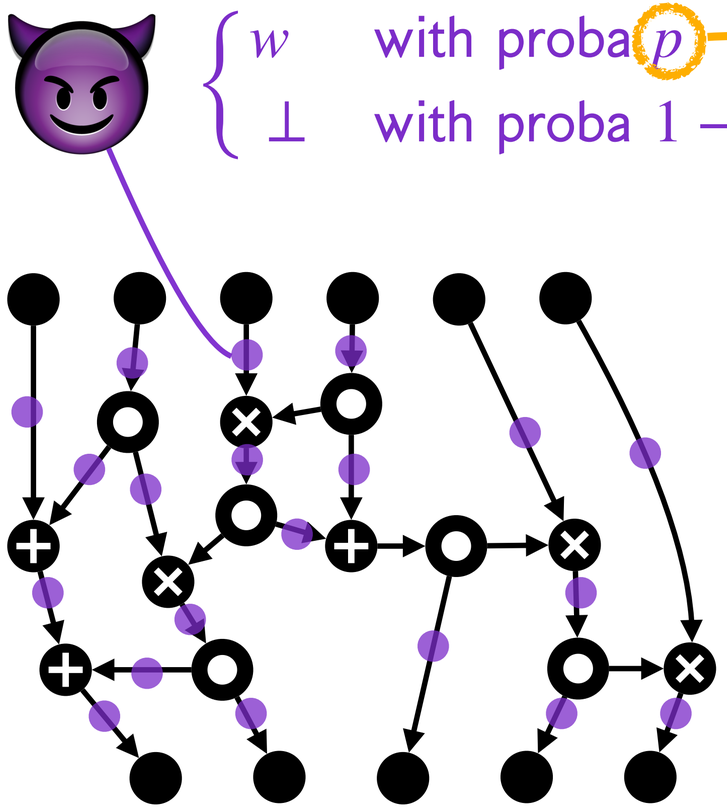


In average $p \cdot |G|$ leaking wires per gadget

Chernoff bound:

$< 2\,p\,|G|$ leaking wires with overwhelming probability

Random probing model



$\begin{cases} w & \text{with proba } p \\ \perp & \text{with proba } 1-p \end{cases}$

leakage probability

In average $p \cdot |G|$ leaking wires per gadget

Chernoff bound:

$< 2\,p\,|G|$ leaking wires with overwhelming probability

r-region probing security

$\Rightarrow$ p-random probing security

with $p = \Theta(r)$

Unifying probing and noisy models

Key lemma:

If f is δ-noisy, then $\exists\, f'$ s.t.

$$f(x) = f'(\phi(x))$$

where

$$\phi(x) := \begin{cases} x & \text{with proba } p \leq \delta \cdot |\mathcal{X}| \\ \perp & \text{with proba } 1 - p \end{cases}$$

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⇒ *δ-noisy leakage can be simulated from p-random probing leakage*

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Random probing leakage

$\phi(w_1),\ \phi(w_2),\ \ldots\ ,\ \phi(w_N)$

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⇒ *δ-noisy leakage can be simulated from p-random probing leakage*

Random probing leakage

$$\phi(w_1),\ \phi(w_2),\ \ldots\ ,\ \phi(w_N)$$

Apply $f_1', \ldots, f_N'$

Noisy leakage

$$f_1(w_1),\ f_2(w_2),\ \ldots\ ,\ f_N(w_N)$$

Unifying probing and noisy models

Key lemma:

If f is δ-noisy, then $\exists\, f'$ s.t.

$$f(x) = f'(\phi(x))$$

where

$$\phi(x) := \begin{cases} x & \text{with proba } p \leq \delta \cdot |\mathcal{X}| \\ \perp & \text{with proba } 1-p \end{cases}$$

p-random probing security
$\Rightarrow$ δ-noisy security with $\delta = \Theta(p)$

$\Rightarrow$ *δ-noisy leakage can be simulated from p-random probing leakage*

Random probing leakage

$$\phi(w_1),\ \phi(w_2),\ \ldots\ ,\ \phi(w_N)$$

Apply f_1', …, f_N'

Noisy leakage

$$f_1(w_1),\ f_2(w_2),\ \ldots\ ,\ f_N(w_N)$$

Unifying probing and noisy models

r-region probing security

$\Rightarrow$

p-random probing security

with $p = \Theta(r)$

$\Rightarrow$

δ-noisy leakage security

with $\delta = \Theta(p)$

Unifying probing and noisy models

r-region probing security

$\Rightarrow$

p-random probing security with $p = \Theta(r)$

$\Rightarrow$

δ-noisy leakage security with $\delta = \Theta(p)$

leakage rate

$\begin{cases} 1 = \textit{lot of leakage (low noise)} \\ 0 = \textit{no leakage (infinite noise)} \end{cases}$

Unifying probing and noisy models

r-region probing security

$\Rightarrow$

p-random probing security with $p = \Theta(r)$

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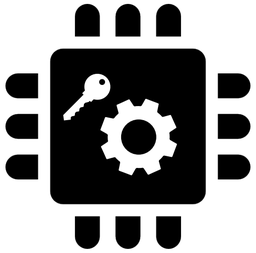

the noise / leakage rate depends on the hardware

Unifying probing and noisy models

r-region probing security

$\Rightarrow$

p-random probing security with $p = \Theta(r)$

$\Rightarrow$

δ-noisy leakage security with $\delta = \Theta(p)$

leakage rate

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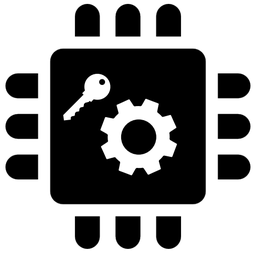

the noise / leakage rate depends on the hardware

efficient masking schemes secure vs. constant (high) leakage rate

Secure schemes

State of the art

- State-of-the-art noisy-leakage-secure schemes
  - most schemes with **at least $\mathcal{O}(n^2)$ complexity**
  - a few schemes with $\mathcal{O}(1)$ leakage rate, but **constant not explicit**
- In what follows
  - region probing security in **quasilinear complexity**
  - random probing security with **explicit constant leakage rate**

Security in quasilinear complexity

Quasilinear masking

A $\overrightarrow{v}$-sharing of x

$$\overrightarrow{x} = (x_0, x_1, \ldots, x_{n-1}) \quad \text{s.t.} \quad \langle \overrightarrow{v}, \overrightarrow{x} \rangle = x$$

Quasilinear masking

A $\overrightarrow{v}$-sharing of x

$$\overrightarrow{x} = (x_0, x_1, \ldots, x_{n-1}) \quad \text{s.t.} \quad \langle \overrightarrow{v}, \overrightarrow{x} \rangle = x = \sum_{i=0}^{n-1} x_i \cdot \omega^i$$

$$\overrightarrow{v} = (1, \omega, \omega^2, \ldots, \omega^{n-1}) \quad \text{for} \quad \omega \xleftarrow{\$} \mathbb{F}$$

Quasilinear masking

Polynomial $P_{\vec{x}}(\omega)$
(shares = coefficients)

A $\vec{v}$-sharing of x

$$\vec{x} = (x_0, x_1, \ldots, x_{n-1}) \quad \text{s.t.} \quad \langle \vec{v}, \vec{x} \rangle = x = \sum_{i=0}^{n-1} x_i \cdot \omega^i$$

$$\vec{v} = (1, \omega, \omega^2, \ldots, \omega^{n-1}) \quad \text{for} \quad \omega \xleftarrow{\$} \mathbb{F}$$

Efficient multiplication

- Let $\vec{t}$ such that

$$P_{\vec{t}} = P_{\vec{x}} \cdot P_{\vec{y}}$$

- We get

$$P_{\vec{t}}(\omega) = \sum_{i=0}^{2n-1} t_i \omega^i = x \cdot y$$

Efficient multiplication

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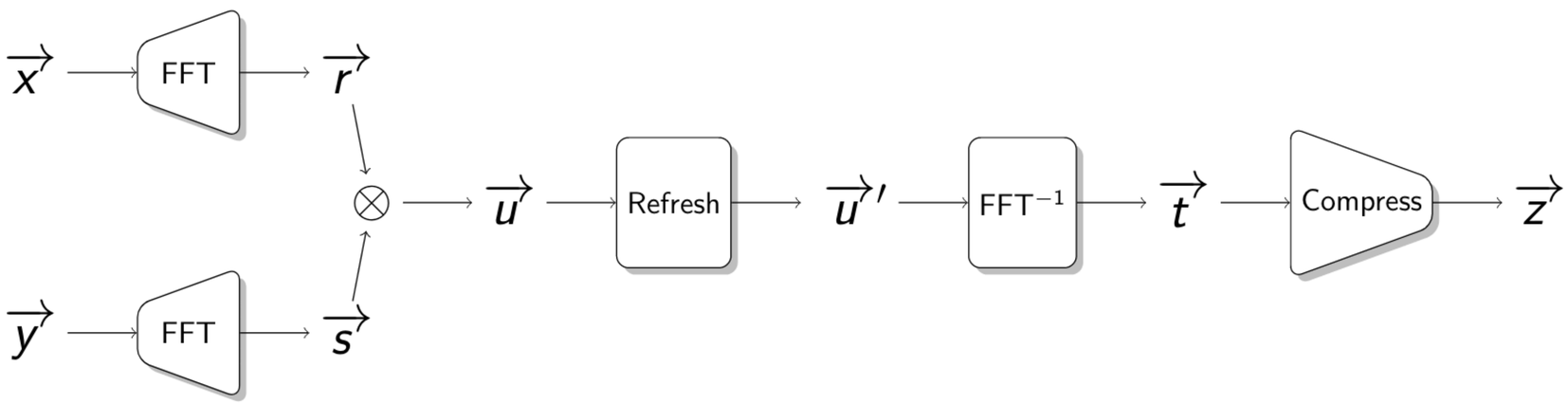
- We get

$$P_{\vec{t}}(\omega) = \sum_{i=0}^{2n-1} t_i \, \omega^i = x \cdot y$$

- Compression:

