Constructions for digital signatures - Part III: Threshold Computation in the Head

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NIST PQC Seminars

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CryptoExperts

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Threshold Computation in the Head

Joint work with Thibauld Feneuil

https://ia.cr/2022/1407

Original TCitH framework (Asiacrypt'23)

https://ia.cr/2023/1573

Improved TCitH framework (preprint)

Roadmap

- MPC-in-the-Head paradigm
- TC-in-the-Head framework (and application to PQ signatures)
  - 🌲 TCitH with Merkle trees
  - 🌲 TCitH with GGM trees
  - ✖ TCitH using multiplication homomorphism
  - 📦 TCitH using packed secret sharing
- Application: post-quantum ring signatures
- Relation to other proof systems

MPC-in-the-Head paradigm

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One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

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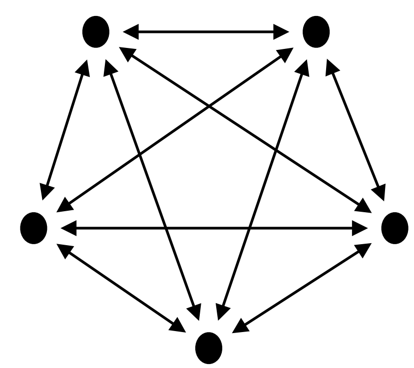
Multiparty computation (MPC)



Input sharing $[\![x]\!]$

Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

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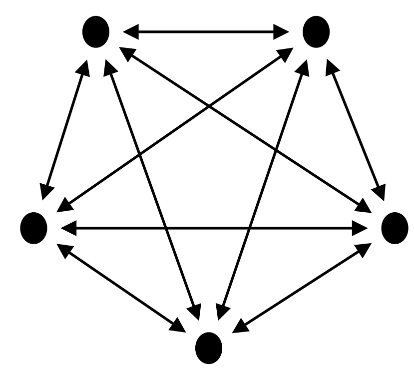

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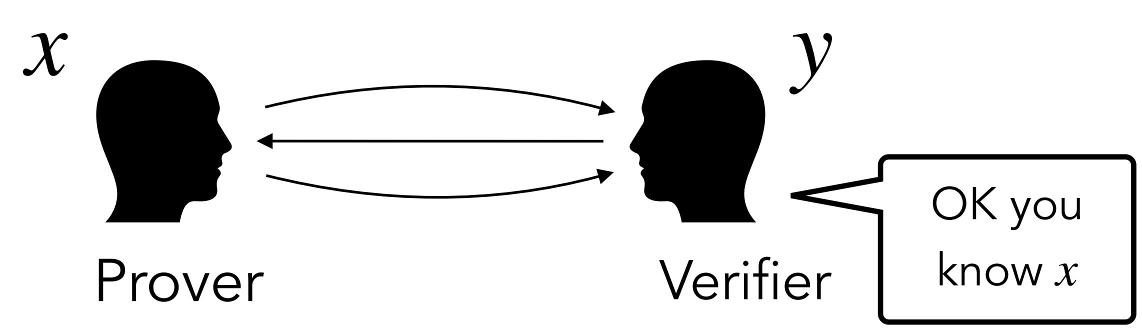
Zero-knowledge proof



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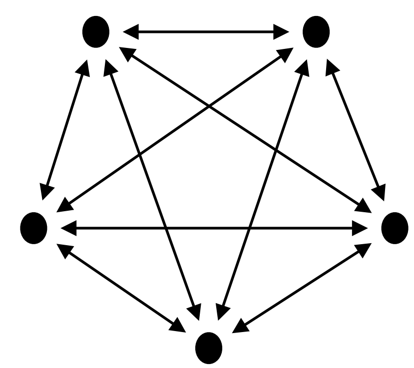

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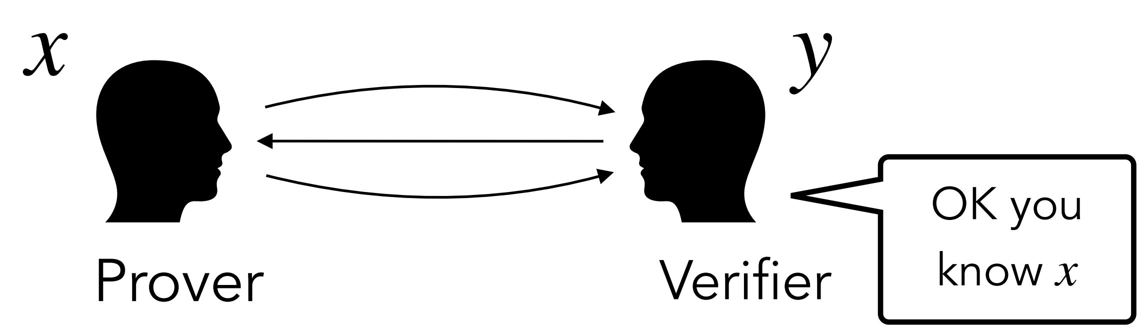
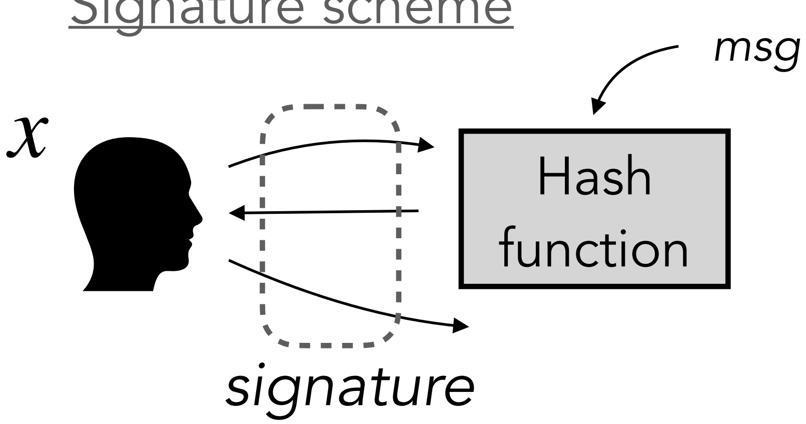
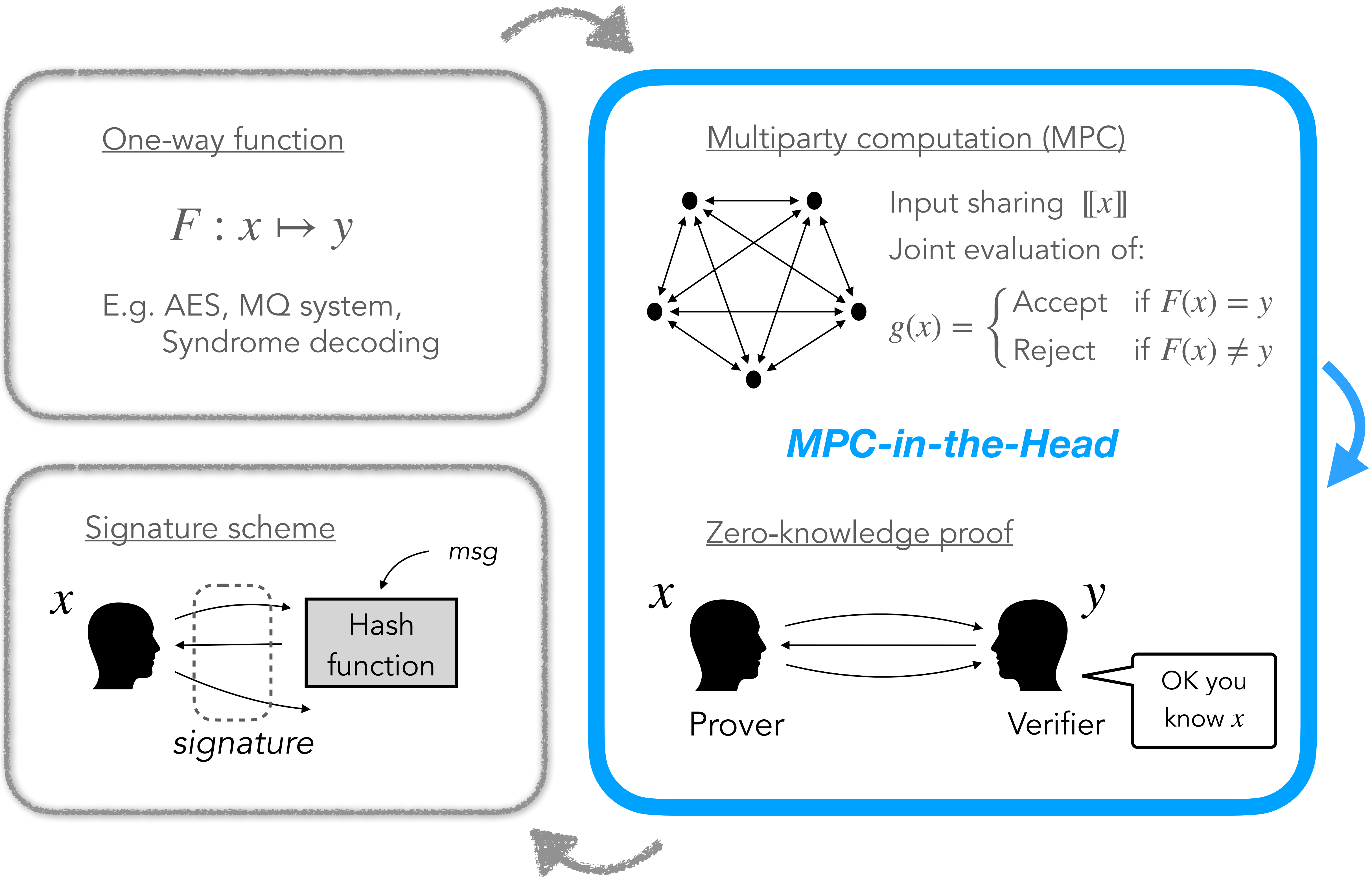


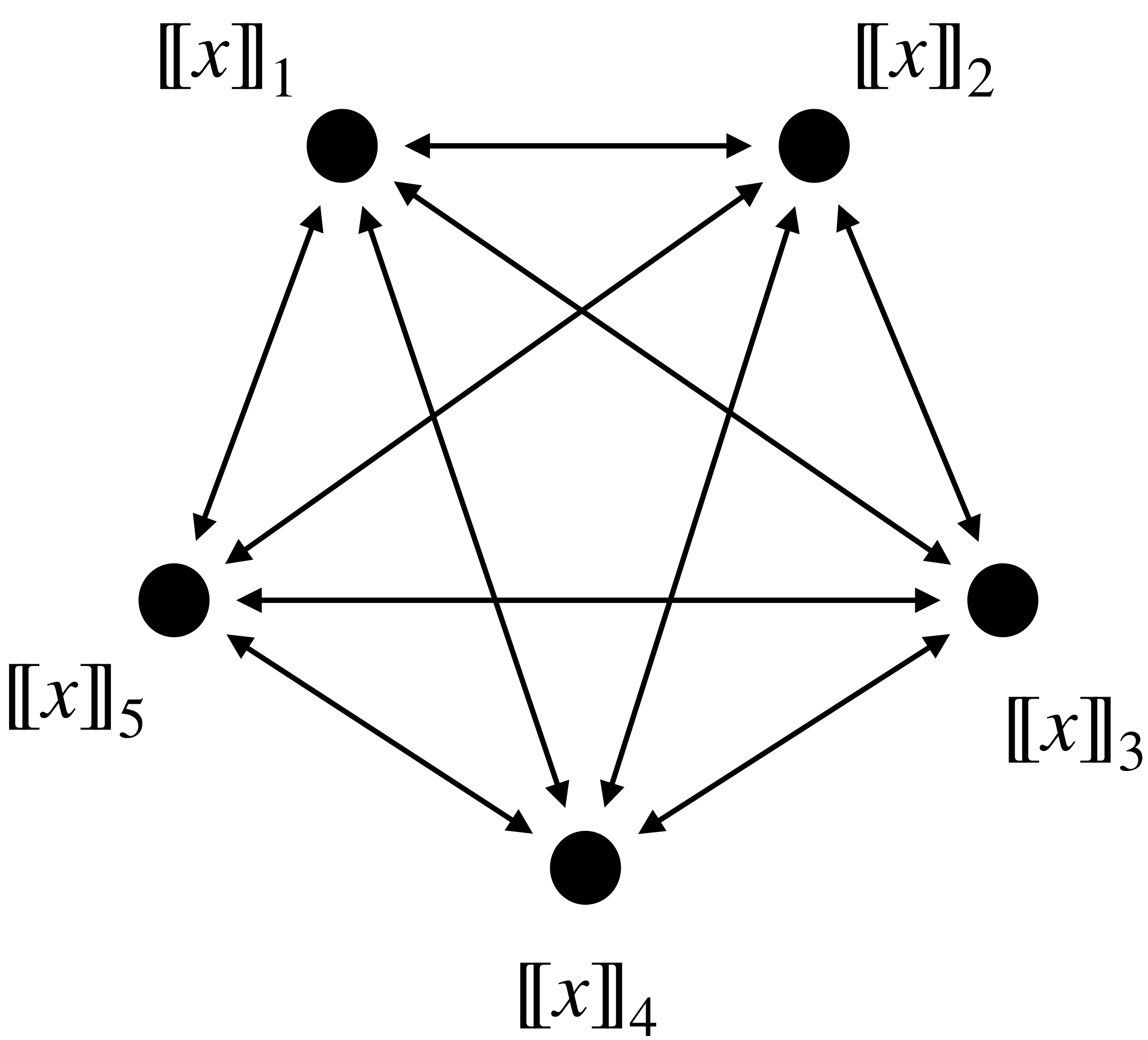
Signature scheme



MPC-in-the-Head paradigm



MPC model



- Jointly compute

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- ℓ-private
- Semi-honest model

