Post-Quantum Signatures from Threshold Computation in the Head

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Joint work with Thibauld Feneuil

NIST 5th PQC Standardization Conference

Washington DC, 11 April, 2024

CryptoExperts

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Roadmap

- MPC-in-the-Head paradigm
- Threshold Computation in the Head
    - Original framework (Asiacrypt 2023)
      https://ia.cr/2022/1407
    - Improved framework (preprint)
      https://ia.cr/2023/1573

MPC-in-the-Head paradigm

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

MPC-in-the-Head paradigm

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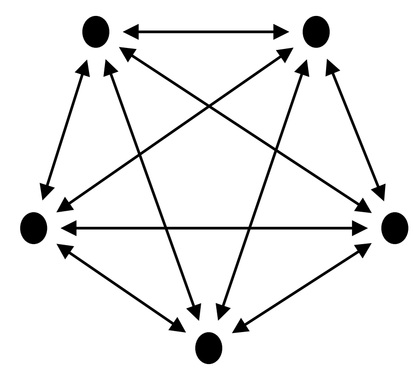
E.g. AES, MQ system,
Syndrome decoding

<u>Multiparty computation (MPC)</u>



Input sharing $[\![x]\!]$

Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

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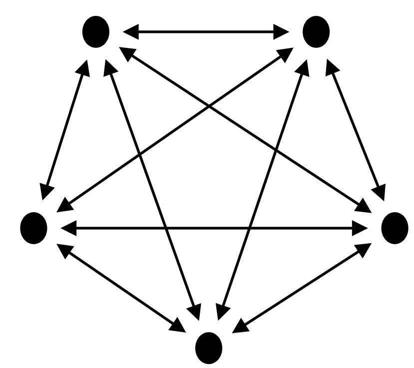

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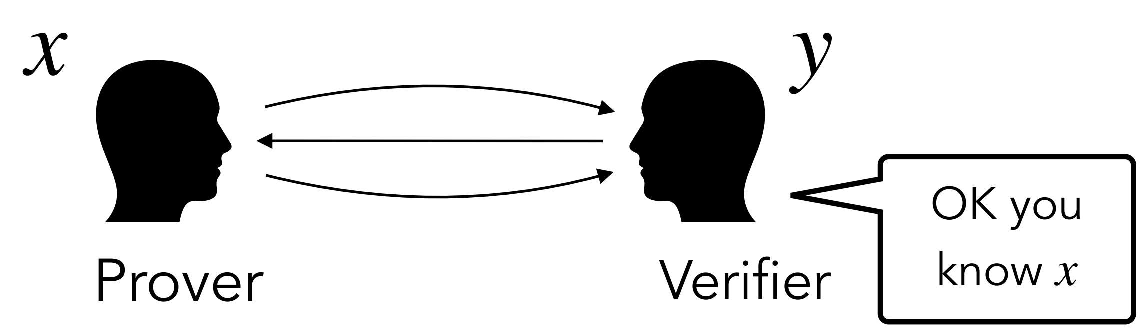
Zero-knowledge proof



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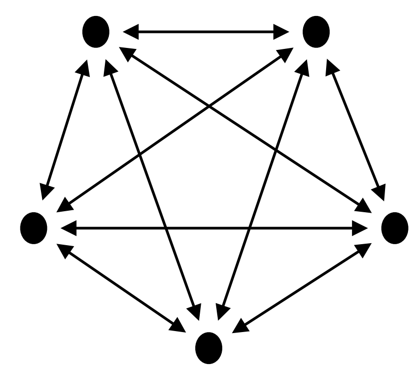