$$\vec{z} = (t_0, \ldots, t_{n-1}) + \omega^n \cdot (t_n, \ldots, t_{2n-1})$$

Multiplication gadget



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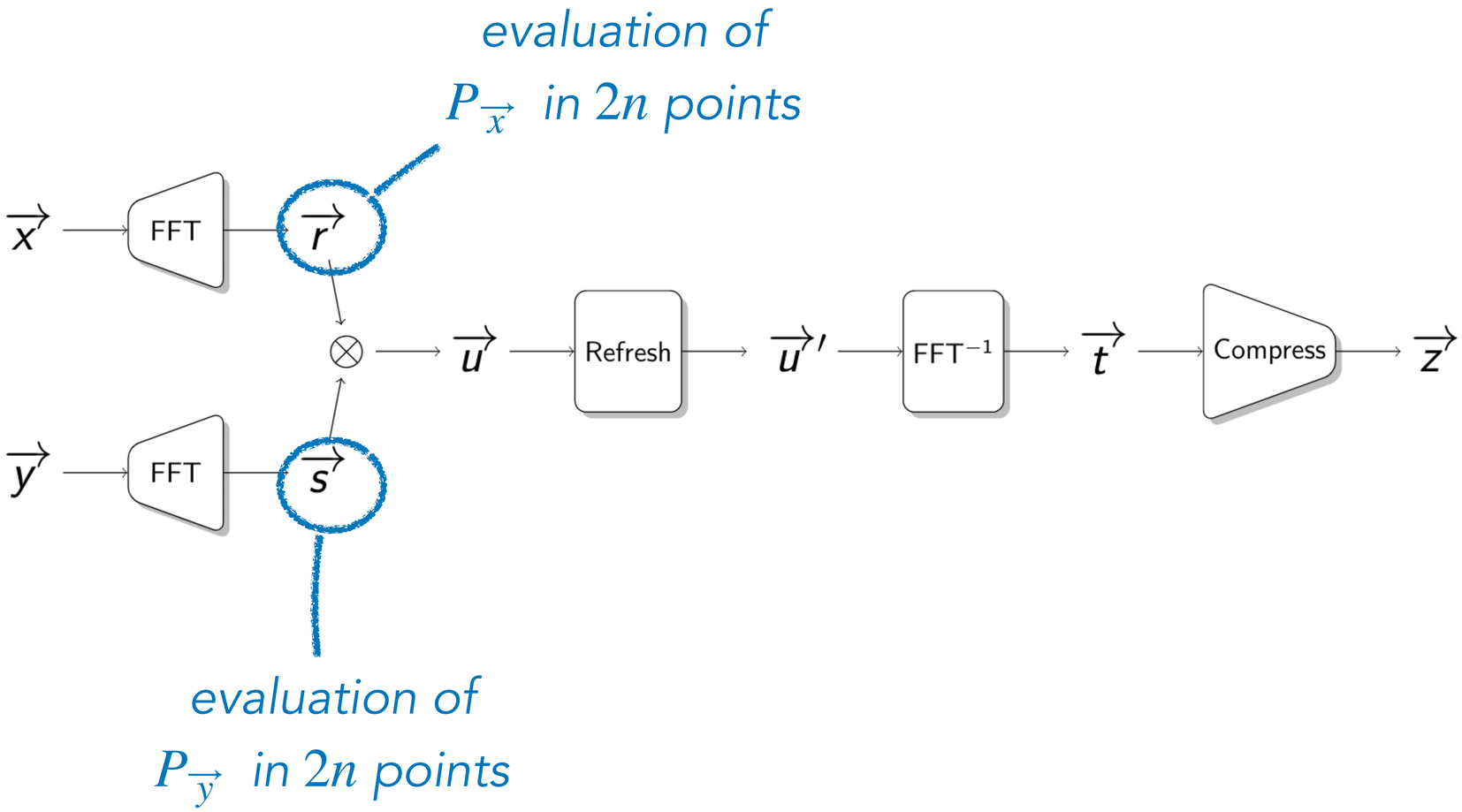
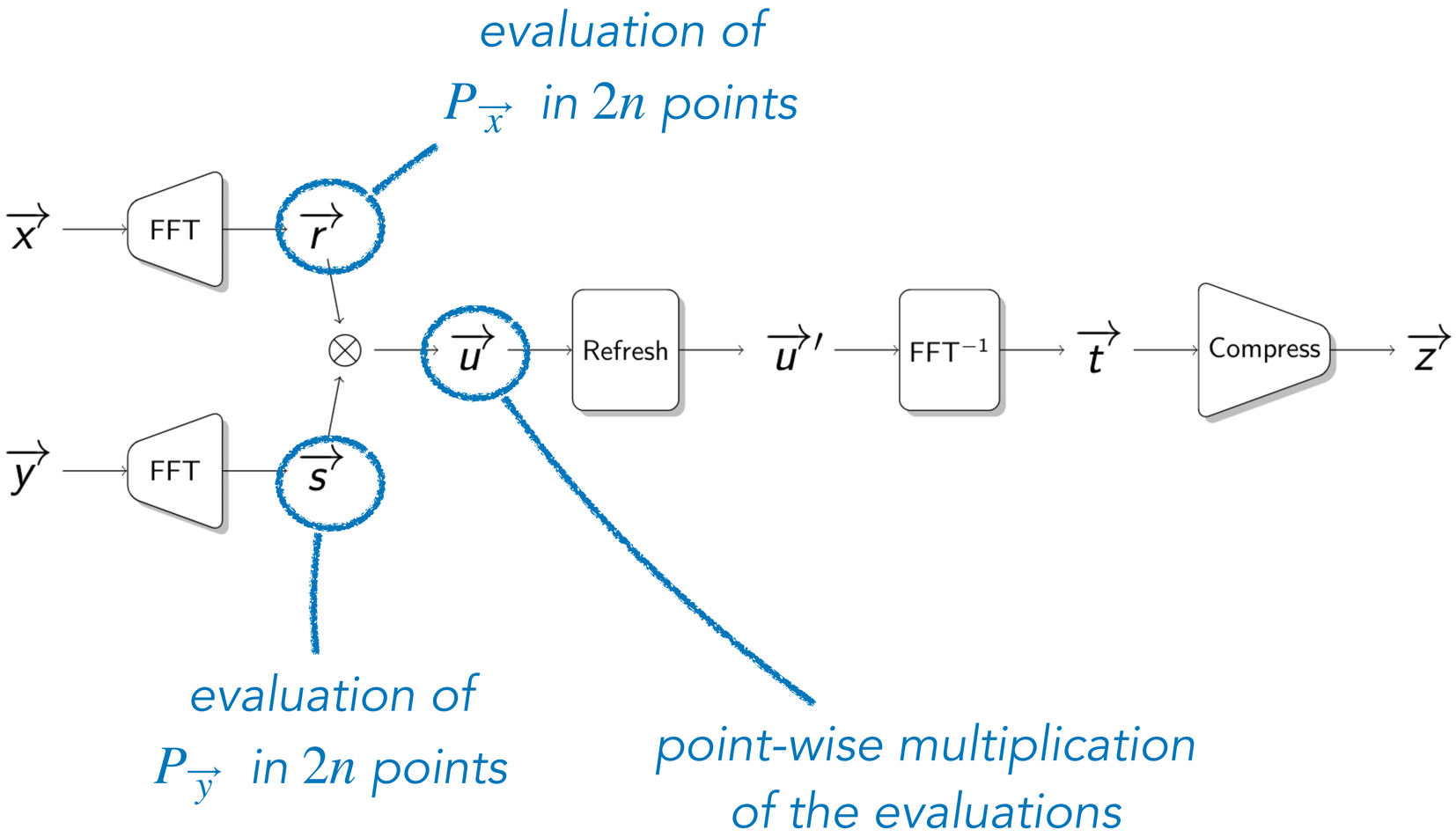
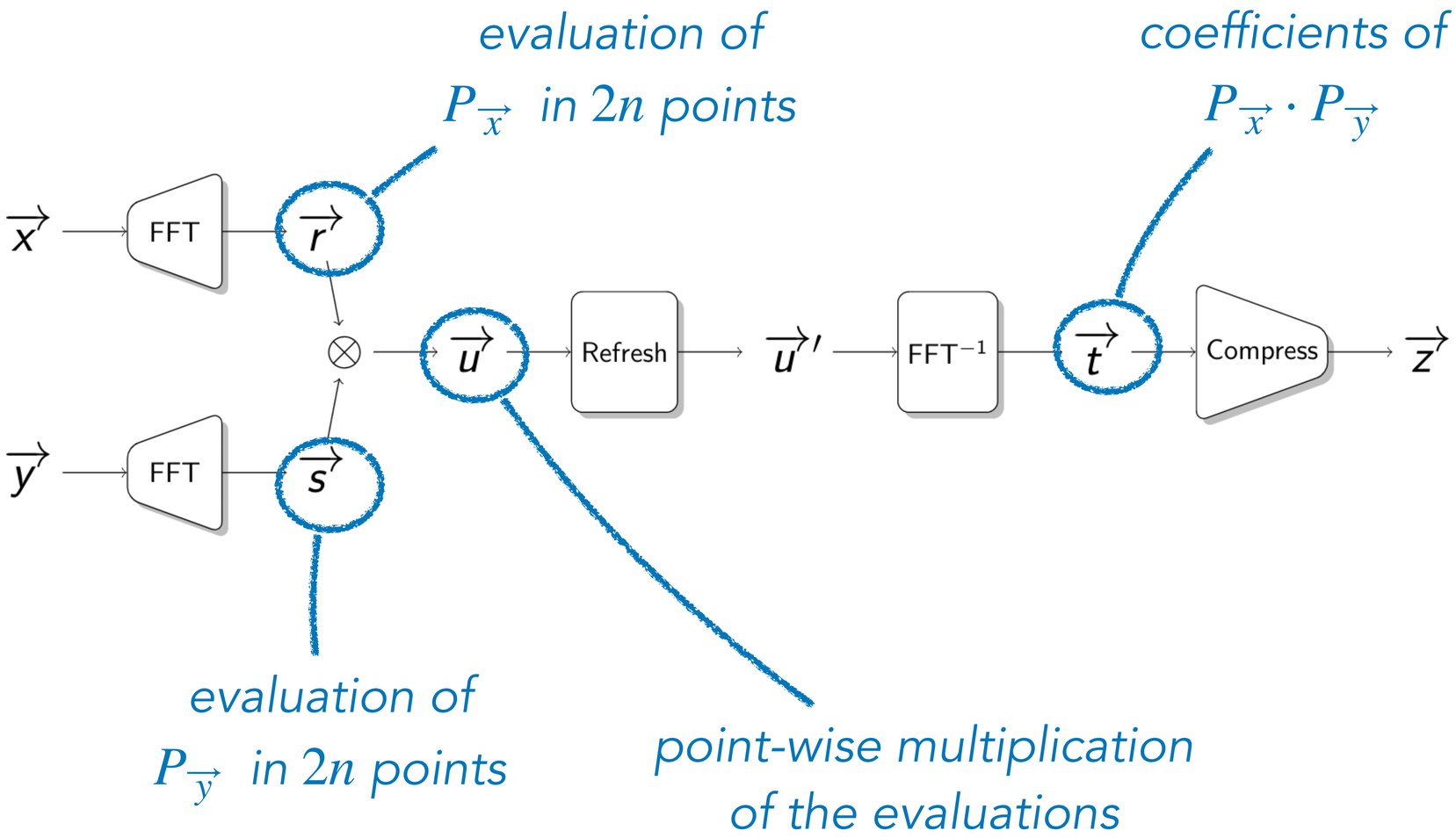