$[\![x]\!]$ is a linear secret sharing of x

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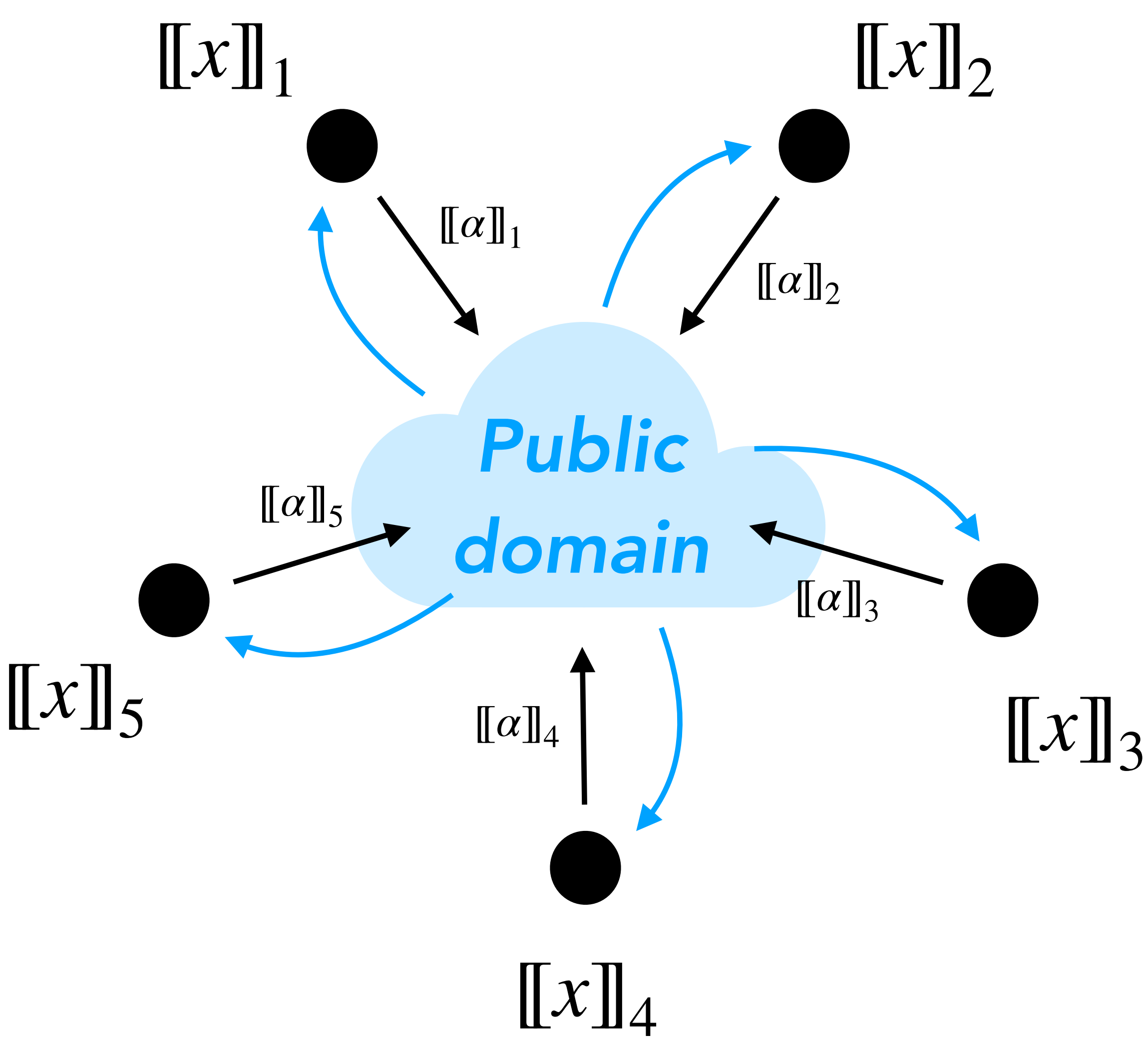

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- ℓ-private
- Semi-honest model
- *Broadcast model*

MPCitH transform

Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

$\text{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\text{Com}^{\rho_N}([\![x]\!]_N)$

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Verifier

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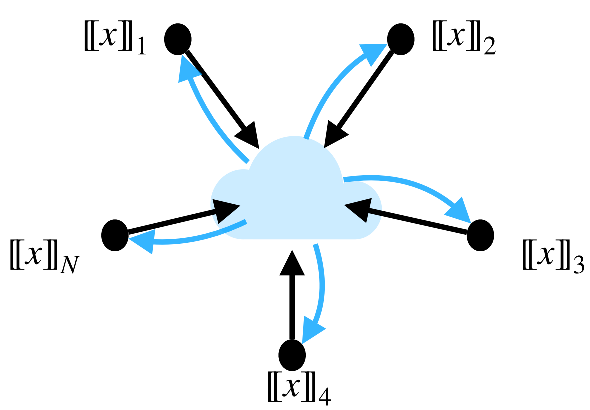

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$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

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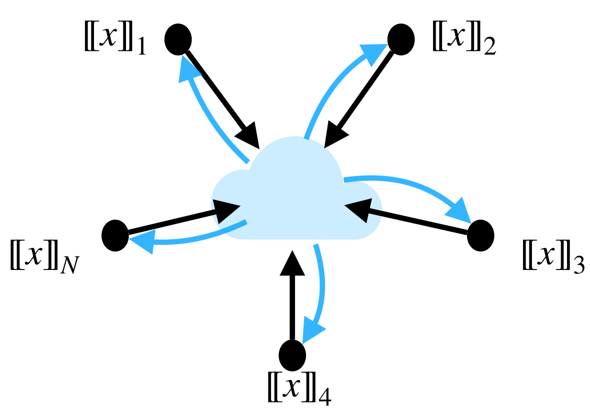

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$I \subseteq \{1,\ldots,N\}$, s.t. $|I| = \ell$.

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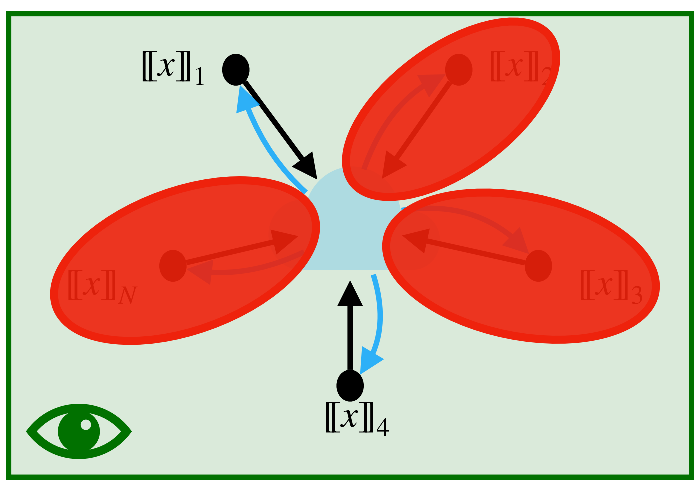

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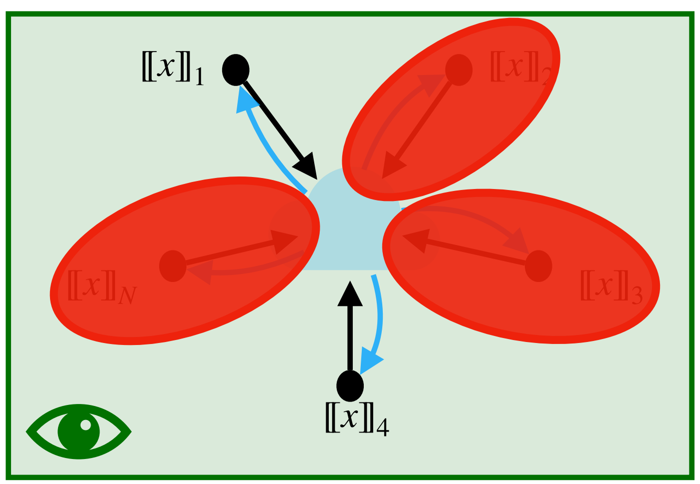

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Check $g(y, \alpha) =$ Accept

<u>Prover</u>

<u>Verifier</u>

MPCitH transform: with additive sharing

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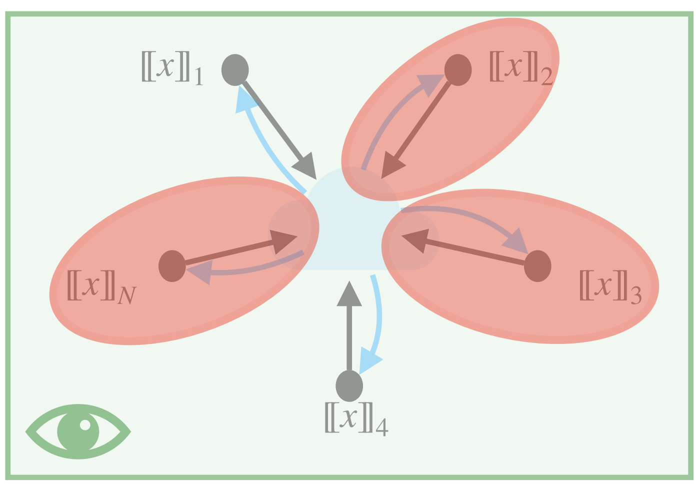

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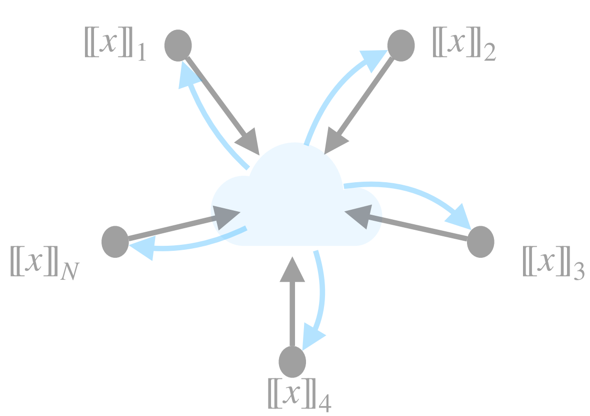

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Additive sharing:

$$x = [\![x]\!]_1 + \cdots + [\![x]\!]_N$$

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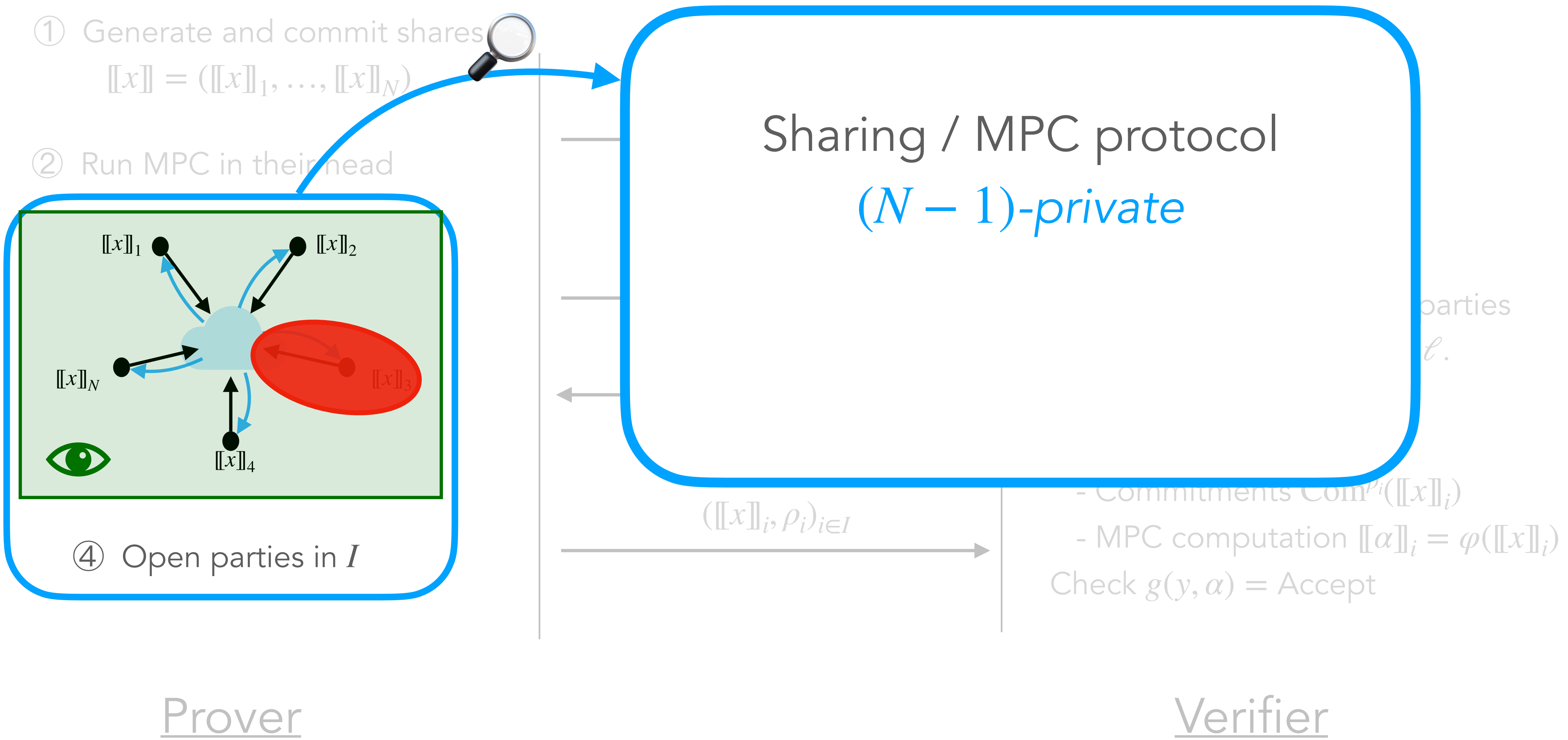

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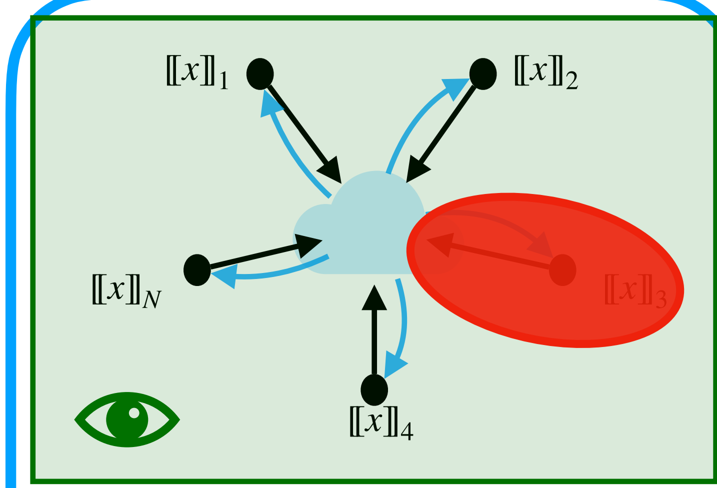

④ Open parties in I

Sharing / MPC protocol
$(N-1)$-*private*

$\Rightarrow$ soundness error $= \dfrac{1}{N}$

parties
ℓ.