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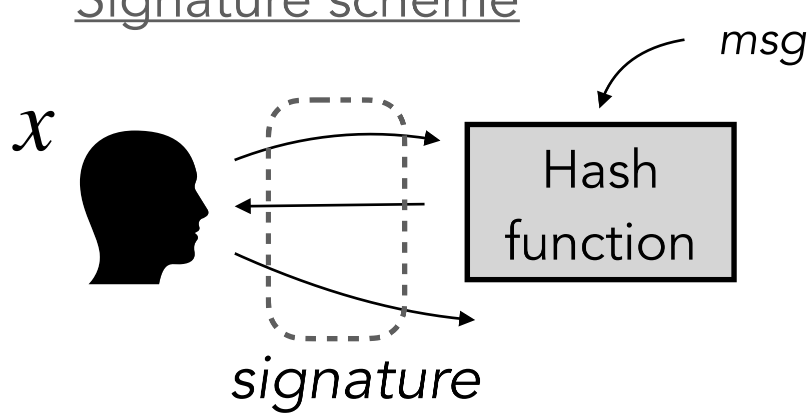
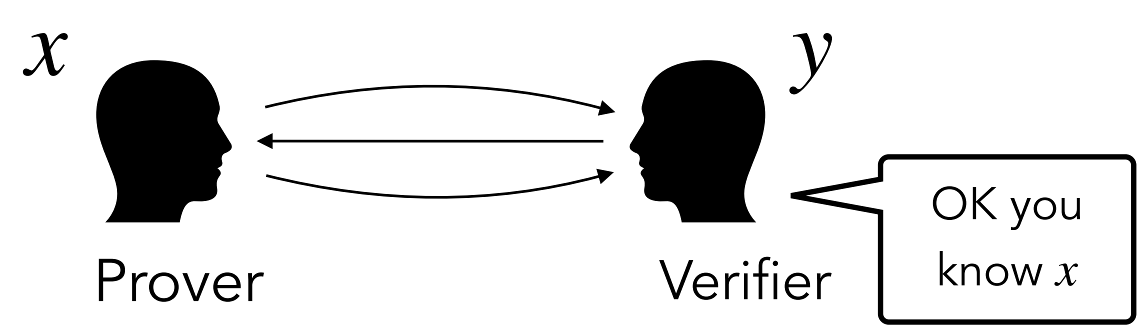
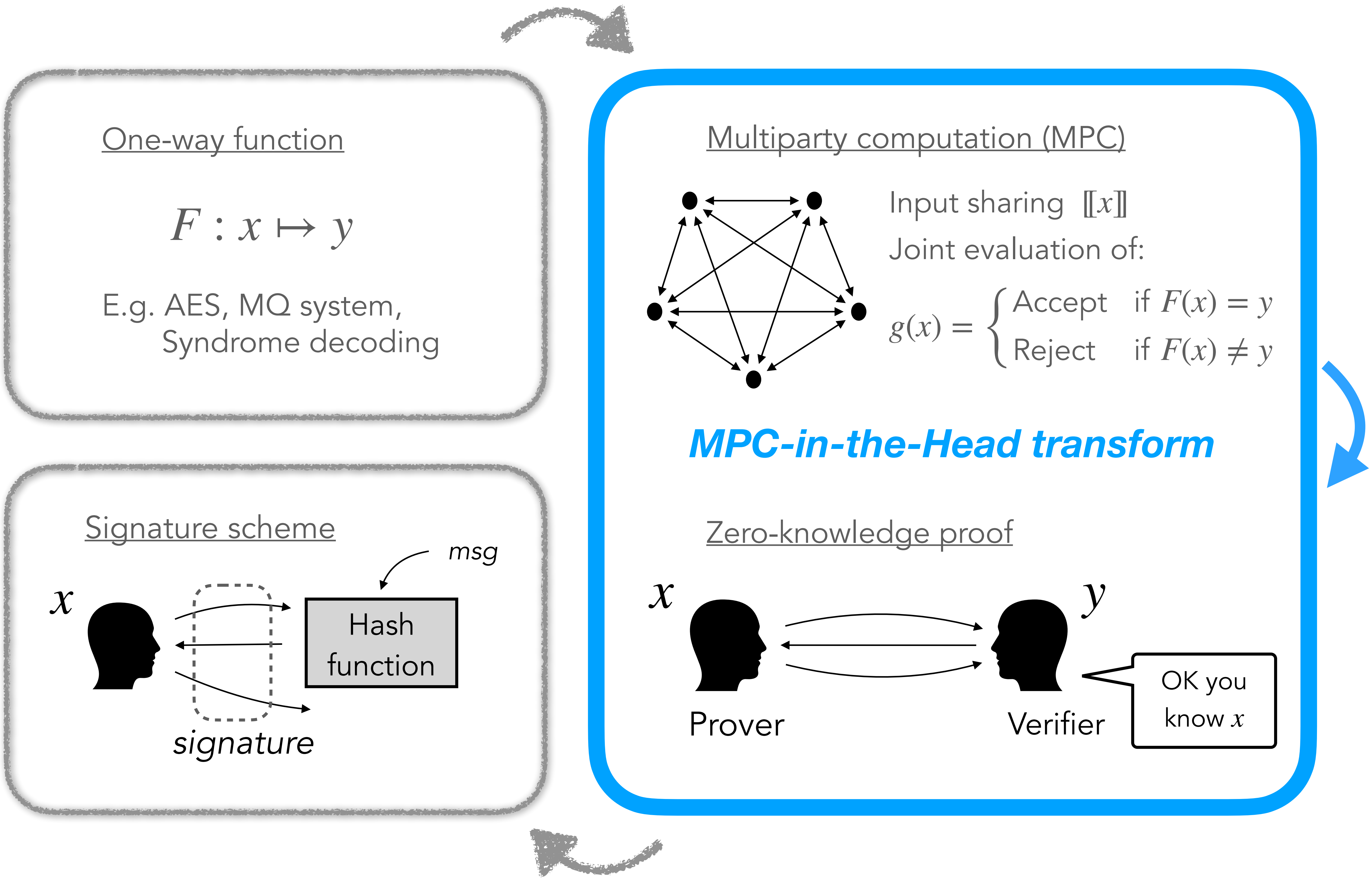
Signature scheme