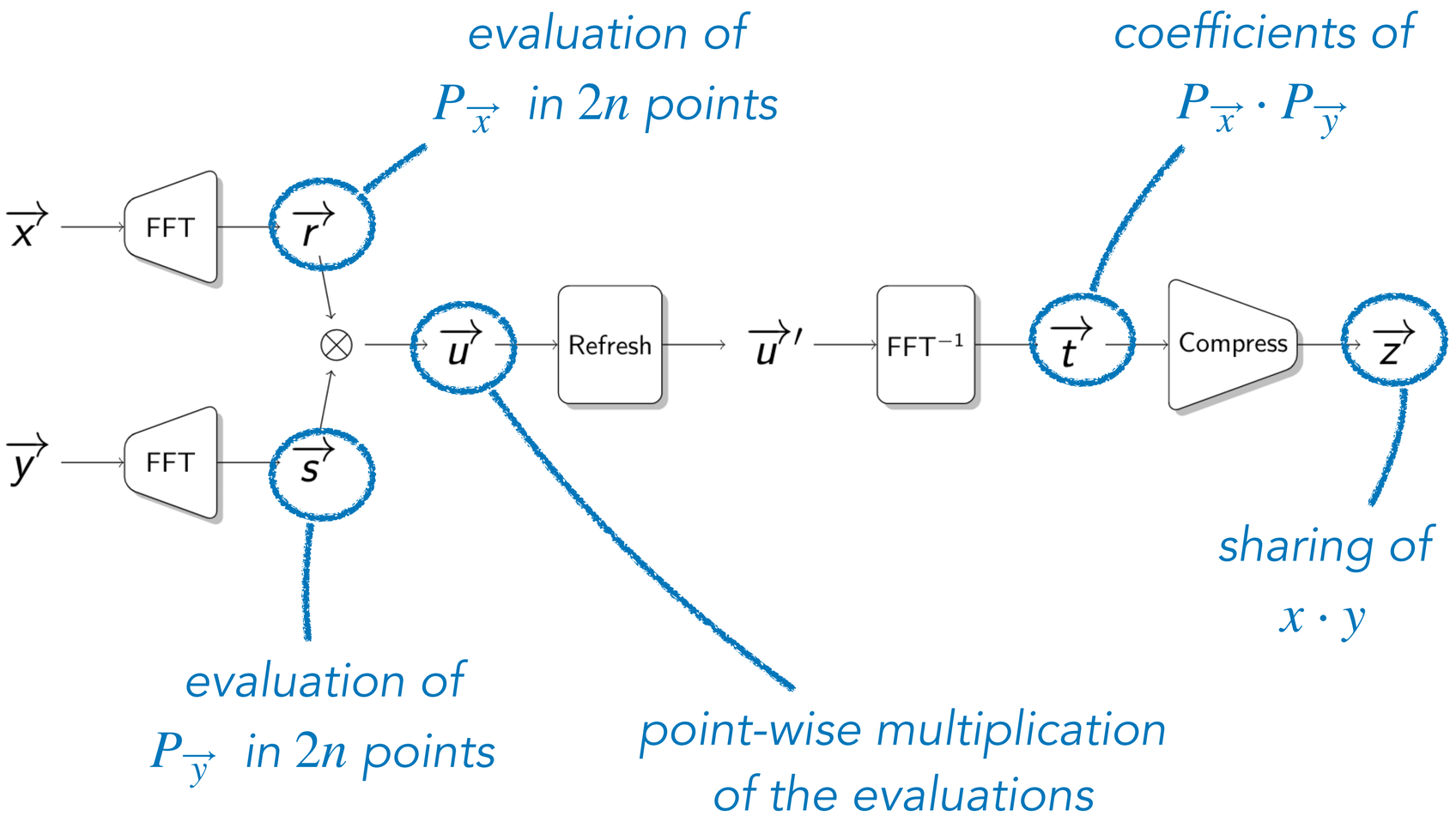
Multiplication gadget



Multiplication gadget



Multiplication gadget



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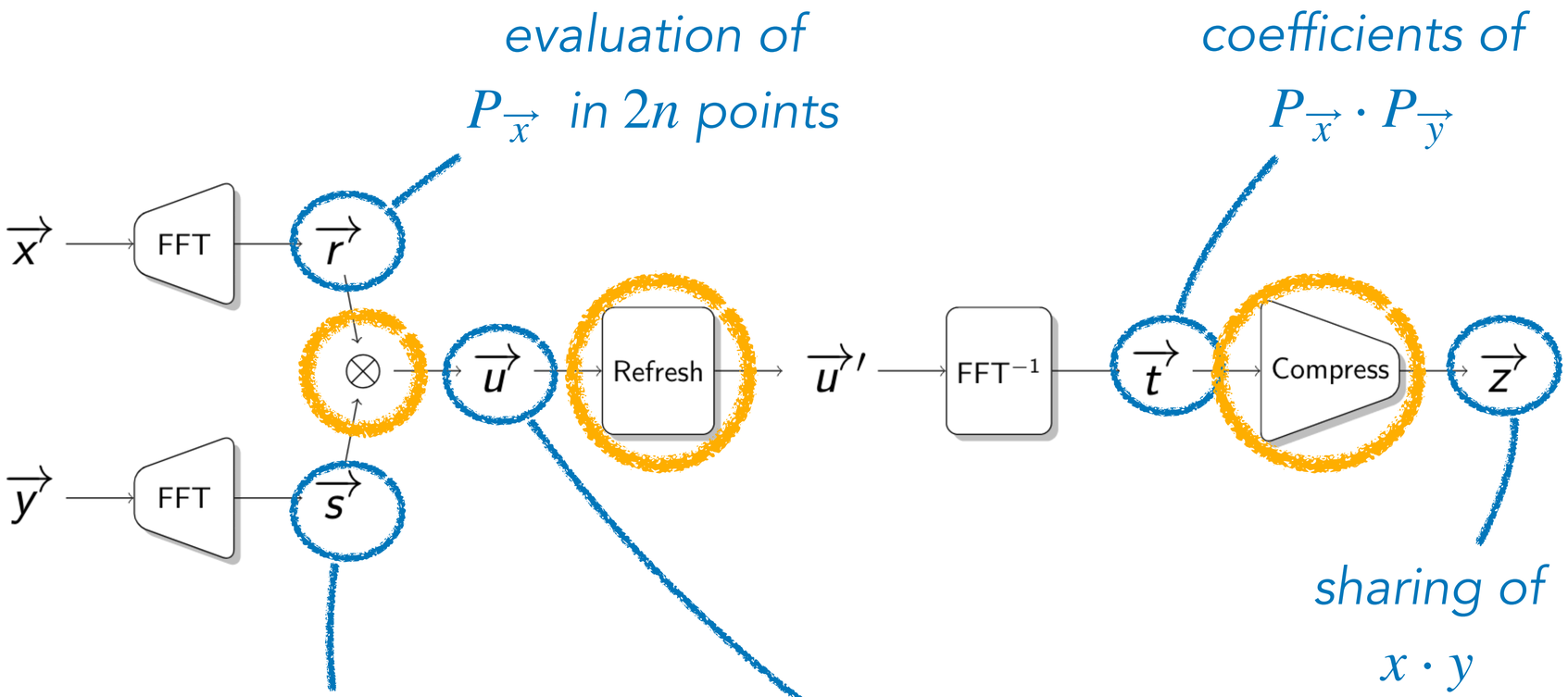