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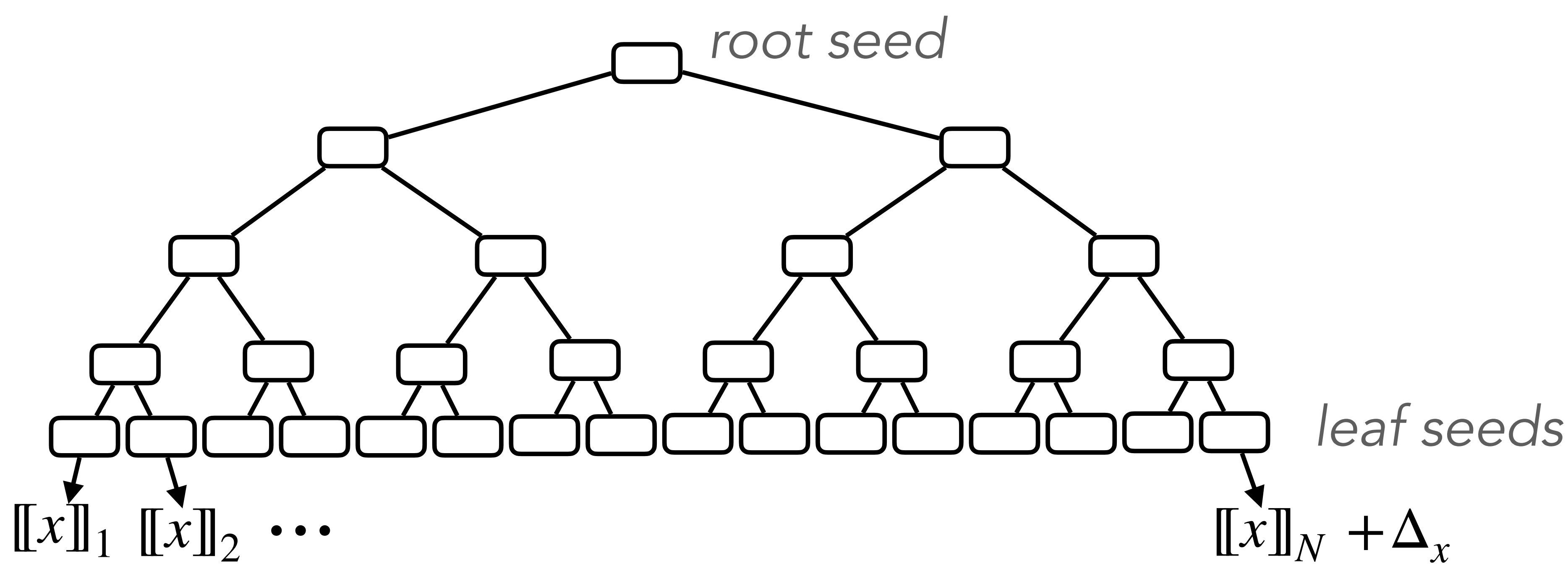
Generated using a GGM seed tree [KKW18]:



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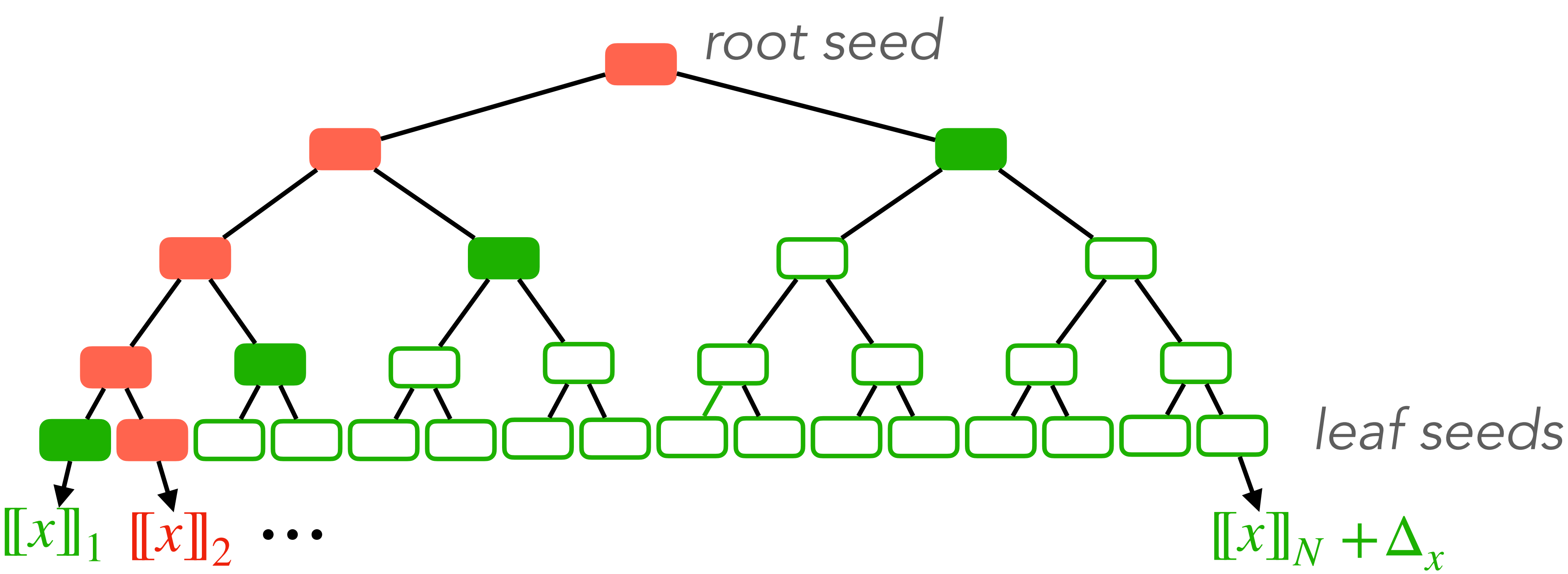

TC-in-the-Head framework
(with Merkle trees)

Threshold Computation in the Head

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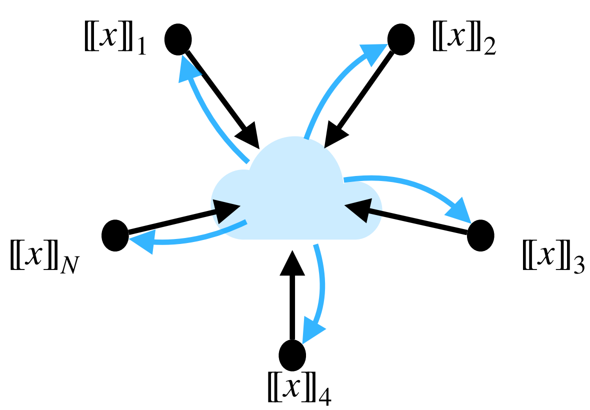
② Run MPC in their head



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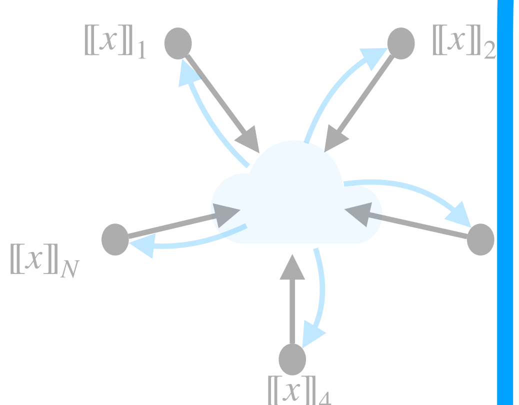

④ Open parties in I

Shamir secret sharing:

$$[\![x]\!]_i := P(e_i) \quad \forall i$$

$$\text{for} \quad P(X) := x + r_1 \cdot X + \cdots + r_\ell \cdot X^\ell$$

m set of parties
t. $|I| = \ell$.

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ccept

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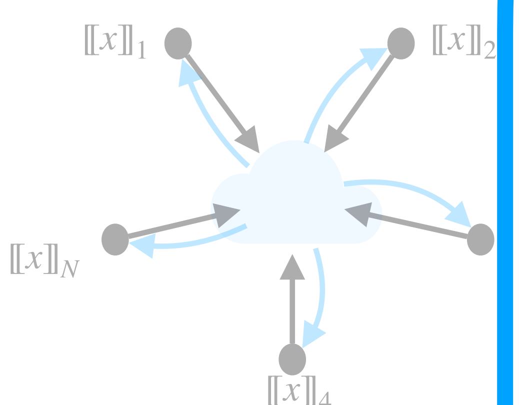

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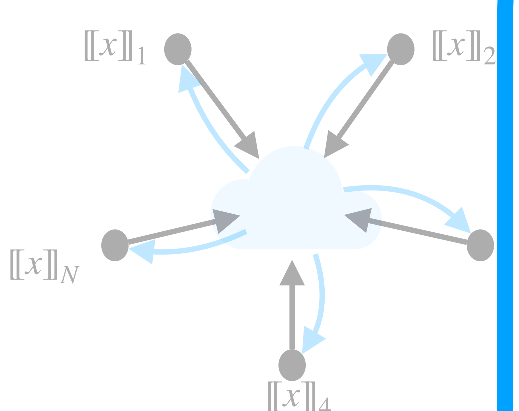

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We use $\ell \ll N$ (e.g. $\ell = 1$)

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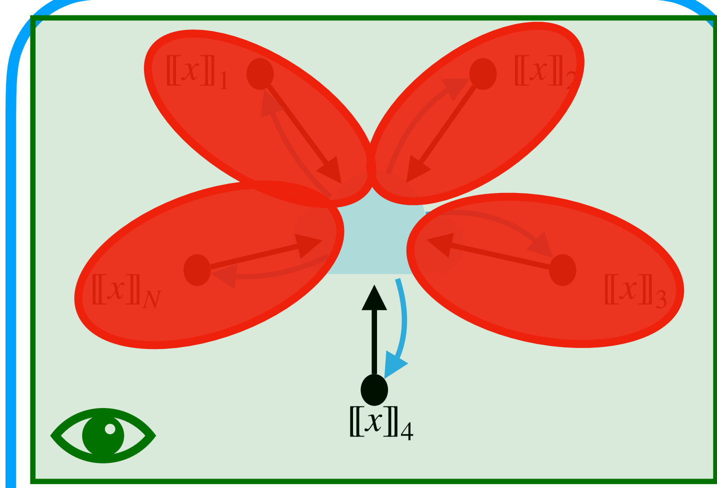

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Sharing / MPC protocol *ℓ-private*

Prover

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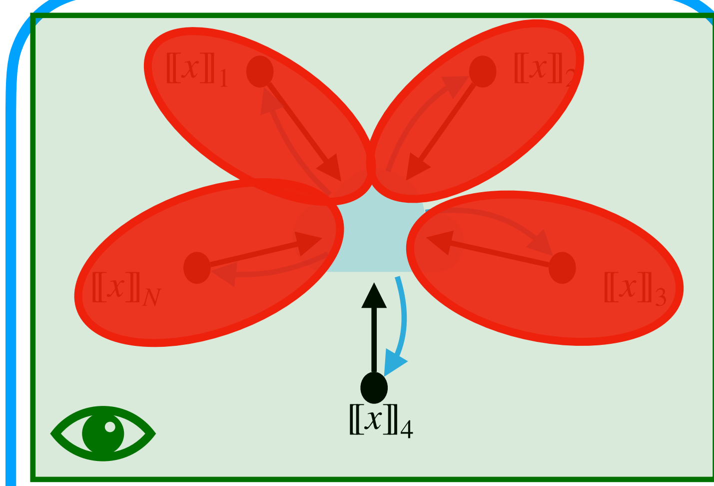

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$\Rightarrow$ soundness error = $(N - \ell)/N$ 🤔

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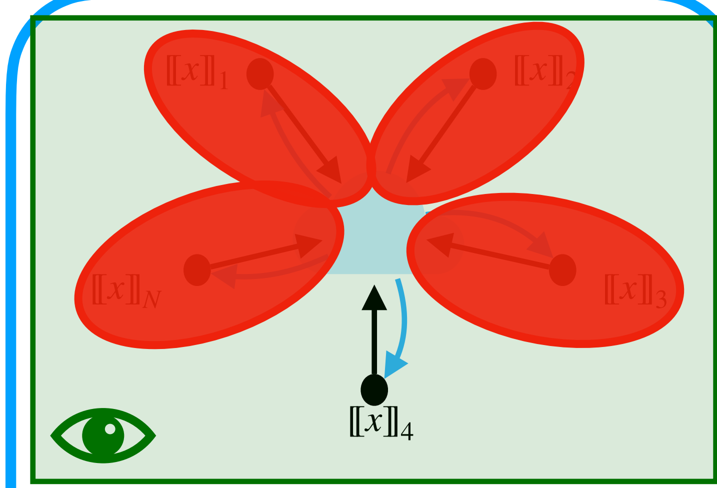

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💡 broadcast messages must be valid Shamir's sharings

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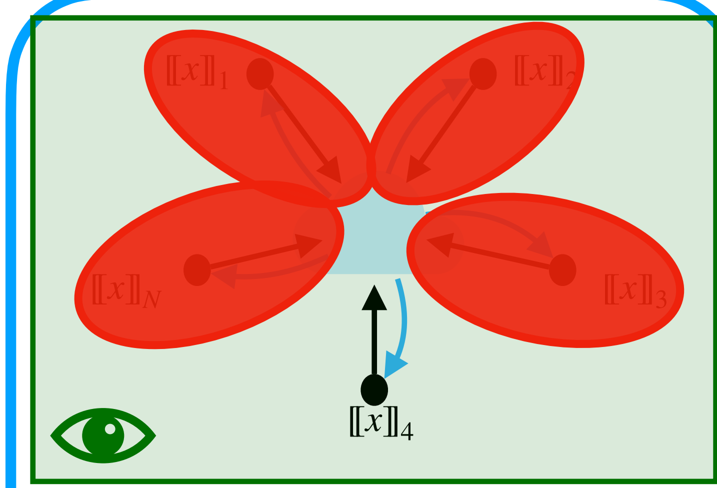