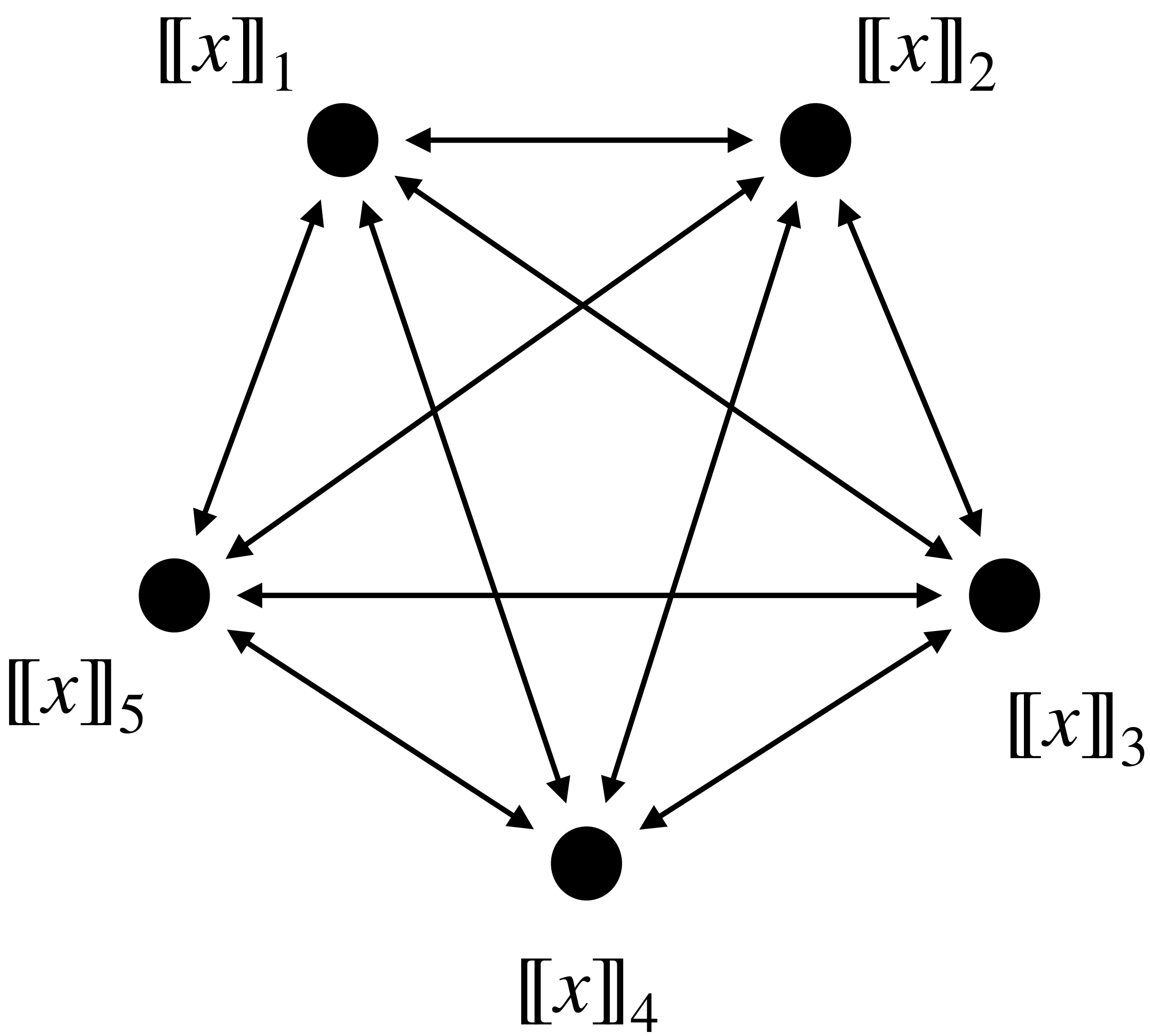
Zero-knowledge proof



MPC-in-the-Head paradigm



MPC model



$[\![x]\!]$ is a linear secret sharing of x