evaluation of $P_{\vec{x}}$ *in* $2n$ *points*

coefficients of $P_{\vec{x}} \cdot P_{\vec{y}}$

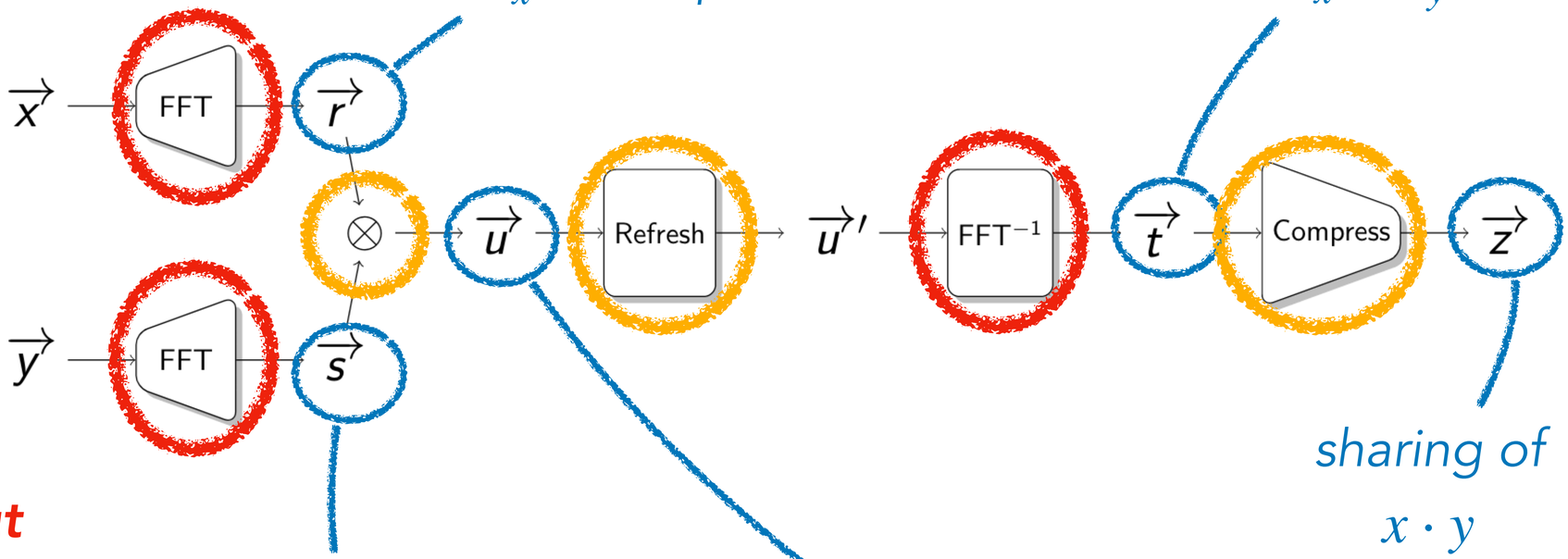
sharing of $x \cdot y$

evaluation of $P_{\vec{y}}$ *in* $2n$ *points*

point-wise multiplication of the evaluations

sharewise operations $\Rightarrow$ ***probing secure***

Multiplication gadget



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coefficients of $P_{\vec{x}} \cdot P_{\vec{y}}$

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evaluation of $P_{\vec{y}}$ *in* $2n$ *points*

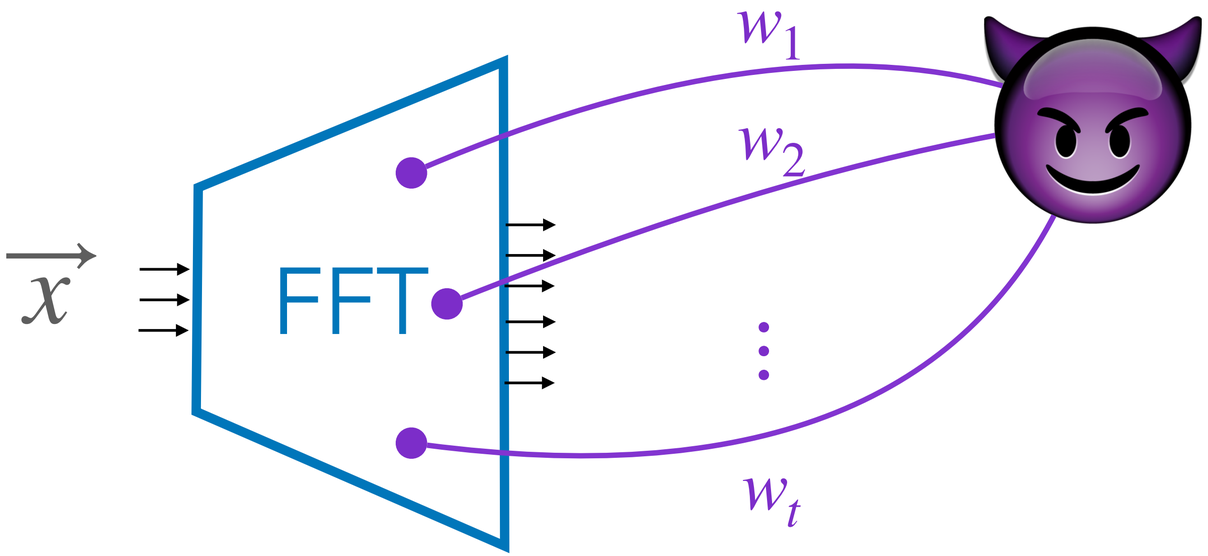
point-wise multiplication of the evaluations

what about the FFT?

🤔

sharewise operations ⇒ ***probing secure***

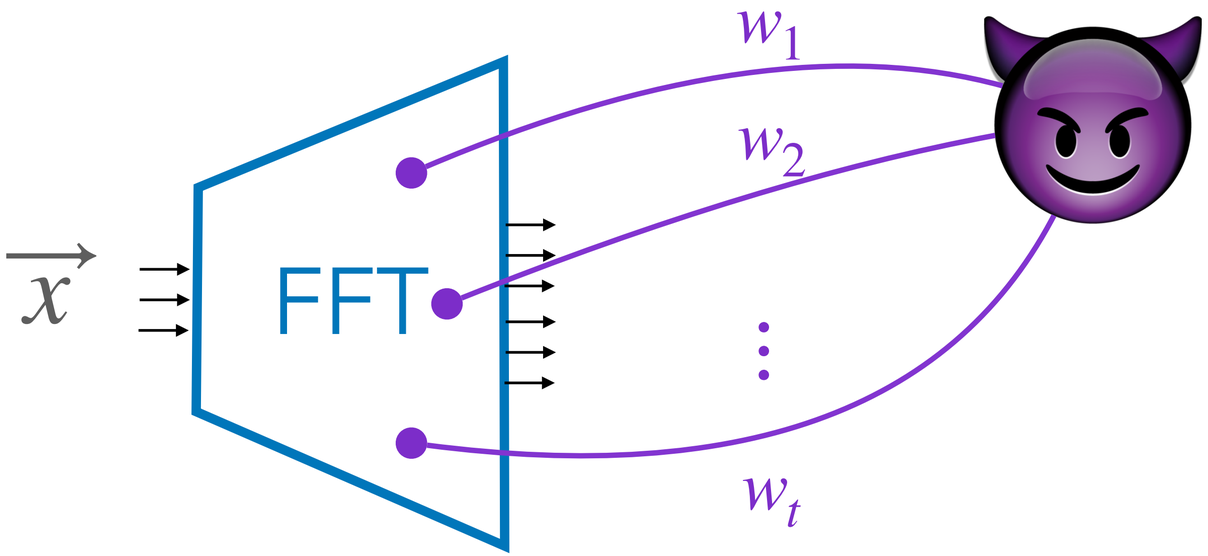
Probing security



💡 *FFT computes linear combinations of the x_i's*

$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_t \end{pmatrix} = \begin{bmatrix} A \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{pmatrix}$$

Probing security



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Lemma 1

If $\vec{v} = \begin{pmatrix} \omega^0 \\ \omega^1 \\ \vdots \\ \omega^{n-1} \end{pmatrix} \notin \left\langle \begin{bmatrix} A \end{bmatrix} \right\rangle$ then $\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_t \end{pmatrix} \sim \mathcal{U}(\mathbb{F}^t)$

(assuming A full rank wlog)

Probing security

Lemma 2

$\exists$ at most t values of $\omega \in \mathbb{F}$ s.t. $\vec{v} = \begin{pmatrix} \omega^0 \\ \omega^1 \\ \vdots \\ \omega^{n-1} \end{pmatrix} \in \langle \left[\; A \; \right] \rangle$

Probing security

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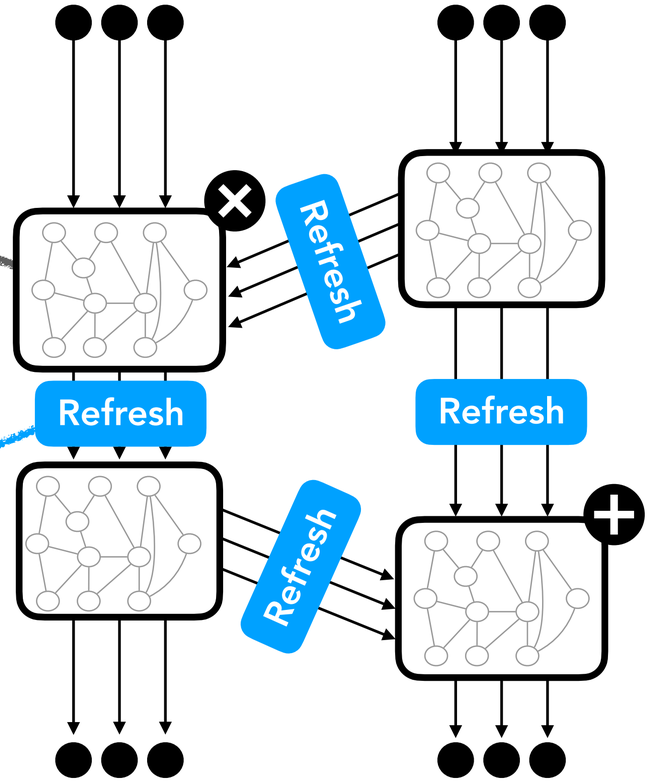
Lemma 1 + Lemma 2

$$P\left[(w_1, \ldots, w_t) \text{ cannot be simulated} \right] \leq \frac{t}{|\mathbb{F}|} < \frac{n}{|\mathbb{F}|}$$