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Prover

Sharing / MPC protocol *ℓ-private*

~~$\Rightarrow$ soundness error $= (N - \ell)/N$~~ 🤔

💡 broadcast messages must be valid Shamir's sharings

$\Rightarrow$ soundness error $= \dfrac{1}{\binom{N}{\ell}}$ 🤩

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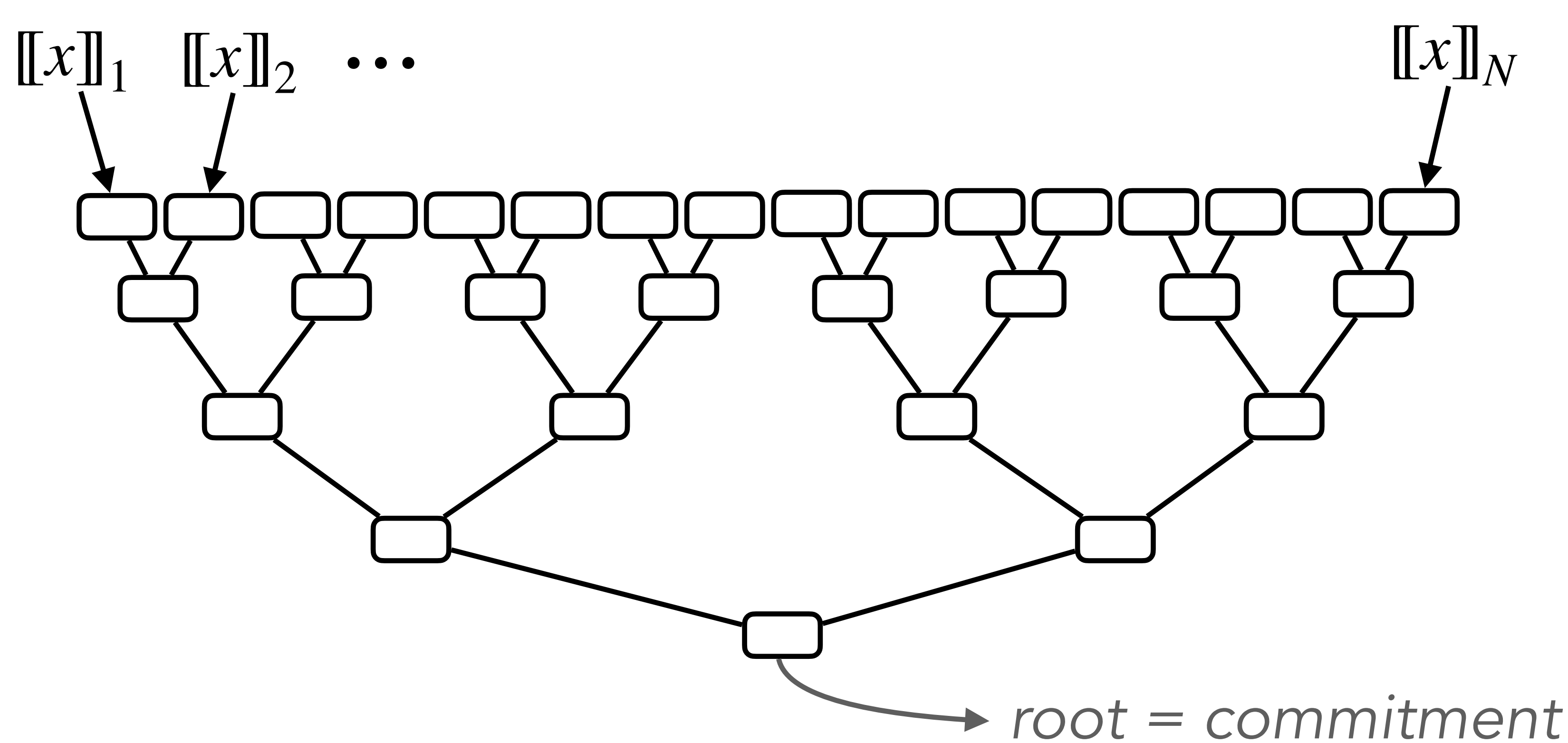
Committed using a Merkle tree:



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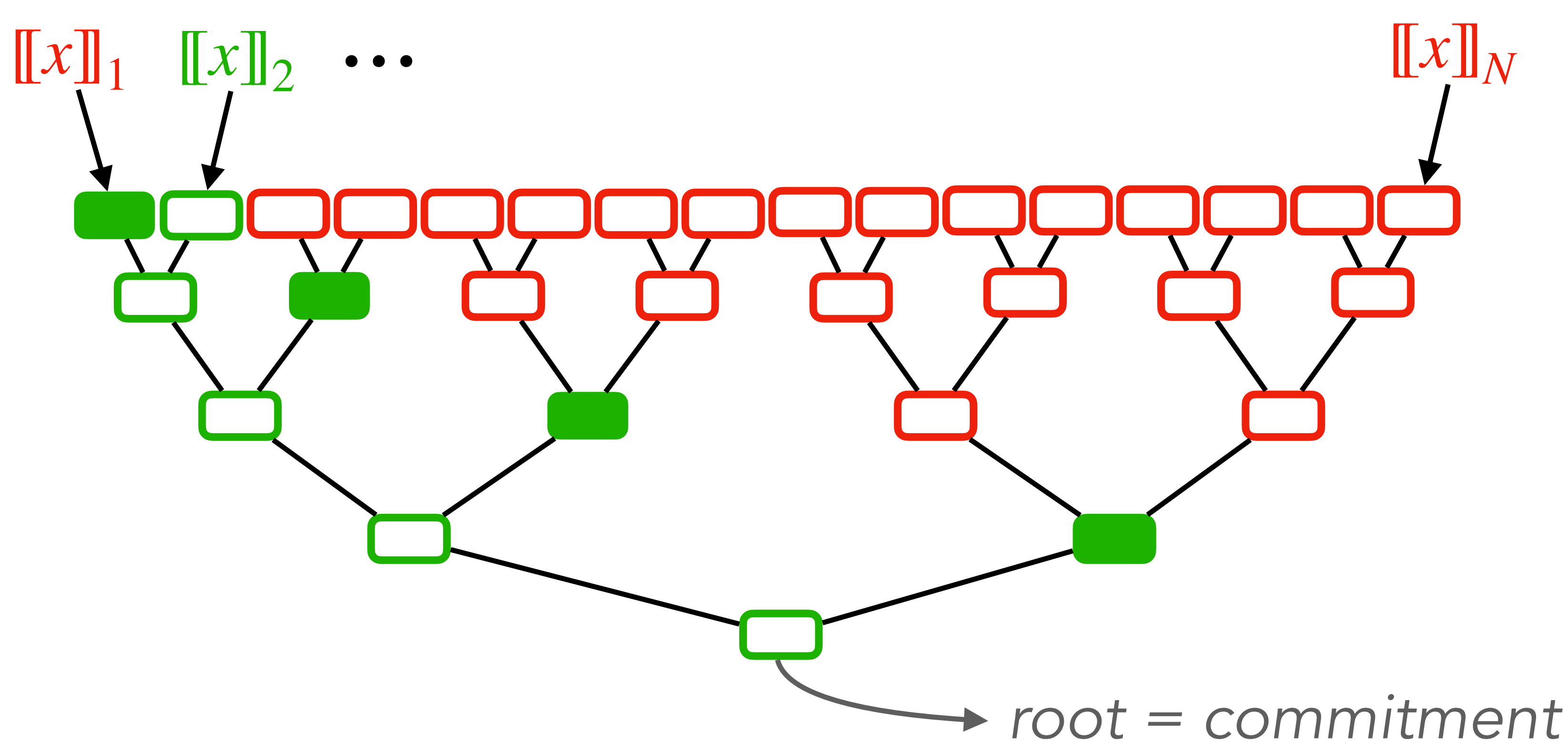

Soundness

$$\begin{aligned} p &= \text{“false positive probability”} \\ &= P\big[\text{MPC protocol accepts } [\![x]\!] \text{ while } f(x) \neq y\big] \end{aligned}$$

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Soundness error of standard MPCitH

$$\boxed{\frac{1}{N} + p}$$

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Soundness error of standard MPCitH

$$\frac{1}{N} + p$$

hope 🙏 →

Soundness error of TCitH

$$\frac{1}{\binom{N}{\ell}} + p$$

Soundness

$$p = \text{"false positive probability"}$$
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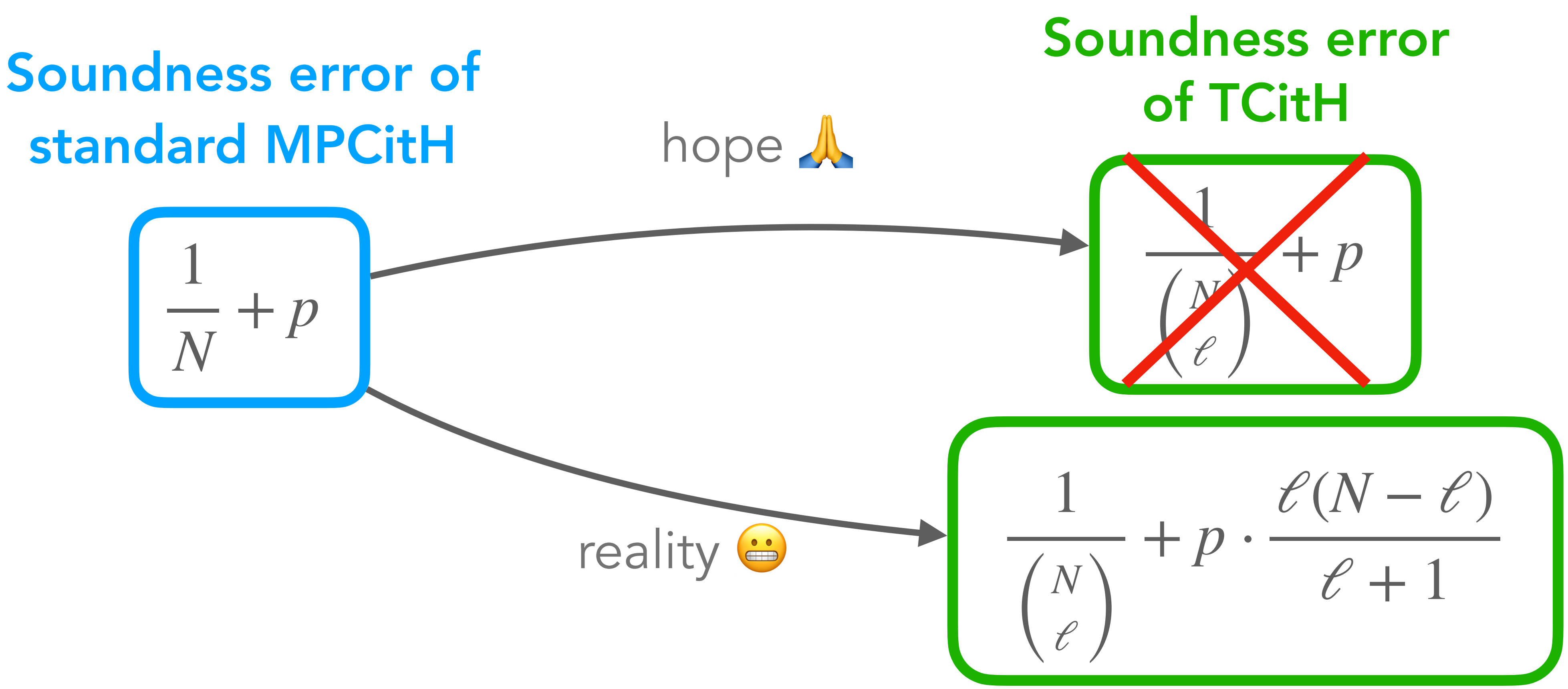

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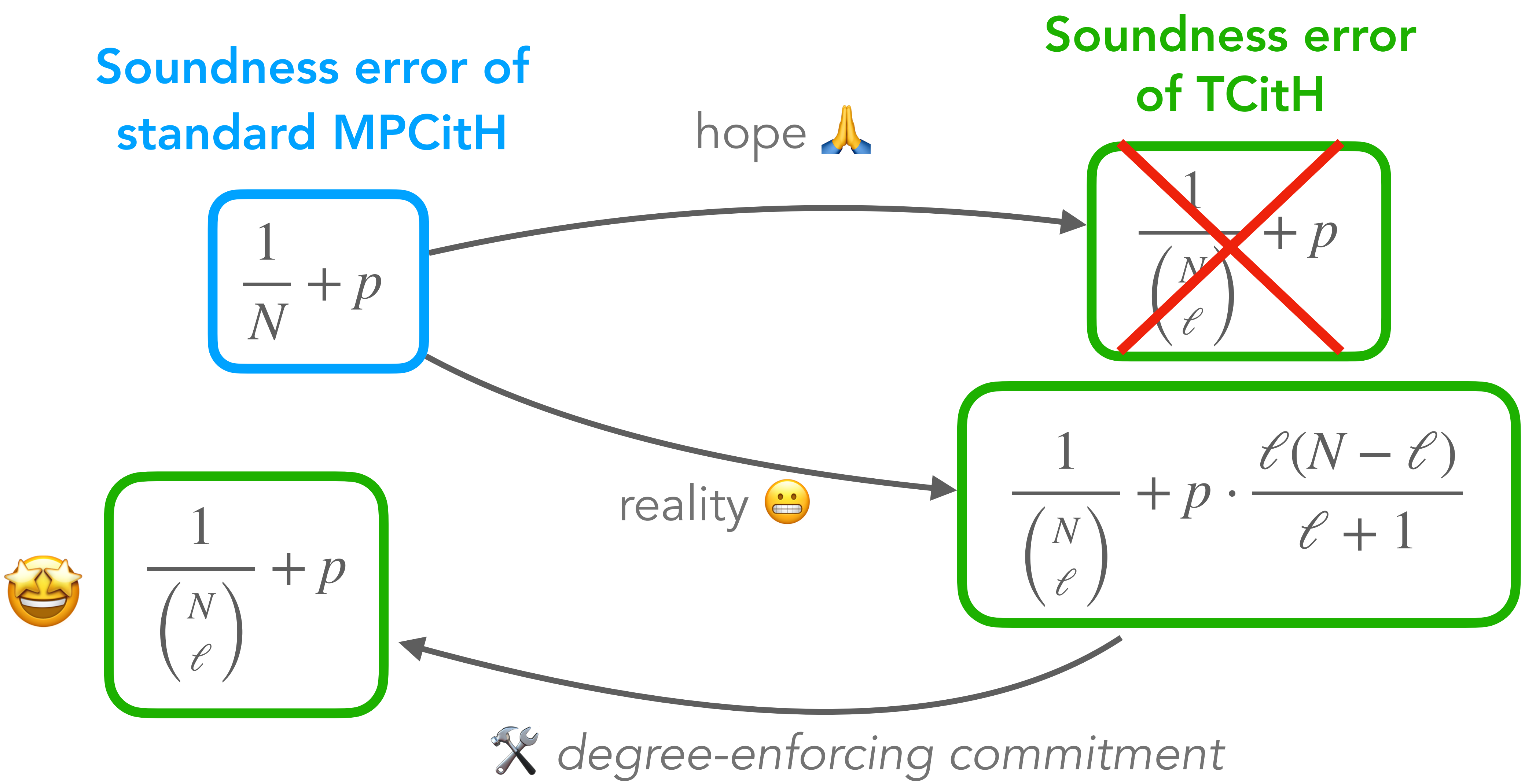

TCitH vs. standard MPCitH

$\ell = 1 \;\Rightarrow\;$ Similar soundness: $\dfrac{1}{N} + p$

TCitH vs. standard MPCitH

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🚀 *much less symmetric crypto*

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×2

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Getting rid of these limitations