- Jointly compute

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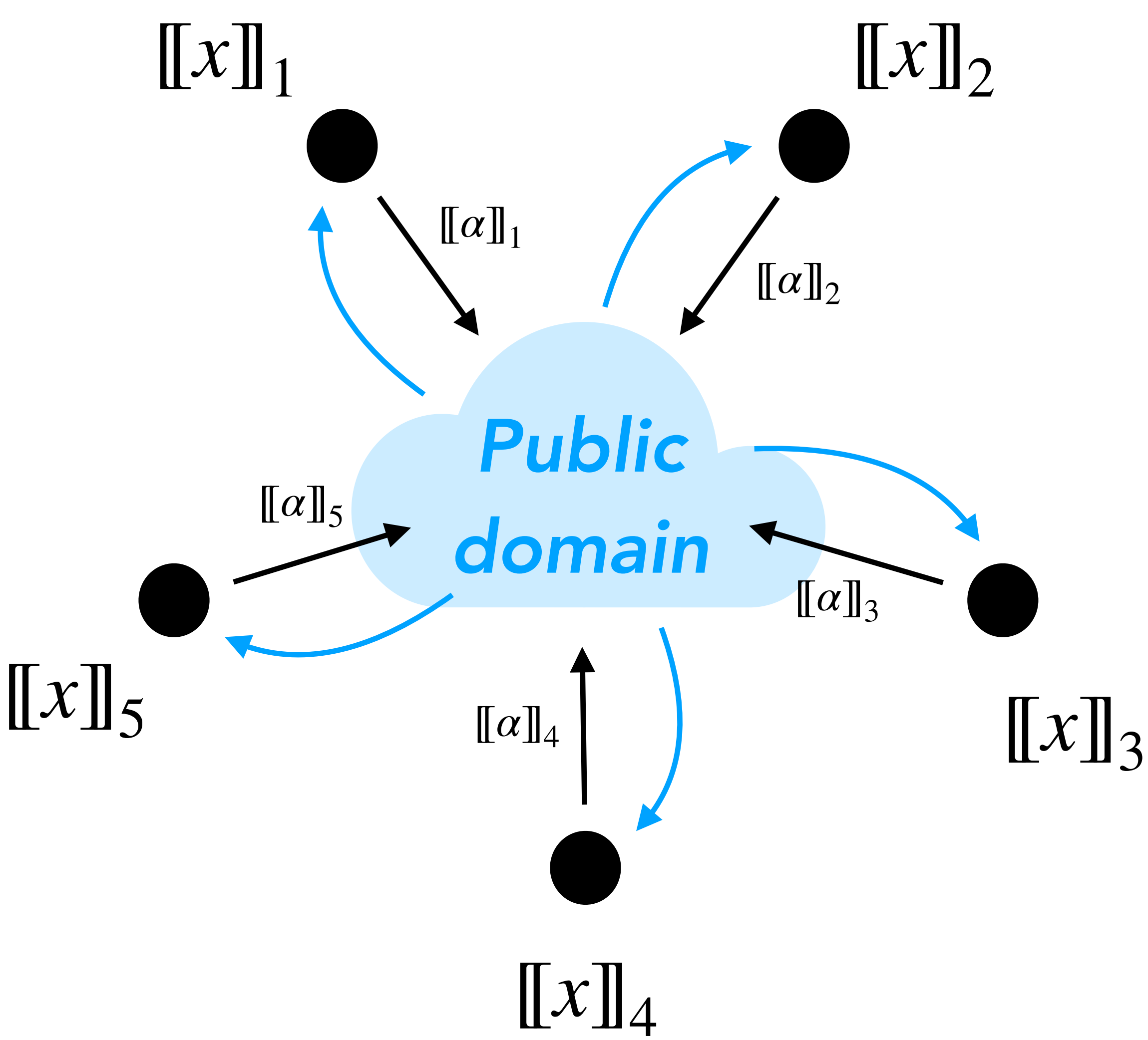
- ℓ-private
- Semi-honest model