Composition security

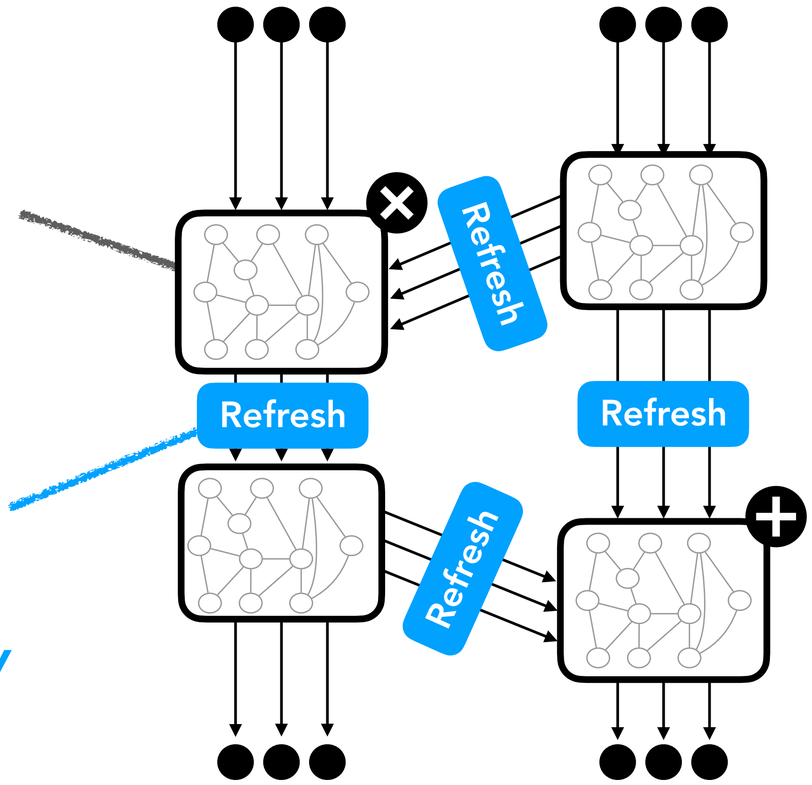
probing security

input-output separation (IOS) property



⇒ region probing security

Composition security



⇒ region probing security

Wrapping up:

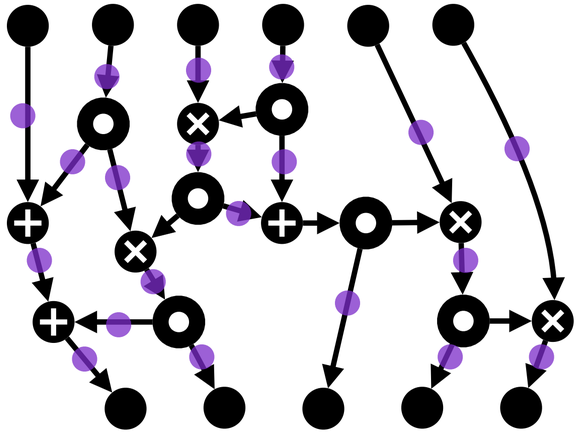
- Gadget complexity: $\Theta(n \log n)$
- Probes per gadget: $\Theta(n)$
- Leakage rate: $\Theta(1/\log n)$

Security with constant leakage rate

Simulation with abort

1. Sample a set of leaking wires

$$W \leftarrow \mathsf{LeakingWires}(\hat{C}, p)$$



$$\begin{cases} w & \text{with proba } p \\ \perp & \text{with proba } 1-p \end{cases}$$

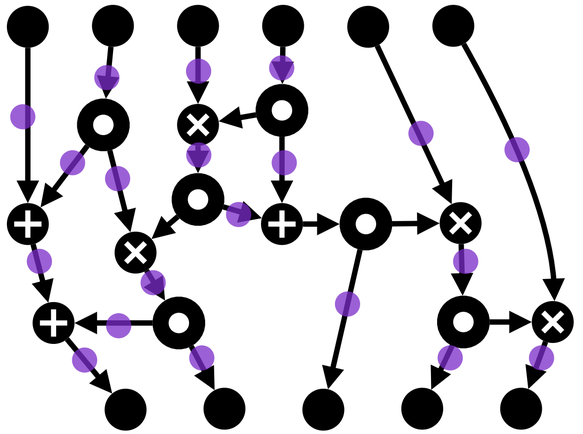
Simulation with abort

1. Sample a set of leaking wires

$$W \leftarrow \mathsf{LeakingWires}(\hat{C}, p)$$

2. Simulate the corresponding wire values

$$\mathsf{Sim} : W \mapsto \begin{cases} \text{perfect simulation} \\ \perp \text{ (abort)} \end{cases}$$



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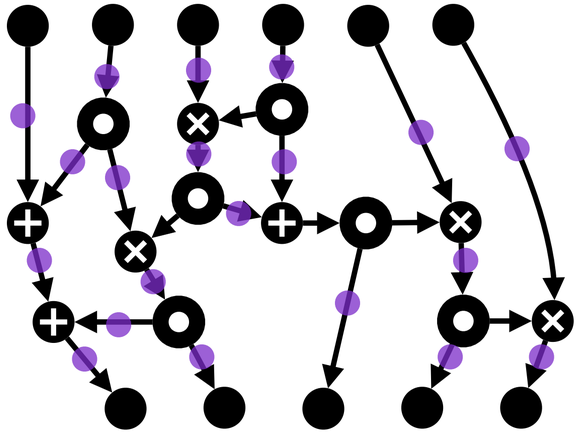
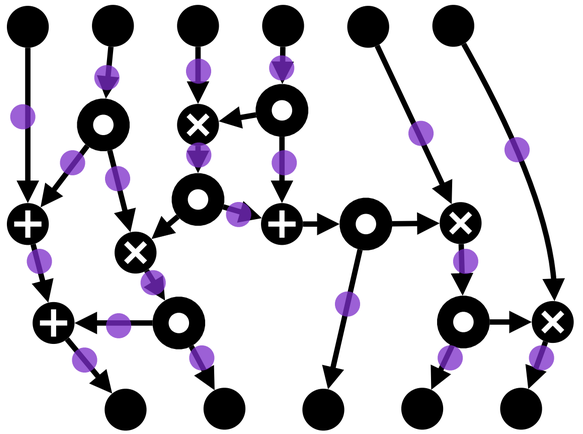


$$\begin{cases} w & \text{with proba } p \\ \perp & \text{with proba } 1-p \end{cases}$$

$$\delta_W = \begin{cases} 1 & \text{if } \mathsf{Sim}(W) = \perp \\ 0 & \text{otherwise} \end{cases}$$

Simulation with abort



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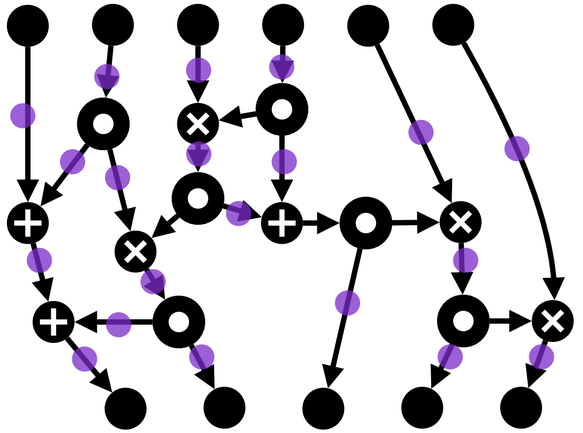
- Failure probability

$$f(p) = \sum_W \delta_W p^{|W|}(1-p)^{s-|W|}$$

Simulation with abort



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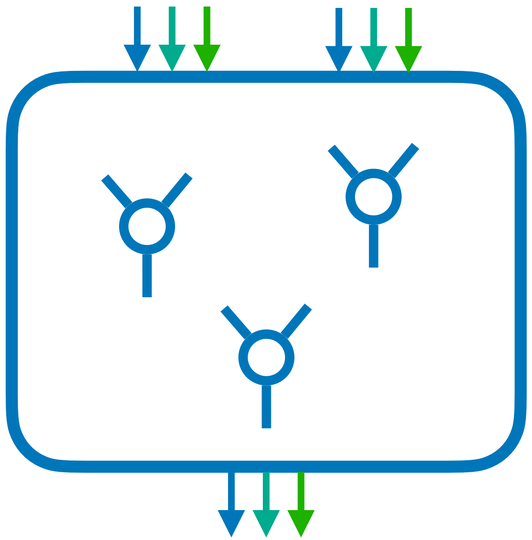
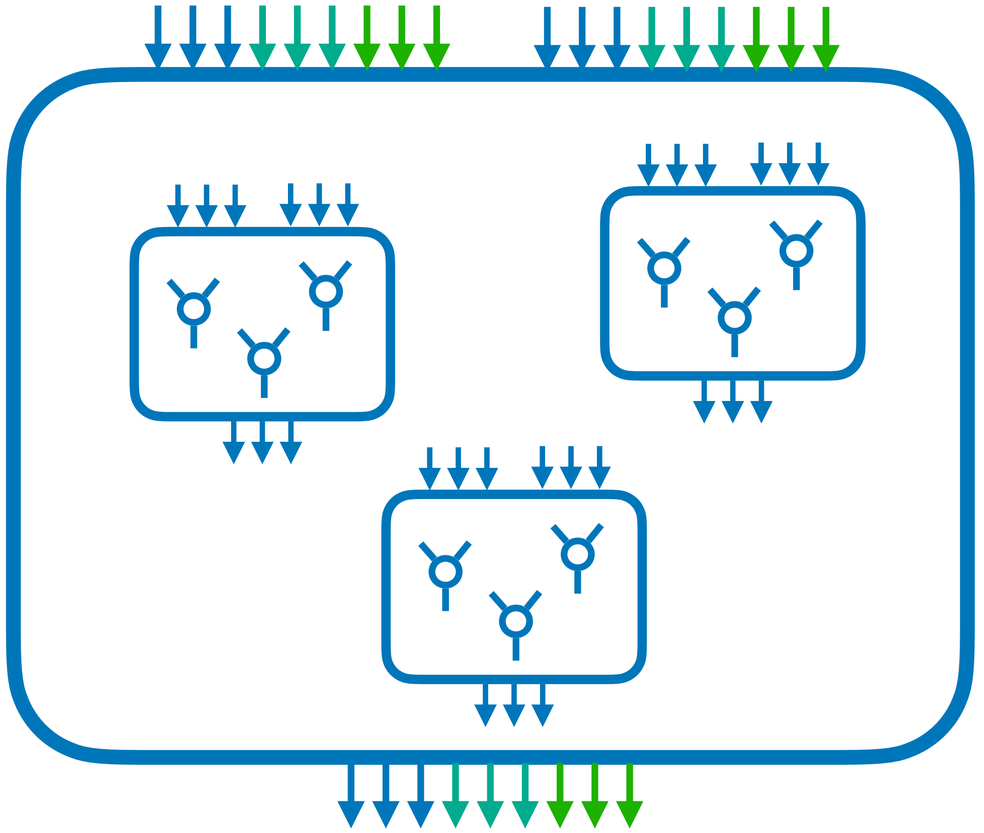
- Failure probability