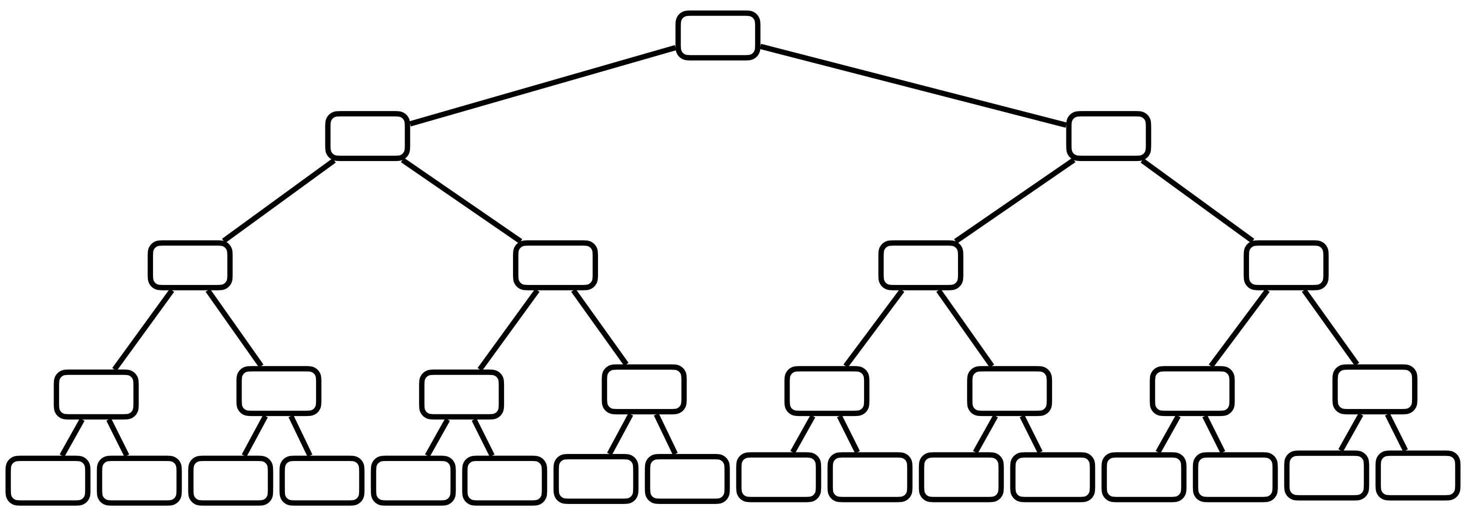
→ TCitH with GGM tree

TC-in-the-Head framework with GGM trees

TCitH with GGM trees

Step 1: Generate a replicated secret sharing [ISN89]

$x = r_1 + r_2 + \cdots + r_N$



$r_1 + r_2 + \quad \cdots \quad \cdots + r_N \quad + \Delta_x = x$

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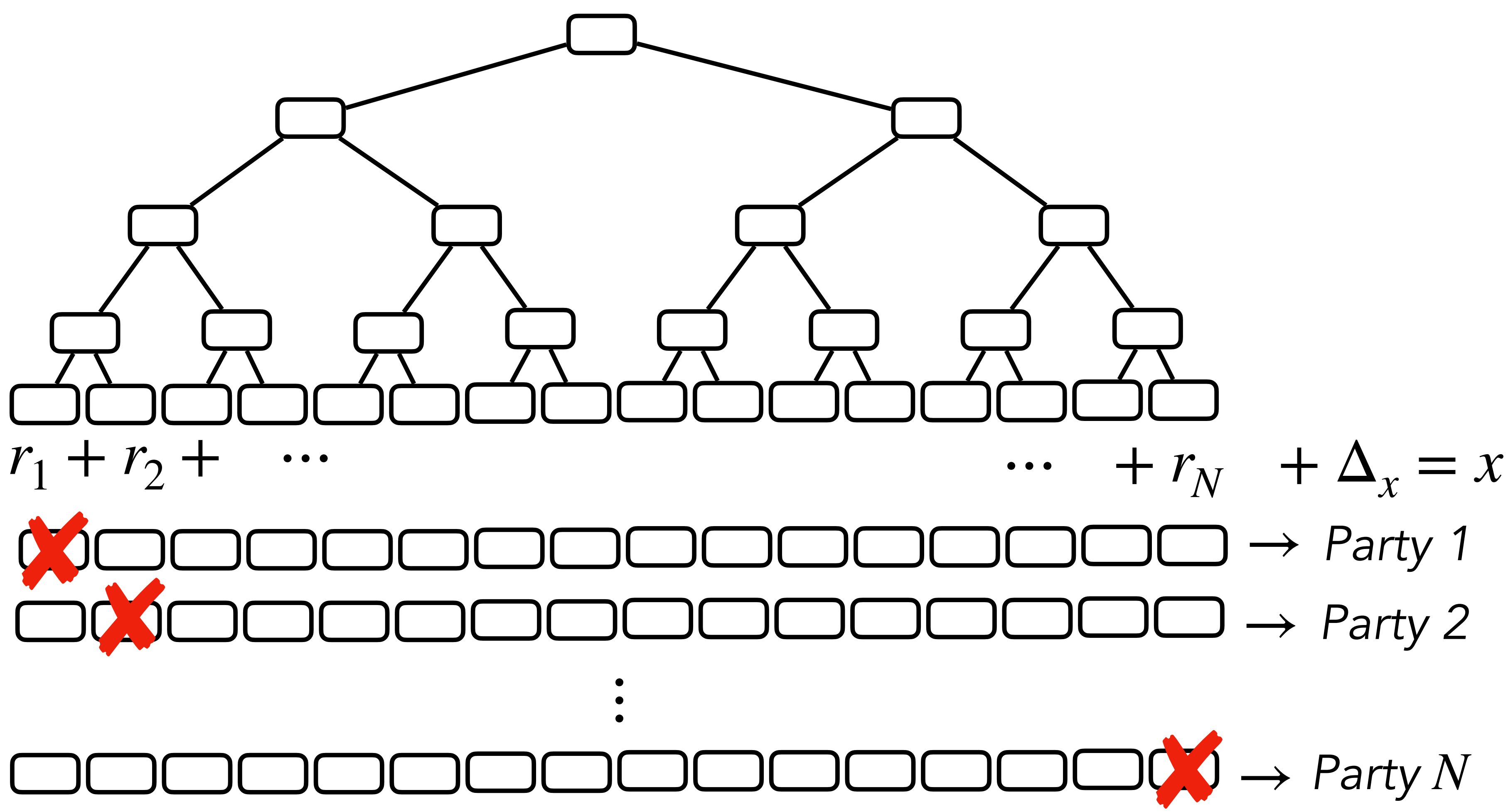

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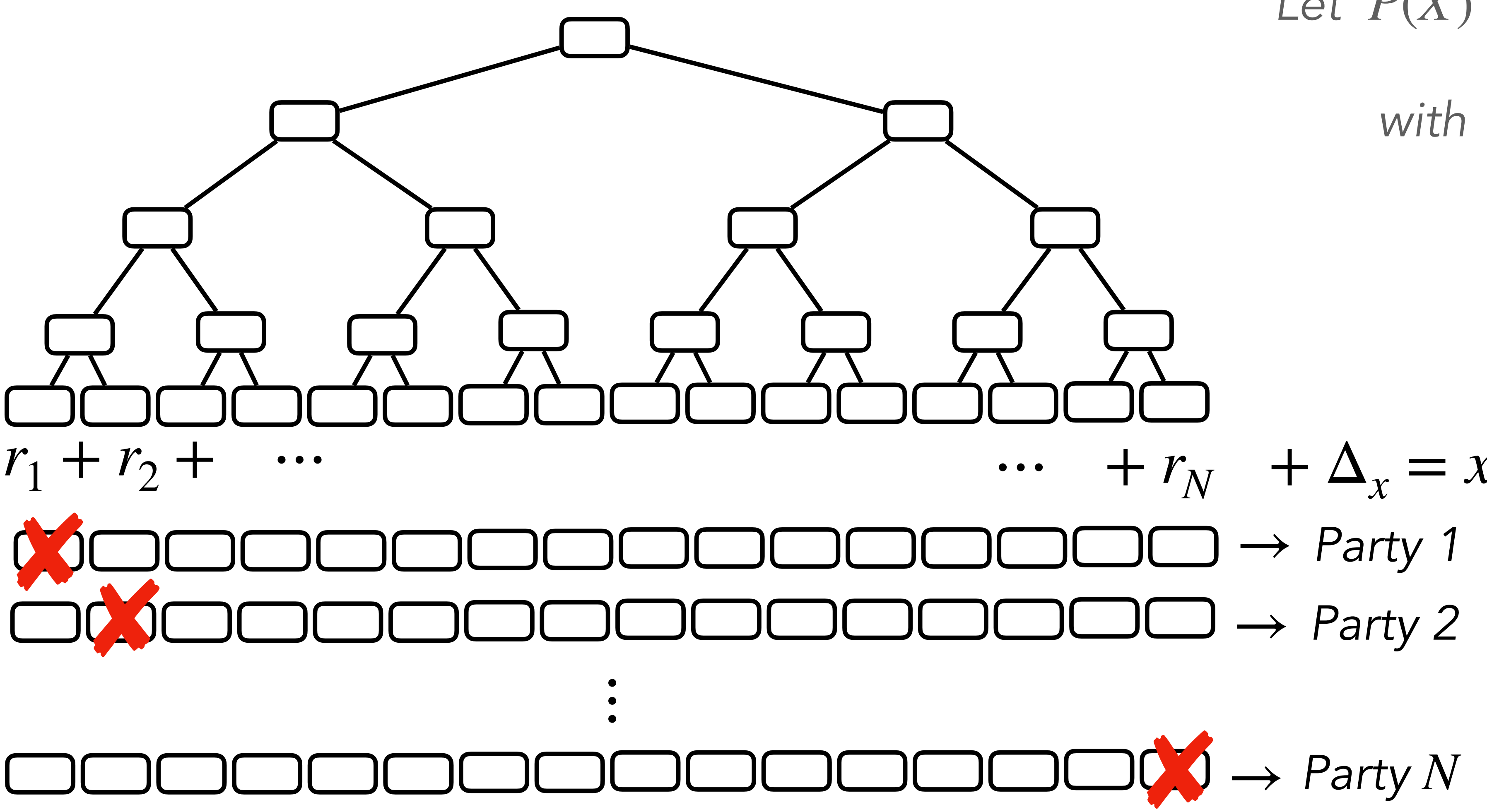

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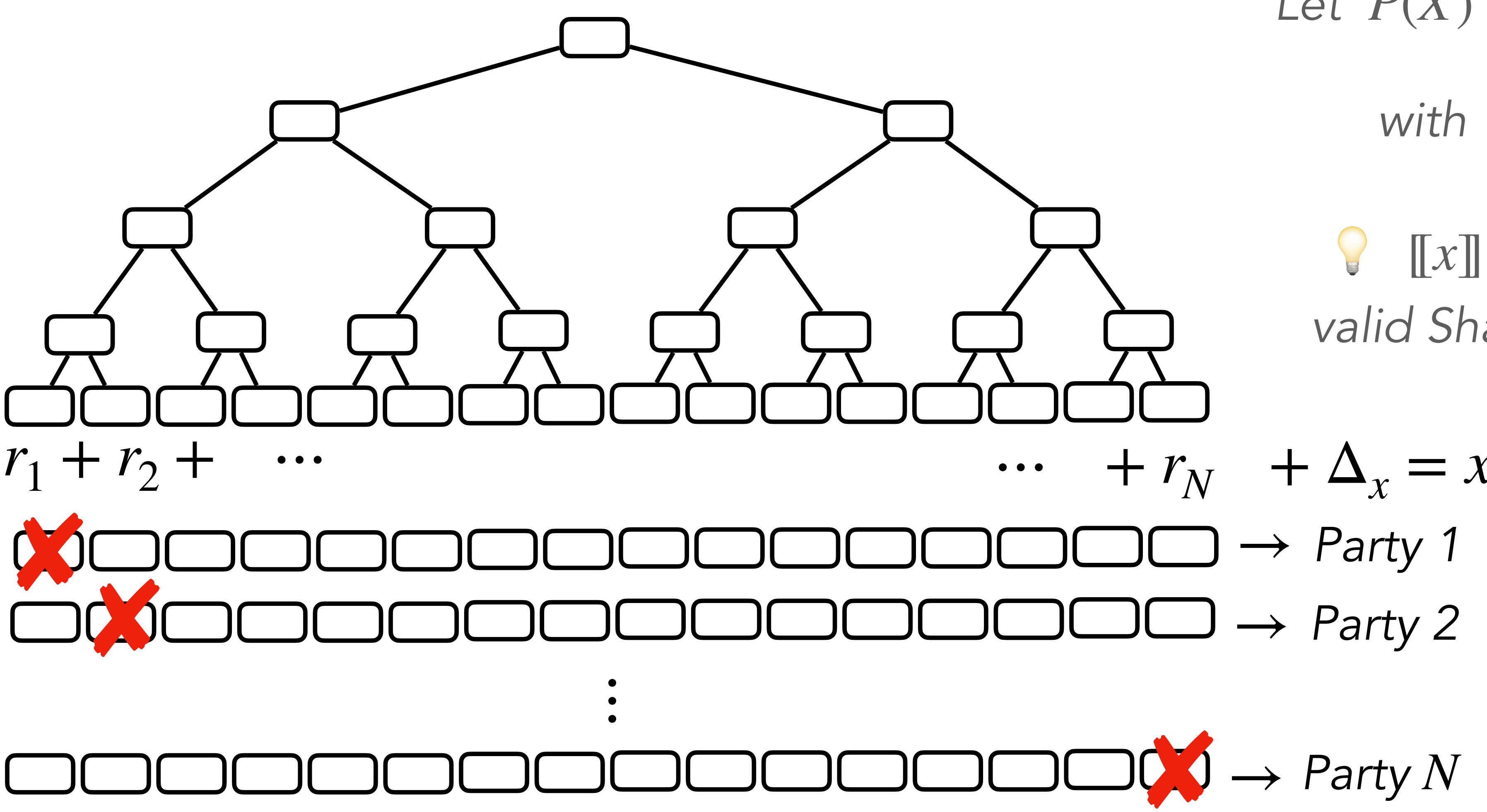