MPC model



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- ℓ-private
- Semi-honest model
- *Broadcast model*

MPCitH transform

Prover

Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\cdots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

Prover

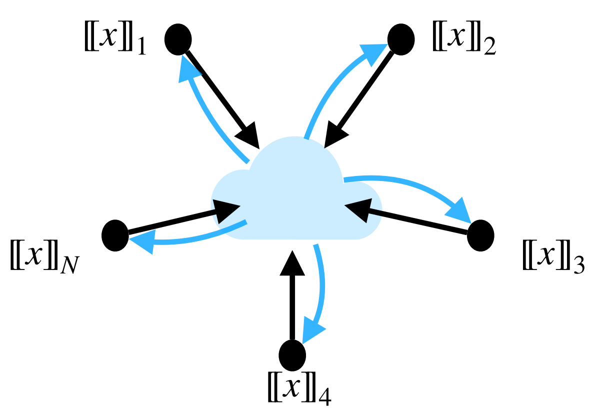
Verifier

MPCitH transform

① Generate and commit shares

$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

② Run MPC in their head



$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

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send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

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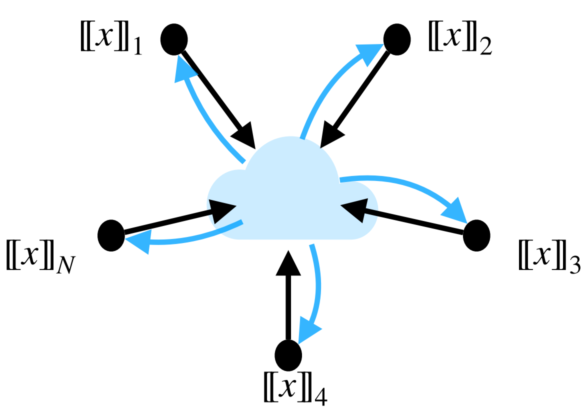

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

$\ldots$

$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

→

send broadcast

$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$

→

I

←

③ Choose a random set of parties

$I \subseteq \{1, \ldots, N\}$, s.t. $|I| = \ell$.