$$f(p) = \sum_W \delta_W p^{|W|}(1-p)^{s-|W|} \leq \sum_i c_i \, p^i$$

The expansion strategy

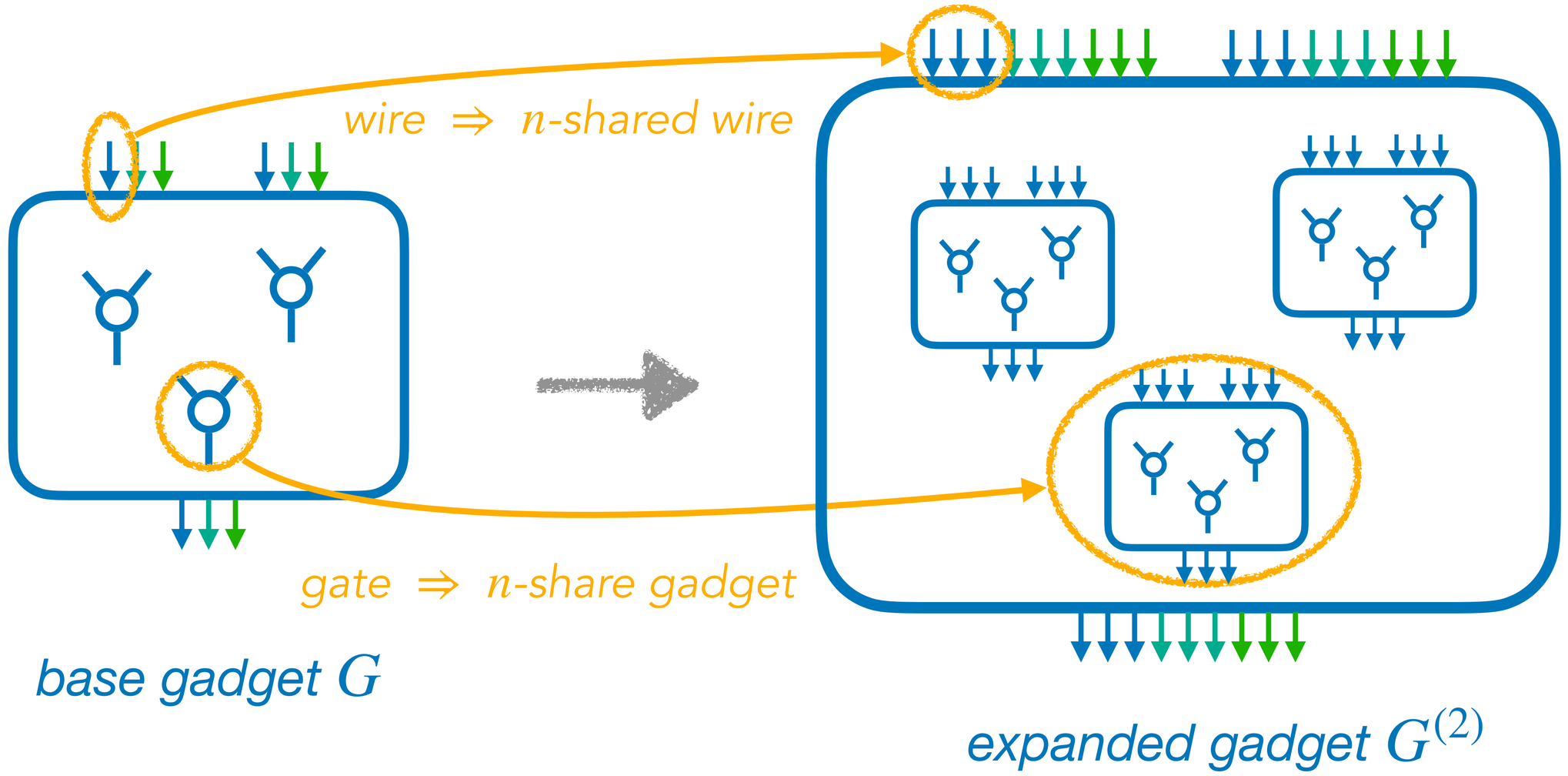


base gadget G



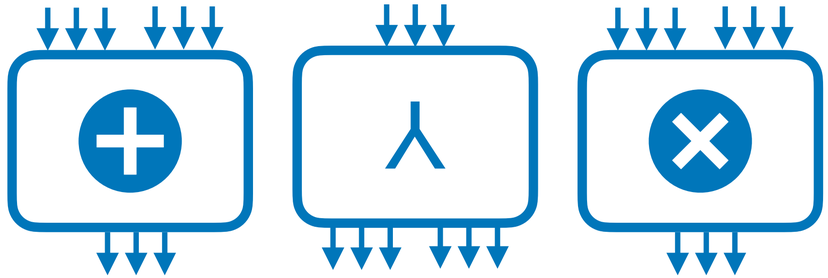
expanded gadget $G^{(2)}$

The expansion strategy



The expansion strategy

Idea: bootstrap constant-size (small) gadgets

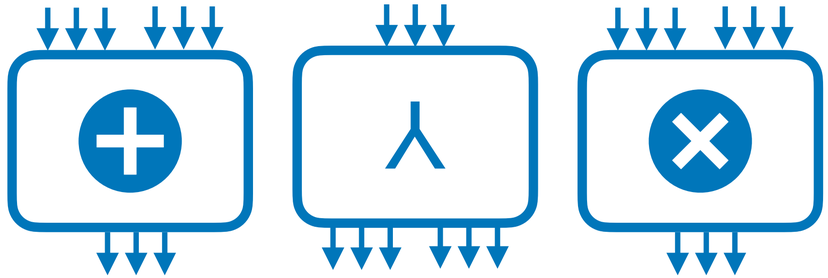


base gadgets

$$\{G\} \to \{G^{(2)}\} \to \cdots \to \{G^{(k)}\}$$

The expansion strategy

Idea: bootstrap constant-size (small) gadgets



base gadgets

$$\{G\} \to \{G^{(2)}\} \to \cdots \to \{G^{(k)}\}$$

Goal: amplification of random probing security

$$p \longrightarrow f(p)$$

The expansion strategy

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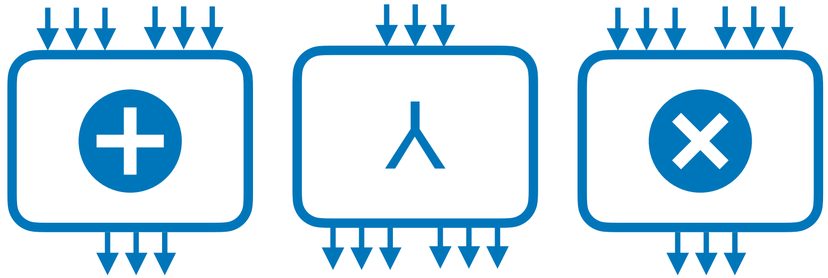

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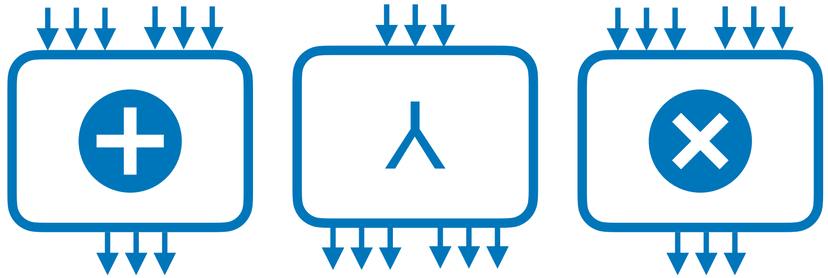