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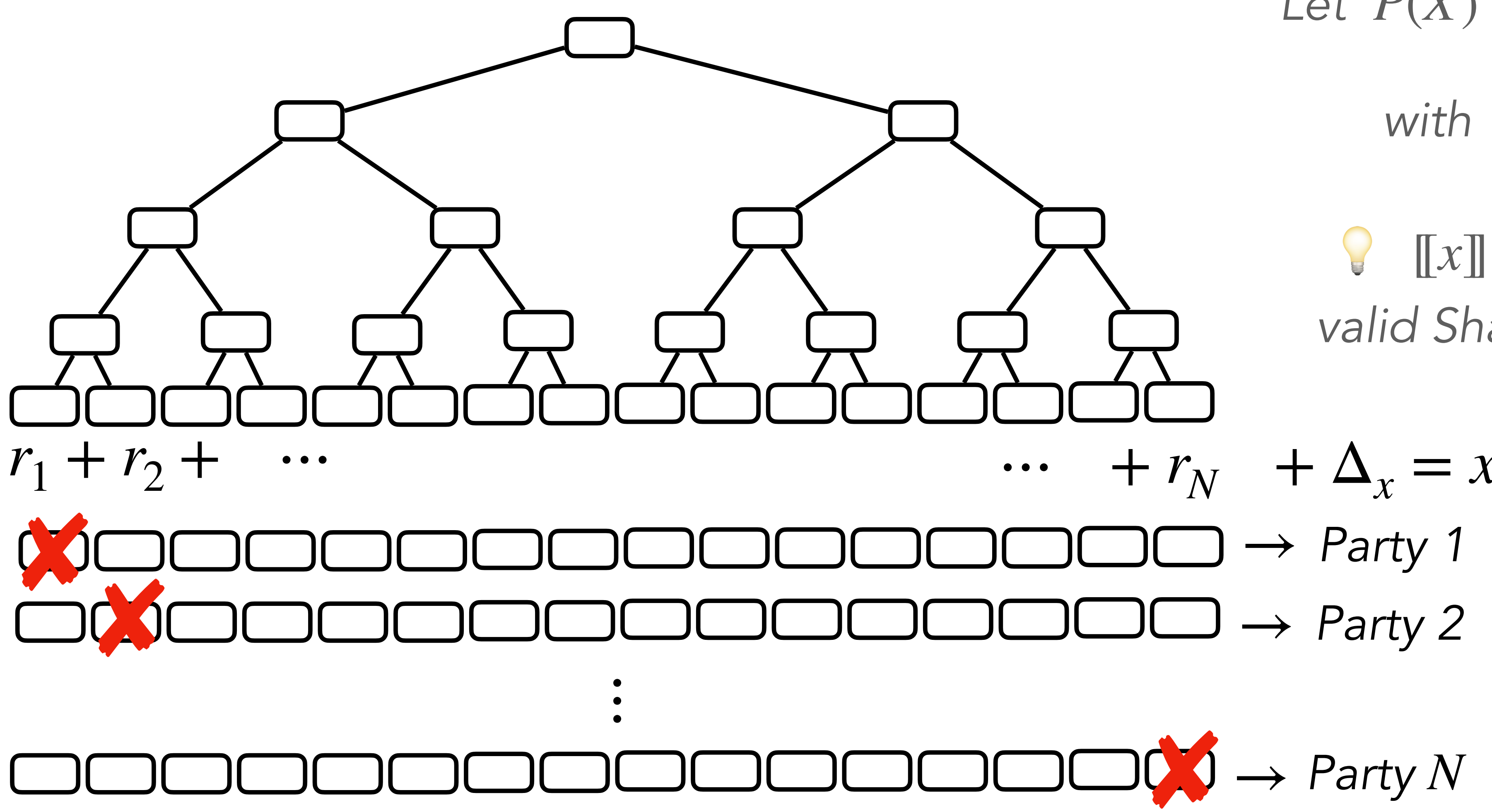

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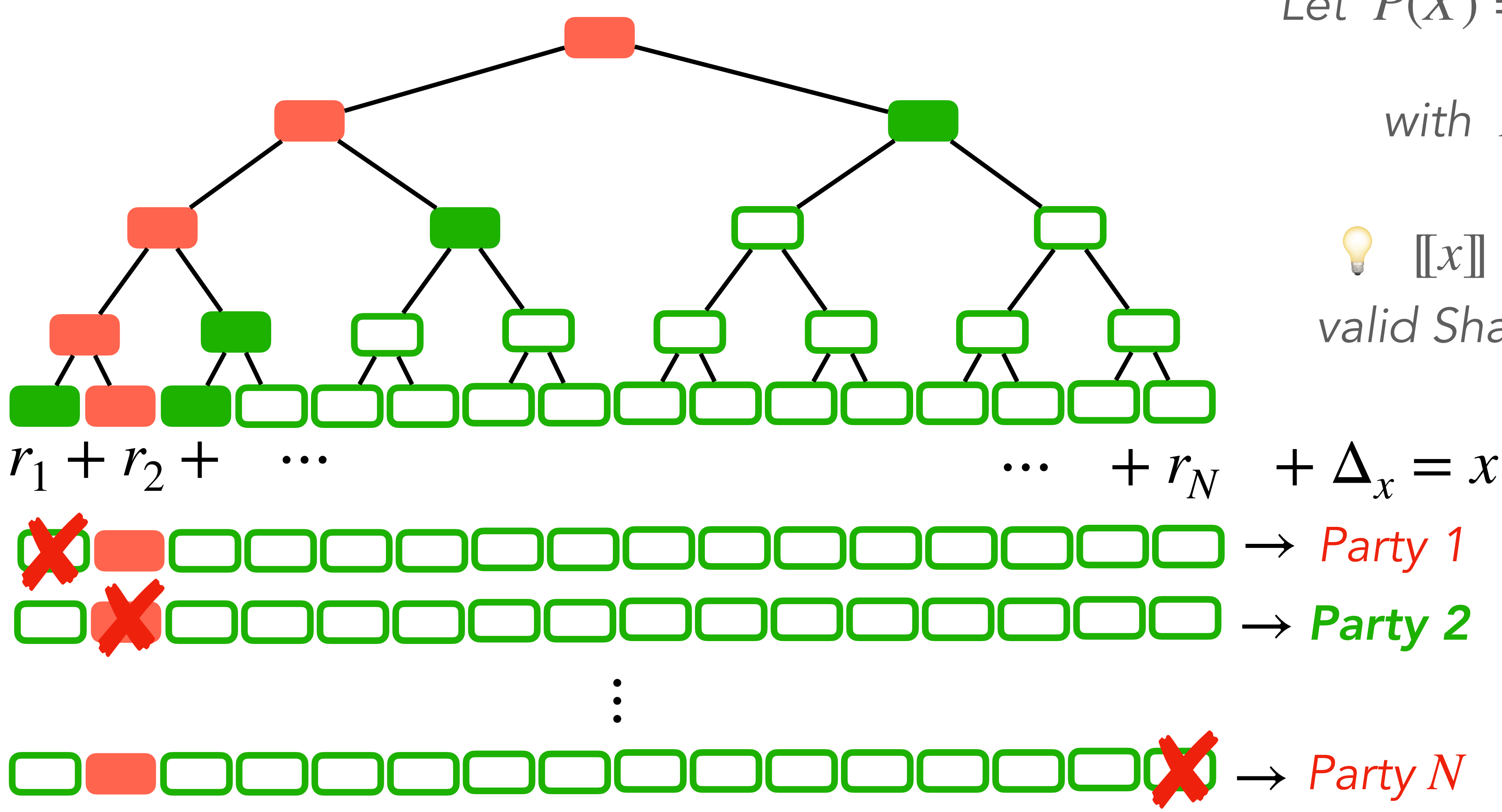

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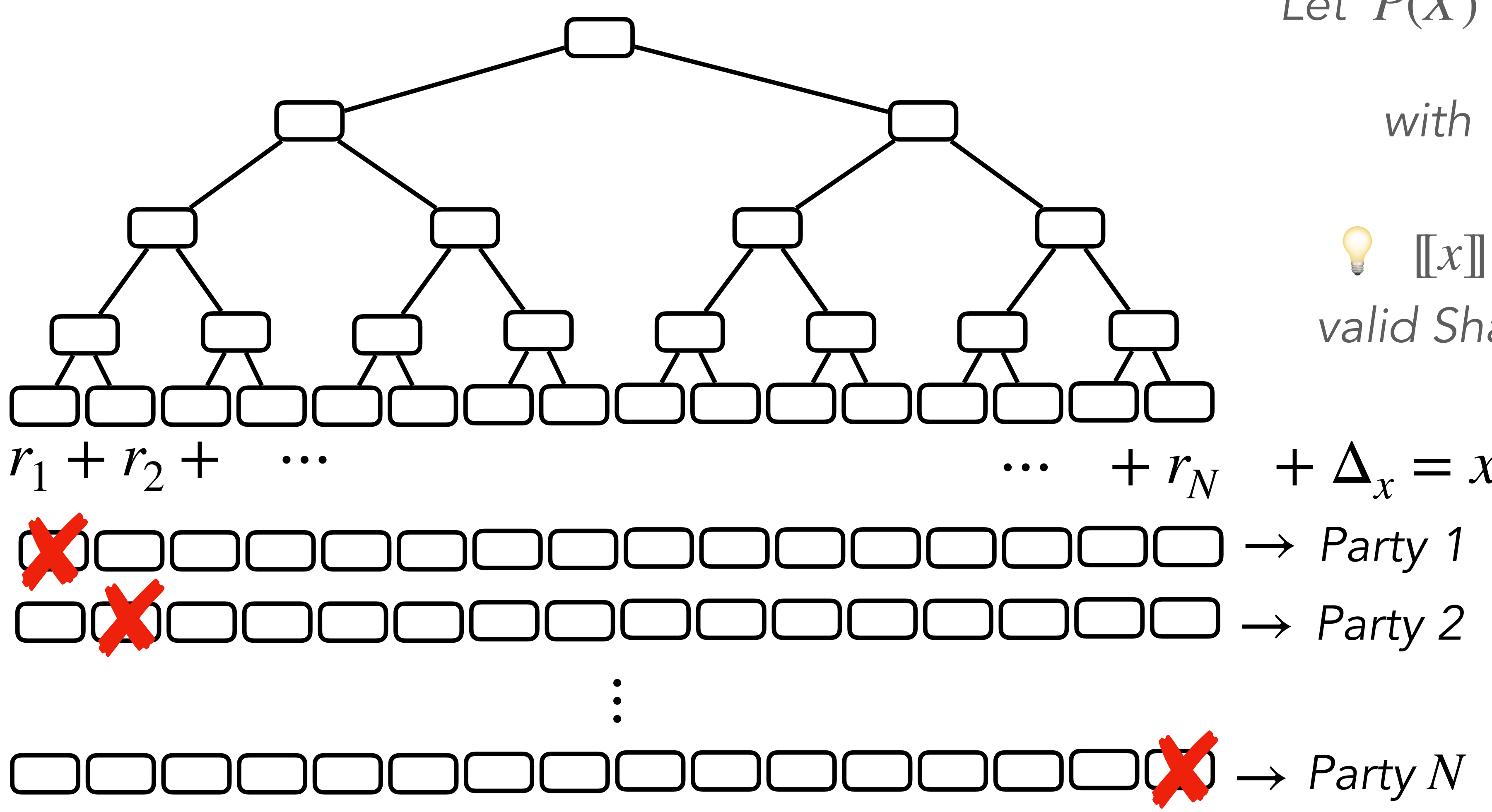

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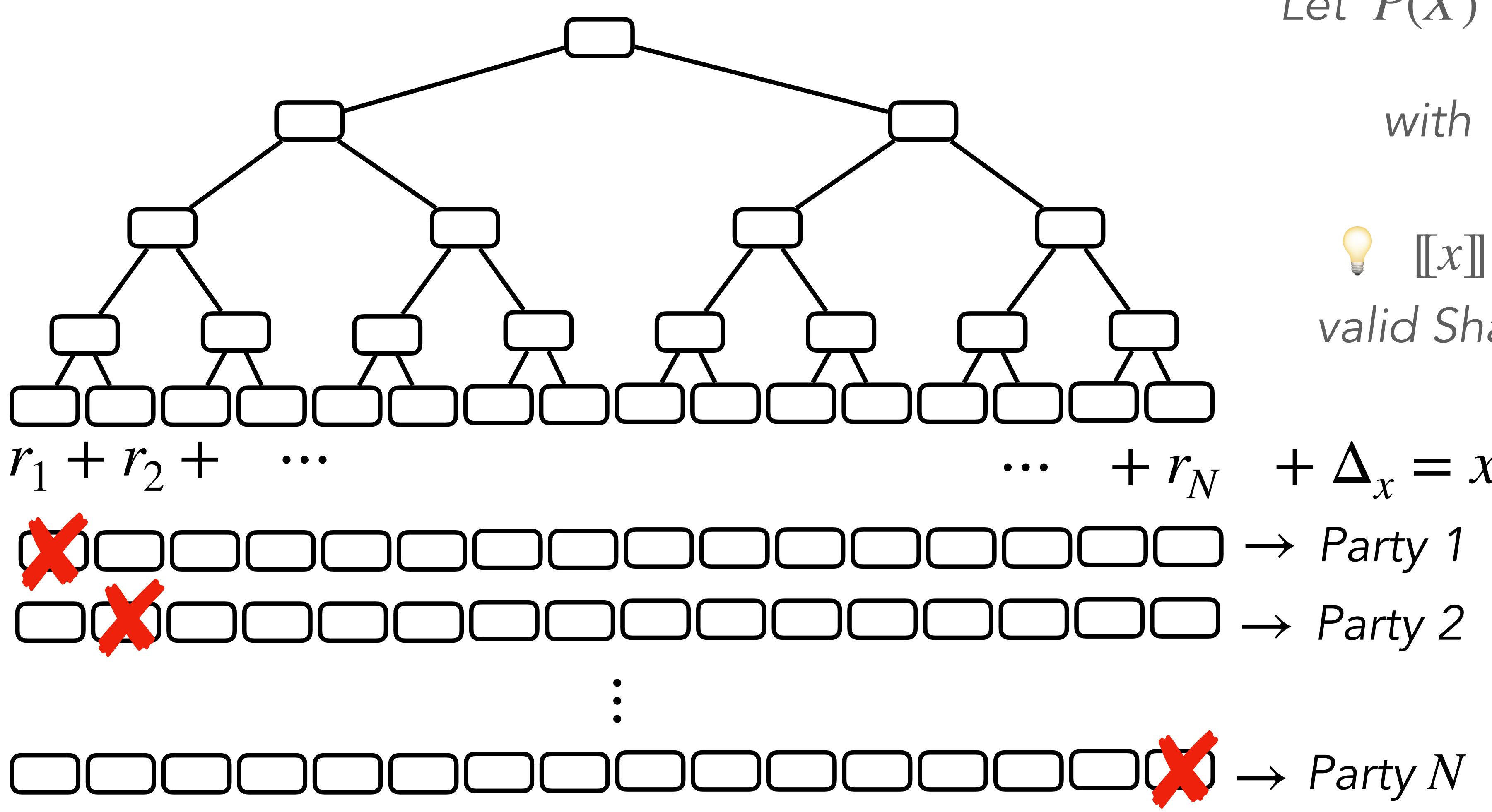