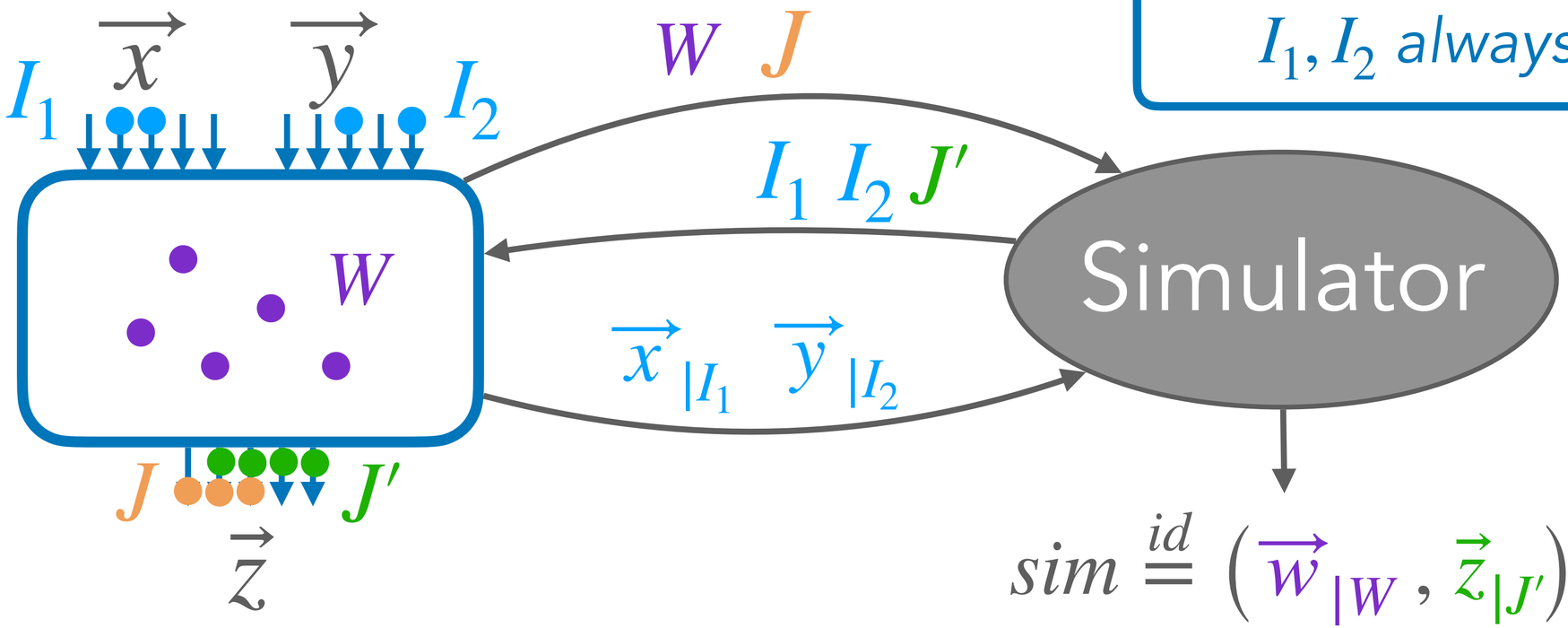
base gadgets

$$\{G\} \to \{G^{(2)}\} \to \cdots \to \{G^{(k)}\}$$

Goal: amplification of random probing security

$$p \longrightarrow f(p) \longrightarrow f(f(p)) \longrightarrow \cdots \longrightarrow f^{(k)}(p)$$

Random probing expandability (RPE)



I_1, I_2 *always exist*

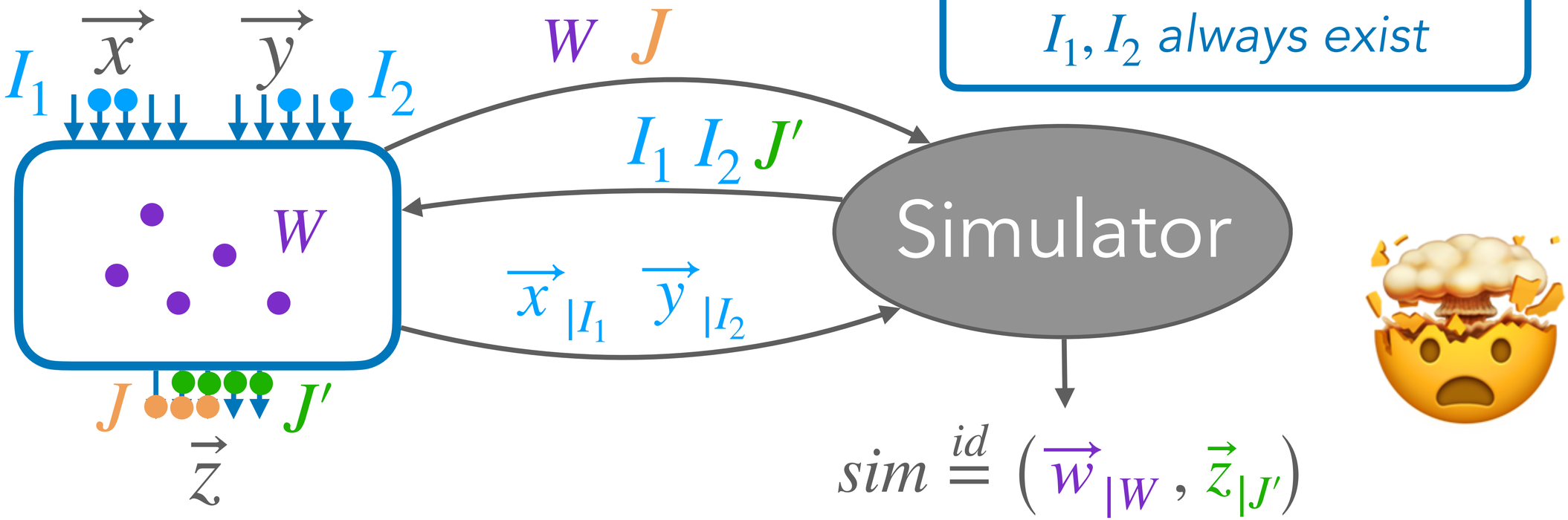
RPE threshold t: $|J| \leq t$,

$(|I_1| > t$ *or* $|I_2| > t)$ = *simulation failure*

if $|J| > t$, *sim. can choose*

J' *s.t.* $|J'| = n - 1$

Random probing expandability (RPE)



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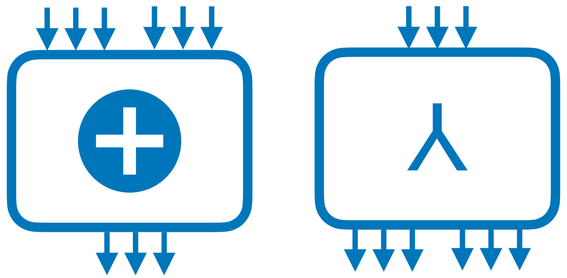
Random probing expandability (RPE)

Base gadgets $\{G\}$ f-RPE $\Rightarrow$ expanded gadgets $\{G^{(2)}\}$ $f^{(2)}$-RPE

$\Rightarrow$ expanded gadgets $\{G^{(k)}\}$ $f^{(k)}$-RPE

$f^{(k)}(p)$ simulation security vs.
p-random probing leakage

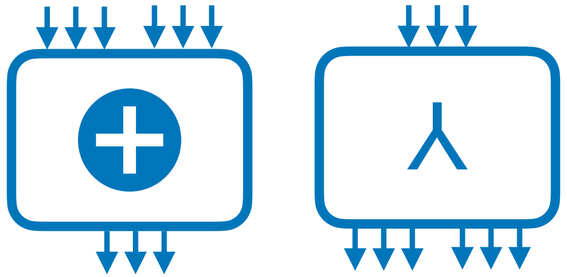
Generic constructions



$$G_{\oplus}(\overrightarrow{x}, \overrightarrow{y}) = G_{\mathsf{R}}(\overrightarrow{x}) + G_{\mathsf{R}}(\overrightarrow{y})$$

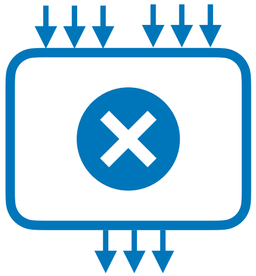
$$G_{\curlywedge}(\overrightarrow{x}) = (G_{\mathsf{R}}(\overrightarrow{x}), G_{\mathsf{R}}(\overrightarrow{x}))$$

Generic constructions



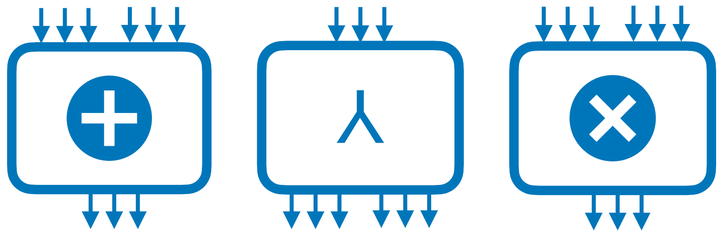
$$G_{\oplus}(\overrightarrow{x}, \overrightarrow{y}) = G_{\mathsf{R}}(\overrightarrow{x}) + G_{\mathsf{R}}(\overrightarrow{y})$$

$$G_{\curlywedge}(\overrightarrow{x}) = (G_{\mathsf{R}}(\overrightarrow{x}), G_{\mathsf{R}}(\overrightarrow{x}))$$



$$G_{\otimes}(\overrightarrow{x}, \overrightarrow{y}) \mapsto \begin{pmatrix} x_1 \cdot G_{\mathsf{R}}(\overrightarrow{y}) \\ x_2 \cdot G_{\mathsf{R}}(\overrightarrow{y}) \\ \vdots \\ x_n \cdot G_{\mathsf{R}}(\overrightarrow{y}) \end{pmatrix} + \text{ greedy use of randomness}$$

Efficient instantiations

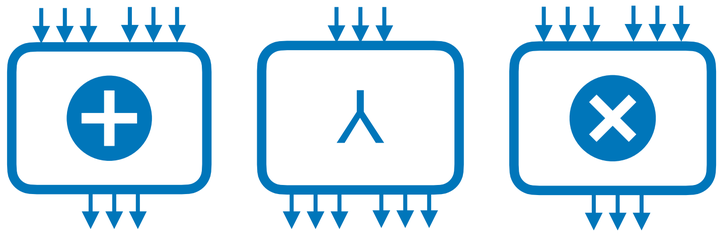


constant-size (small) gadgets

Your gadget is f-RPE

IronMask

Efficient instantiations



constant-size (small) gadgets

IronMask

Your gadget is f-RPE

Maximum tolerated leakage probability

$p_{max} \in [0,1)$ such that $f(p_{max}) < p_{max}$

Efficient instantiations

3-share gadgets

$$\left.\begin{aligned} G_{\mathsf{R}} : z_1 &\leftarrow r_1 + x_1 \\ z_2 &\leftarrow r_2 + x_2 \\ z_3 &\leftarrow (r_1 + r_2) + x_3 \end{aligned}\right\} \Rightarrow \quad \mathcal{O}(|C|\kappa^{3.9}) \ , \quad p_{max} = 2^{-7.5}$$