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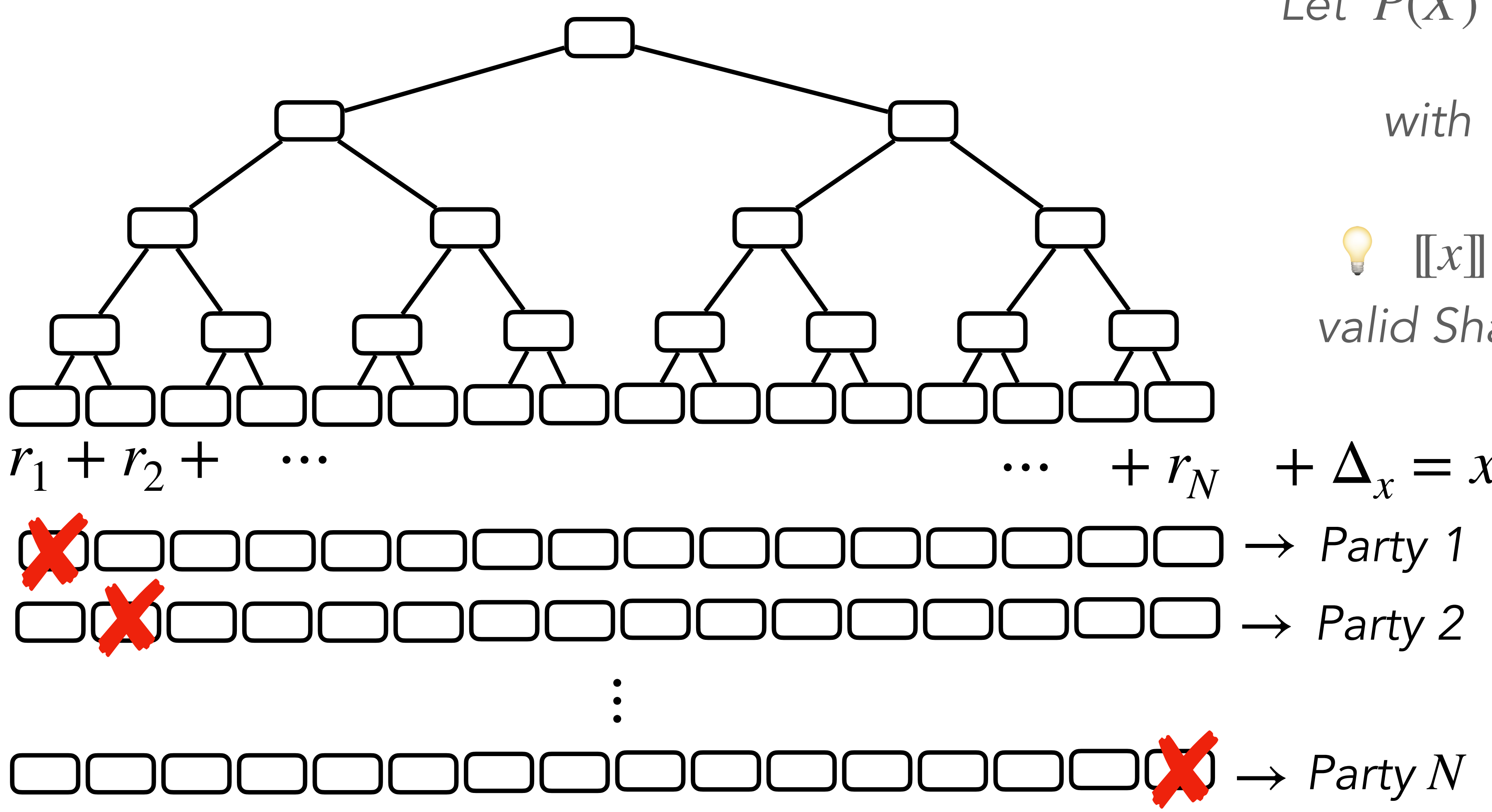

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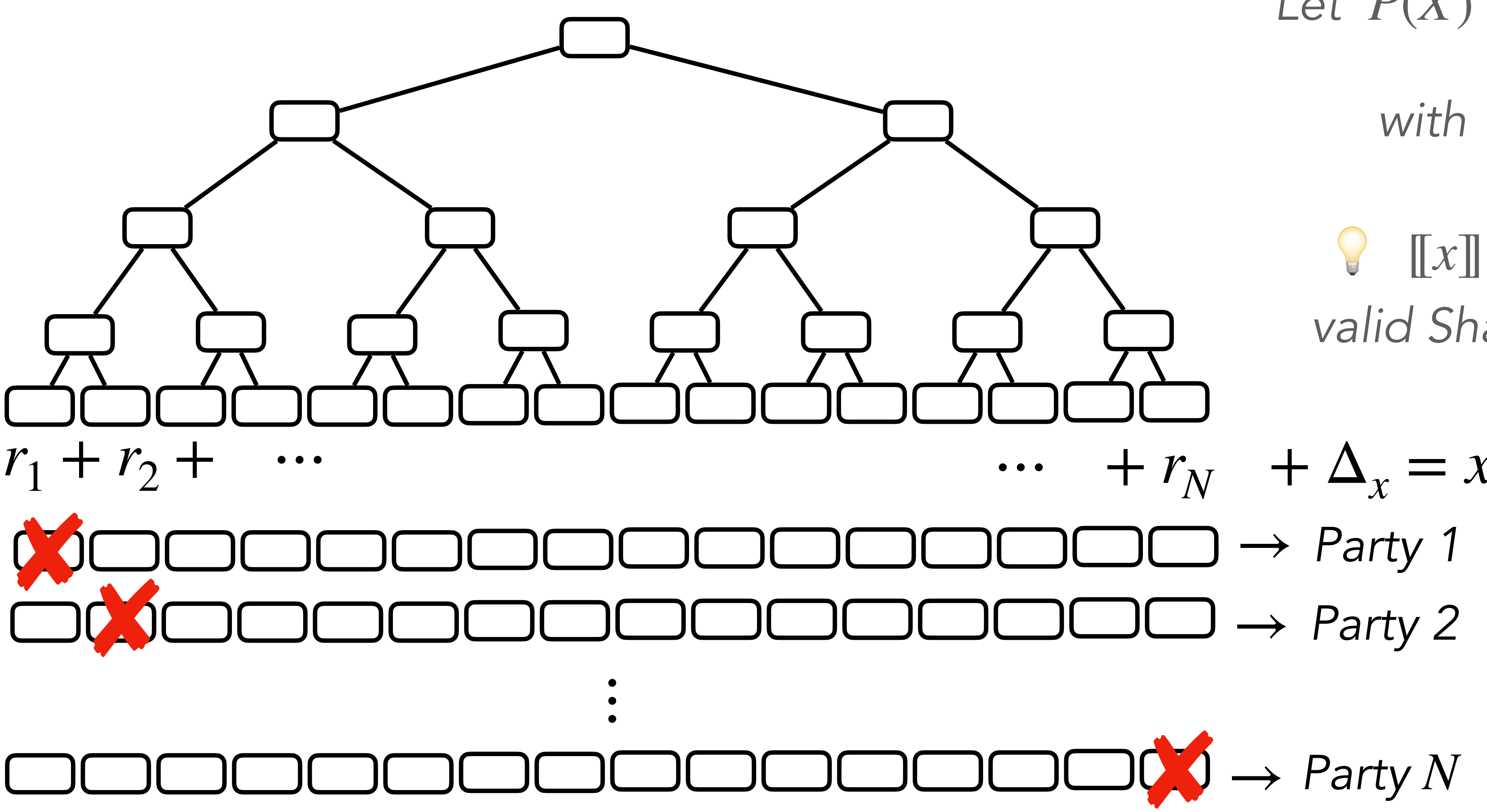

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- (Virtually) increase # party emulations:

$$1 + \left\lceil \frac{\log_2 N}{\log_2 |\mathbb{F}|} \right\rceil = \begin{cases} 2 & \text{if } |\mathbb{F}| \geq N \\ & \vdots \\ 1 + \log_2 N & \text{if } |\mathbb{F}| = 2 \end{cases}$$

Speedups for MPCitH candidates

	Additive MPCitH		TCitH (GGM tree)	
	Traditional (ms)	Hypercube (ms)	TCitH (ms)	Saving
Party emulations / repetition	N	$1 + \log_2 N$	$1 + \left\lceil \frac{\log_2 N}{\log_2 \lvert \mathbb{F} \rvert} \right\rceil$	
AIMer	4.53	3.22	3.22	-0 %
Biscuit	17.71	4.65	4.24	-16 %
MIRA	384.26	20.11	9.89	-51 %
MiRitH-Ia	54.15	6.60	5.42	-18 %
MiRitH-Ib	89.50	8.66	6.66	-23 %
MQOM-31	96.41	11.27	8.74	-21 %
MQOM-251	44.11	7.56	5.97	-21 %
RYDE	12.41	4.65	4.65	-0 %
SDitH-256	78.37	7.23	5.31	-27 %
SDitH-251	19.15	7.53	6.44	-14 %

- Comparison based on a generic MPCitH library (libmpcith)
- Code for MPC protocols fetched from the submission packages

Using multiplication homomorphism
& packed secret sharing

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Soundness error

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γ_j: randomness from the verifier

Soundness error

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Here: $d_\alpha = \ell \cdot \deg f_j$, $p = \left(\frac{1}{|\mathbb{F}|}\right)^{\#\alpha}$

Shorter signatures for MPCitH-based candidates

	Original Size	*Our Variant*	*Saving*
Biscuit	4 758 B	4 048 B	-15 %
MIRA	5 640 B	5 340 B	-5 %
MiRitH-Ia	5 665 B	4 694 B	-17 %
MiRitH-Ib	6 298 B	5 245 B	-17 %
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RYDE	5 956 B	4 274 B	
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Shorter signatures for MPCitH-based candidates

Two very recent works :

- Baum, Beullens, Mukherjee, Orsini, Ramacher, Rechberger, Roy, Scholl. *One Tree to Rule Them All: Optimizing GGM Trees and OWFs for Post-Quantum Signatures.* https://ia.cr/2024/490
  - General techniques to reduce the size of GGM trees
  - **Apply to TCitH-GGM** (gain of ~500 B at 128-bit security)

- Bidoux, Feneuil, Gaborit, Neveu, Rivain. *Dual Support Decomposition in the Head: Shorter Signatures from Rank SD and MinRank.* https://ia.cr/2024/541
  - New MPC protocols for TCitH / VOLEitH signatures based on **MinRank & Rank SD**

Using packed secret sharing

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  - $P(\omega_1) = x_1 , \quad \ldots , \quad P(\omega_s) = x_s$
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Here: $(\ell + s - 1) \cdot \deg f_j$

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  $\Rightarrow$ interesting for statements with "medium size" witness
- E.g. an ISIS statement $\vec{t} = A \cdot \vec{e}$ with $\| \vec{e} \|_\infty \leq \beta$

Here: $(\ell + s - 1) \cdot \deg f_j$

$$\frac{\binom{d_\alpha}{\ell}}{\binom{N}{\ell}} + p$$

Soundness error

TCitH-GGM vs. TCitH-MT

TCitH-GGM	*TCitH-MT*
🎄 Smaller tree	🌲 Larger tree (~x2)

TCitH-GGM vs. TCitH-MT

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☑️ Naturally enforce degree of committed sharings	🔁 Need degree enforcing commitment (+1 round)

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📦❌ No advantage of packed sharing	📦 Takes advantage of packed sharing
☑️ Naturally enforce degree of committed sharings	🔁 Need degree enforcing commitment (+1 round)
🎯 Better for "small-size" statements	🎯 Better for "medium-size" statements

Application: post-quantum ring signatures

Post-quantum ring signatures

- Secret key w
- One-way function f
- Public key $y = f(w)$
- MPC protocol $\Pi : [\![w]\!] \mapsto 0/1$

TCitH → *FS*

✍️ signature scheme

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TCitH

FS

✍️ signature scheme

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- Public keys $y_1, \ldots, y_r$
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$$\Pi' : [\![w_{j^*}]\!], [\![j^*]\!] \mapsto 0/1$$

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TCitH → FS

💍✍️ ring signature scheme

Post-quantum ring signatures

💡 Idea:

- One-hot encoding of $j*$

$$s = (0,\ldots,0, s_{j*} := 1, 0,\ldots,0)$$

Post-quantum ring signatures

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Post-quantum ring signatures

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Post-quantum ring signatures

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🛠 Solution: $[\![s^{(1)}]\!], \ldots, [\![s^{(d)}]\!]$ s.t. $s = s^{(1)} \otimes \cdots \otimes s^{(d)}$

$\Rightarrow \mathcal{O}\left(d\sqrt[d]{r}\right)$ signature size $\Rightarrow \mathcal{O}(\log r)$

Post-quantum ring signatures

Protocol Π'

Input: $[\![w_{j*}]\!], [\![s^{(1)}]\!], \ldots, [\![s^{(d)}]\!]$

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Post-quantum ring signatures

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🛠️ Simple
MPC protocol

Post-quantum ring signatures

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🛠 Simple MPC protocol

⚠️ Π must be adapted to use $[\![y_{j^*}]\!]$ instead of y_{j^*}

⚠️ Sharing degrees increase

Post-quantum ring signatures

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TCitH / FS

💍✍️ ring signature scheme

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Post-quantum ring signatures

#users		2^3	2^6	2^8	2^{10}	2^{12}	2^{20}	Assumption	Security
Our scheme	2023	4.41	4.60	4.90	5.48	5.82	8.19	MQ over $\mathbb{F}_{251}$	NIST I
Our scheme	2023	4.30	4.33	4.37	4.45	4.60	5.62	MQ over $\mathbb{F}_{256}$	NIST I
Our scheme	2023	7.51	8.40	8.72	9.36	10.30	12.81	SD over $\mathbb{F}_{251}$	NIST I
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Our scheme	2023	7.87	7.90	7.94	8.02	8.18	9.39	AES128	NIST I
Our scheme	2023	6.81	6.84	6.88	6.96	7.12	8.27	AES128-EM	NIST I
KKW [KKW18]	2018	-	250	-	-	456	-	LowMC	NIST V
GGHK [GGHAK22]	2021	-	-	-	56	-	-	LowMC	NIST V
Raptor [LAZ19]	2019	10	81	333	1290	5161	-	MSIS / MLWE	100 bit
EZSLL [EZS+19]	2019	19	31	-	-	148	-	MSIS / MLWE	NIST II
Falafl [BKP20]	2020	30	32	-	-	35	-	MSIS / MLWE	NIST I
Calamari [BKP20]	2020	5	8	-	-	14	-	CSIDH	128 bit
LESS [BBN+22]	2022	11	14	-	-	20	-	Code Equiv.	128 bit
MRr-DSS [BESV22]	2022	27	36	64	145	422	-	MinRank	NIST I

Post-quantum ring signatures

Application to MQ, SD, AES

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Size range: 5–13 kB
for |ring|=2^{20}

Post-quantum ring signatures

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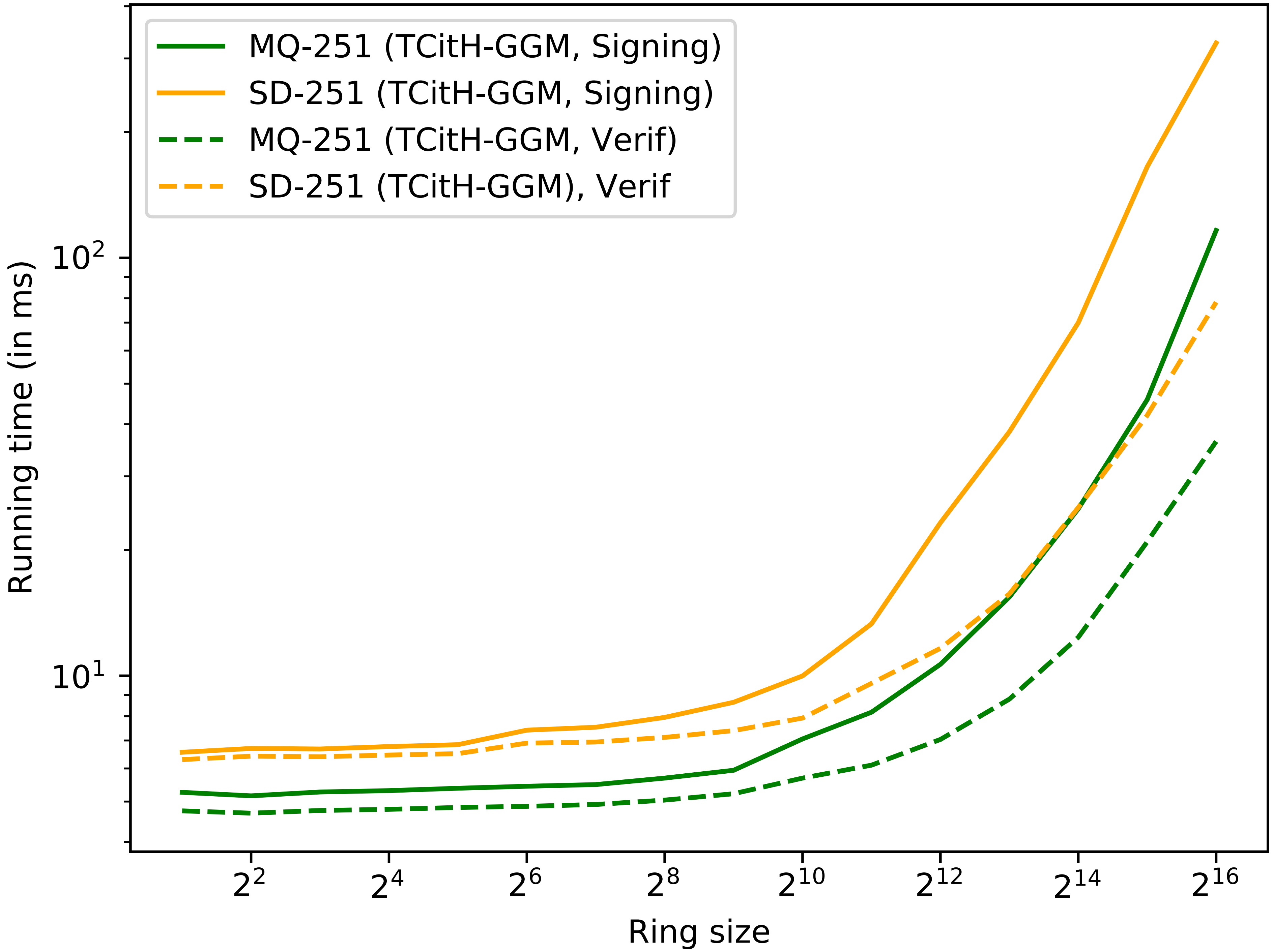
Size range: 5–13 kB for |ring|=2^{20}

Previous works:

$\geq$ 14 kB for |ring|=2^{10}

no / slow implementations

Post-quantum ring signatures

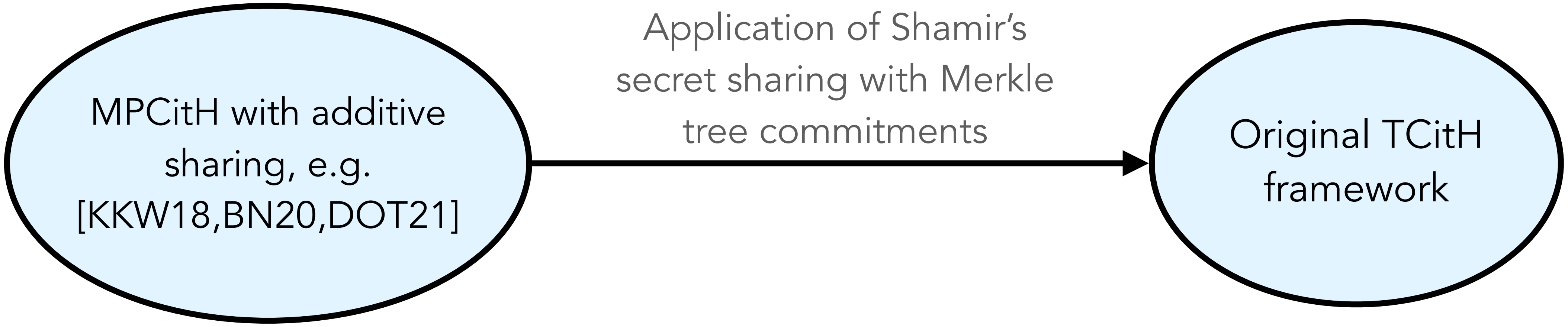


Relation to other proof systems

Connections to other proof systems

MPCitH with additive sharing, e.g. [KKW18,BN20,DOT21]

Connections to other proof systems



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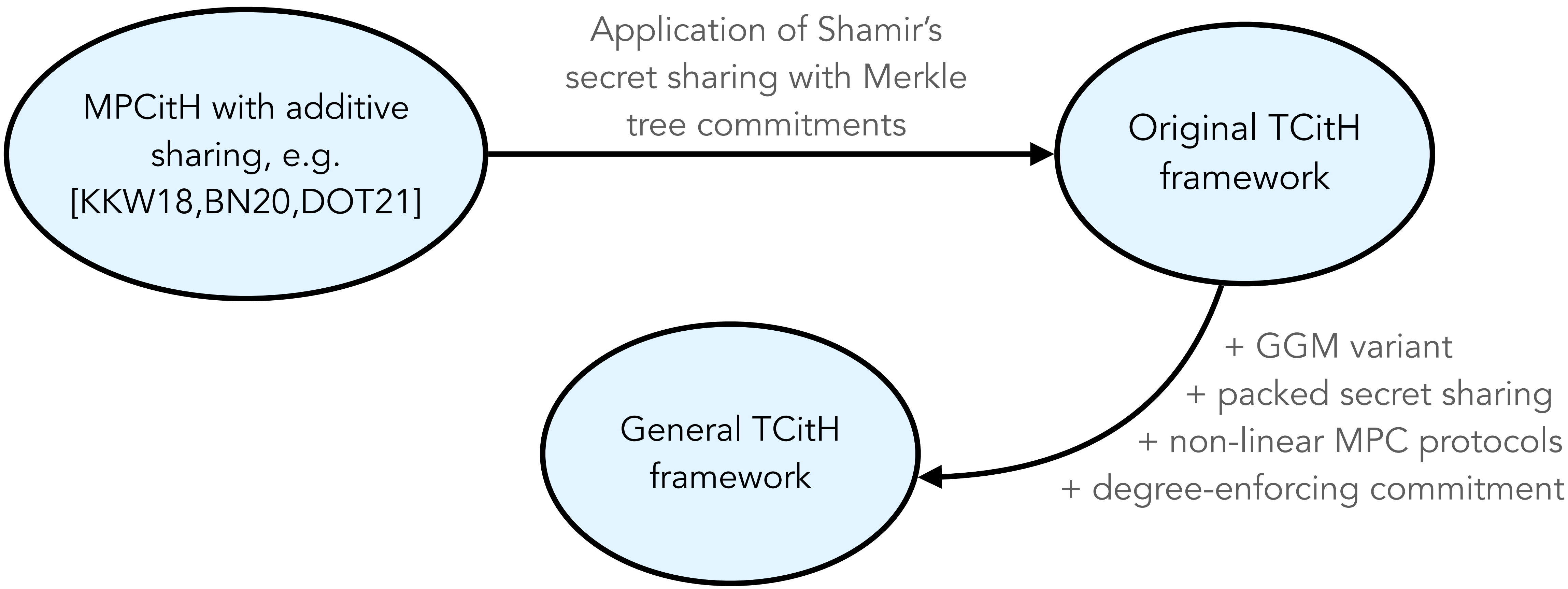

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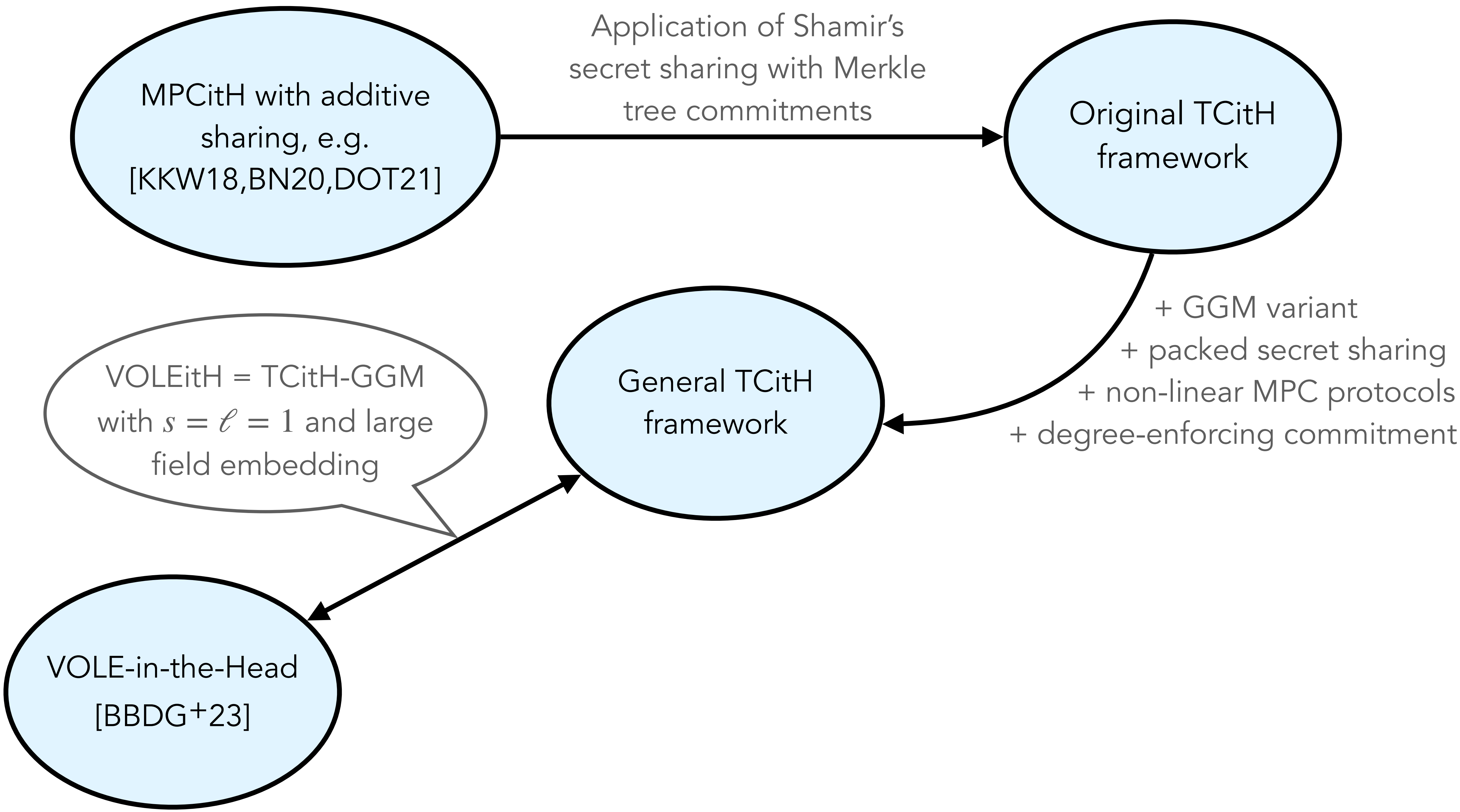

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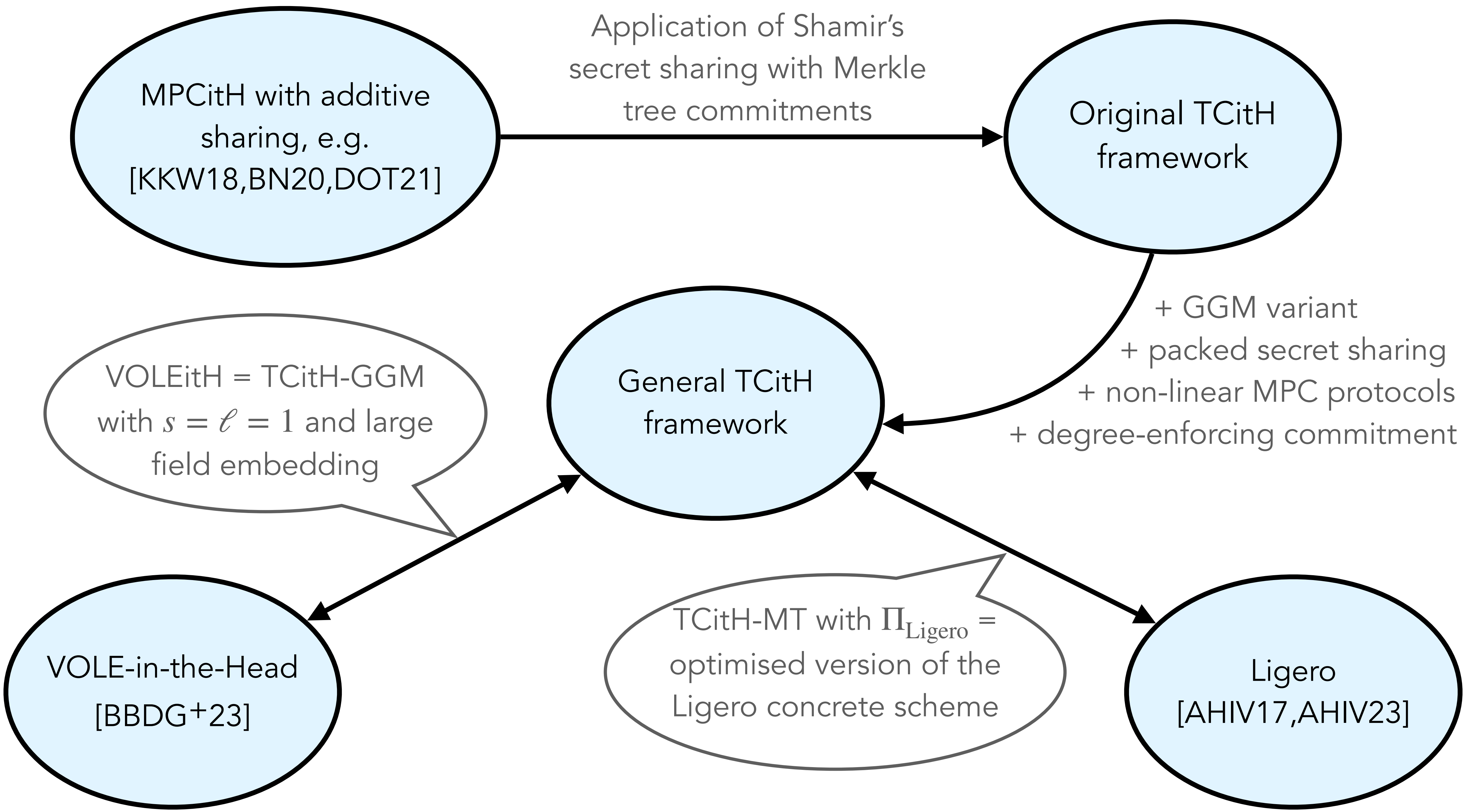

Thank you!

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