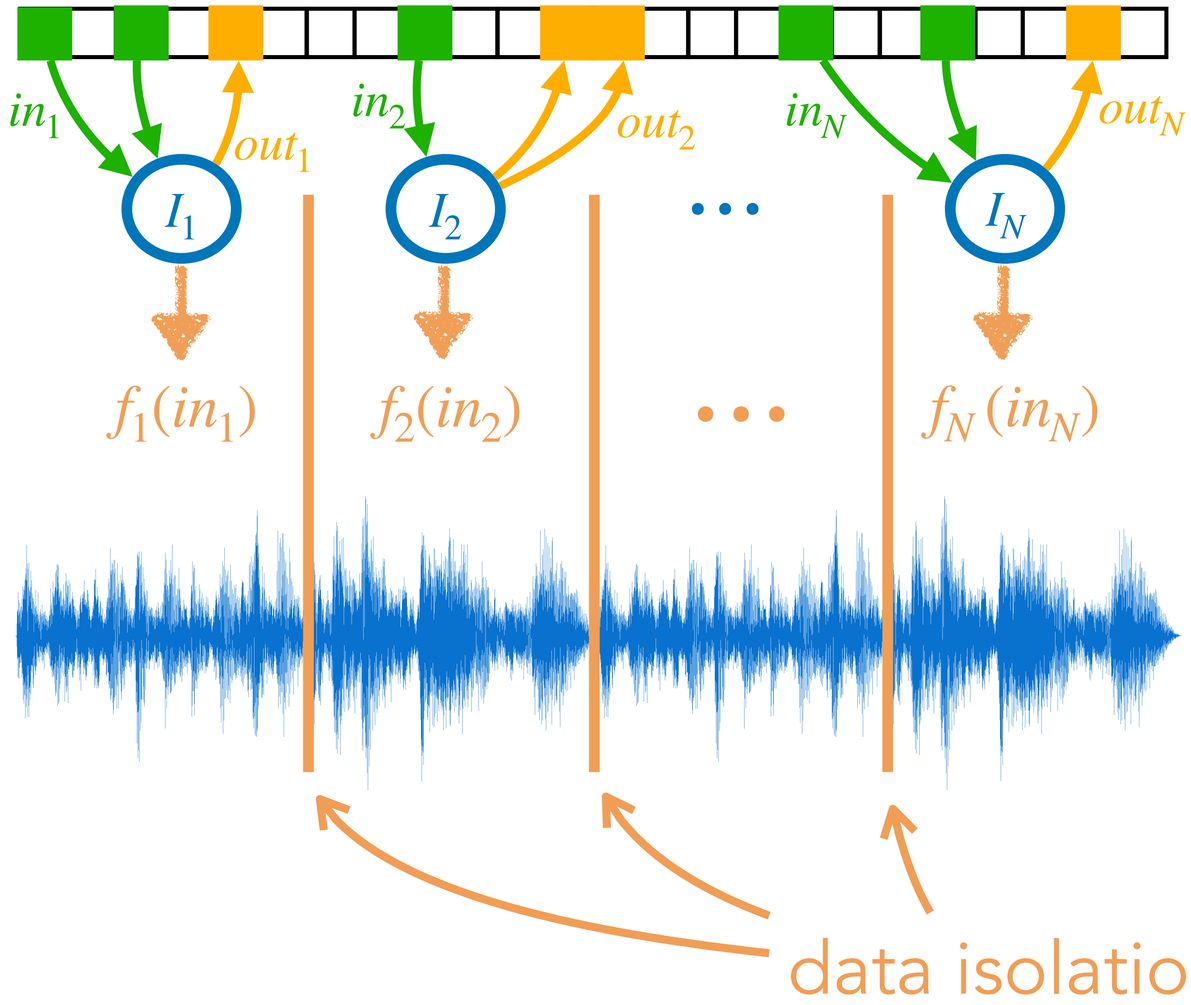
5-share gadgets

$$\left.\begin{aligned} G_{\mathsf{R}} : z_1 &\leftarrow (r_1 + r_2) + x_1 \\ z_2 &\leftarrow (r_2 + r_3) + x_2 \\ z_3 &\leftarrow (r_3 + r_4) + x_3 \\ z_4 &\leftarrow (r_4 + r_5) + x_4 \\ z_5 &\leftarrow (r_5 + r_1) + x_5 \end{aligned}\right\} \Rightarrow \quad \mathcal{O}(|C|\kappa^{3.2}) \ , \quad p_{max} = 2^{-12}$$

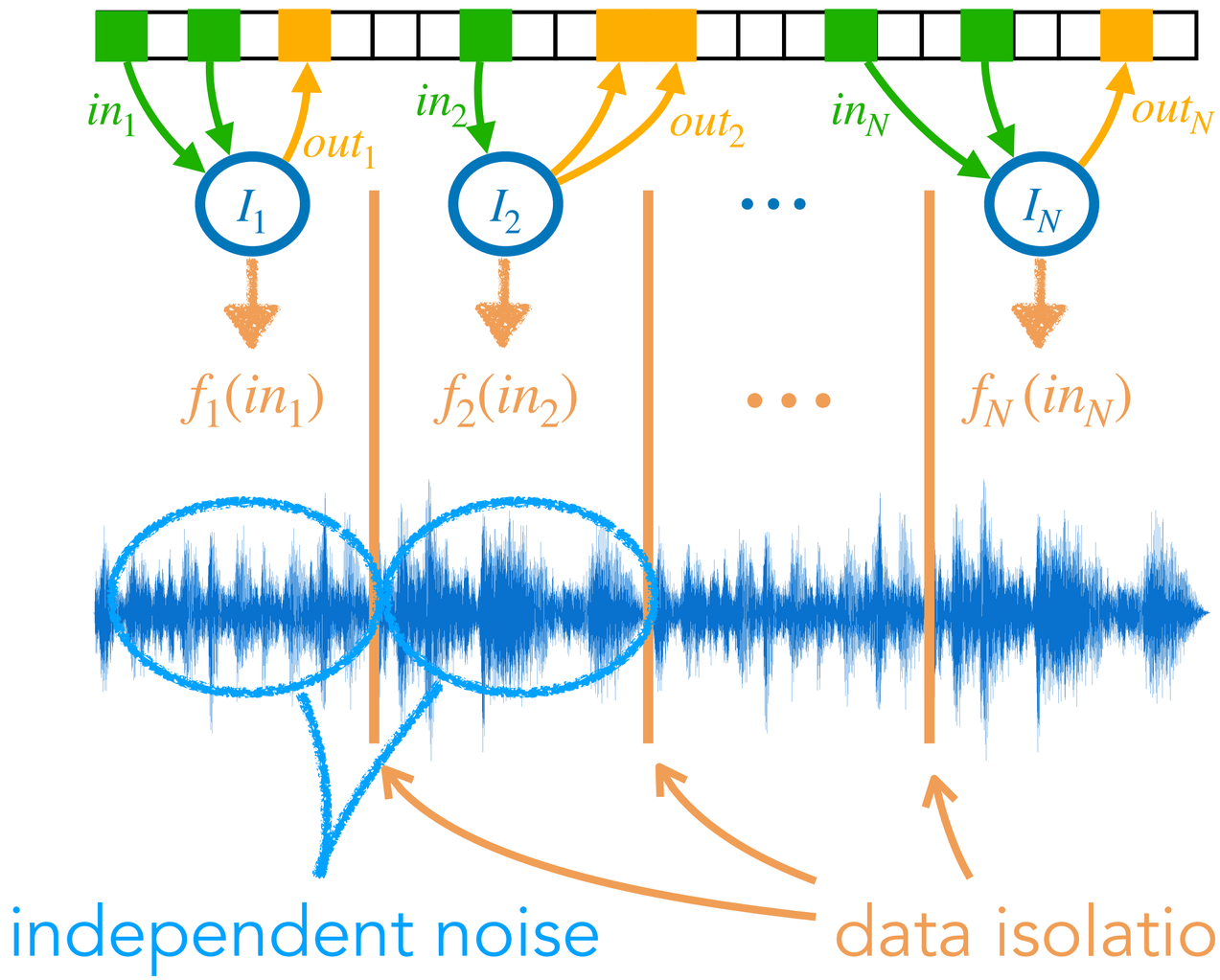
Conclusion

- [x] Provable security against side-channel attacks
- [ ] (Fully) bridging theory and practice

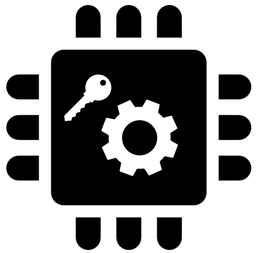
Physical assumptions



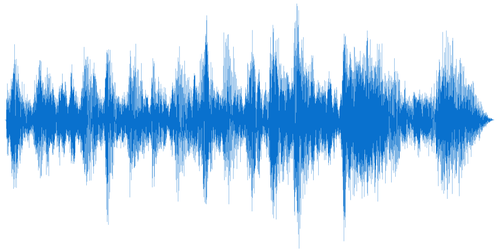
Physical assumptions



Noise parameters



device /
implementation



leakage
distribution

δ

noise
parameter

p

leakage
probability

Performances

3-share gadgets

$$G_R : z_1 \leftarrow r_1 + x_1$$
$$z_2 \leftarrow r_2 + x_2$$
$$z_3 \leftarrow (r_1 + r_2) + x_3$$

5-share gadgets

$$G_R : z_1 \leftarrow (r_1 + r_2) + x_1$$
$$z_2 \leftarrow (r_2 + r_3) + x_2$$
$$z_3 \leftarrow (r_3 + r_4) + x_3$$
$$z_4 \leftarrow (r_4 + r_5) + x_4$$
$$z_5 \leftarrow (r_5 + r_1) + x_5$$

$\Rightarrow$ $\mathcal{O}(|C|\kappa^{3.9})$ $p_{max} = 2^{-7.5}$

→ improved complexity

$\Rightarrow$ $\mathcal{O}(|C|\kappa^{3.2})$ $p_{max} = 2^{-12}$

→ optimised implementations

VeriSiCC Seminar 2022

Verification and Generation of Side-Channel Countermeasures

September 22, 2022, Paris

https://cryptoexperts.com/verisicc/seminaire-2022.html

Thank you!

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