Prover

Verifier

MPCitH transform

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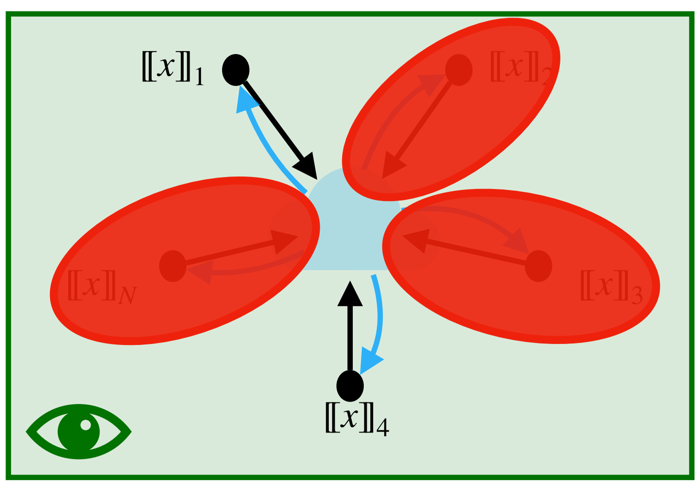

④ Open parties in I

$\mathrm{Com}^{\rho_1}([\![x]\!]_1)$
...
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$ →

send broadcast
$[\![\alpha]\!]_1, \ldots, [\![\alpha]\!]_N$ →

← I

$([\![x]\!]_i, \rho_i)_{i \in I}$ →

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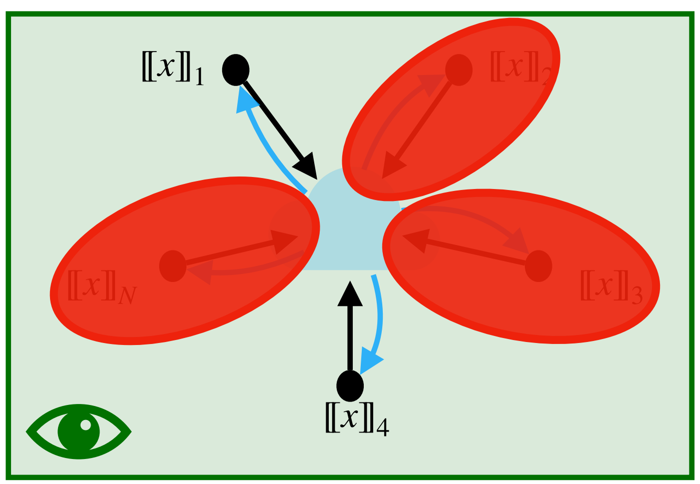

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Check $g(y, \alpha) =$ Accept

Prover

Verifier

MPCitH transform: with additive sharing

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$[\![x]\!] = ([\![x]\!]_1, \ldots, [\![x]\!]_N)$

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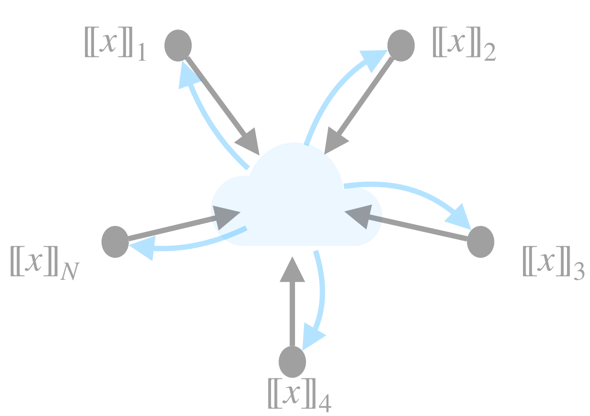

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…
$\mathrm{Com}^{\rho_N}([\![x]\!]_N)$

send broadcast

Additive sharing:

$$x = [\![x]\!]_1 + \cdots + [\![x]\!]_N$$

$([\![x]\!]_i, \rho_i)_{i \in I}$

Choose a random set of parties $I \subseteq \{1,\ldots,N\}$, s.t. $|I| = \ell$.

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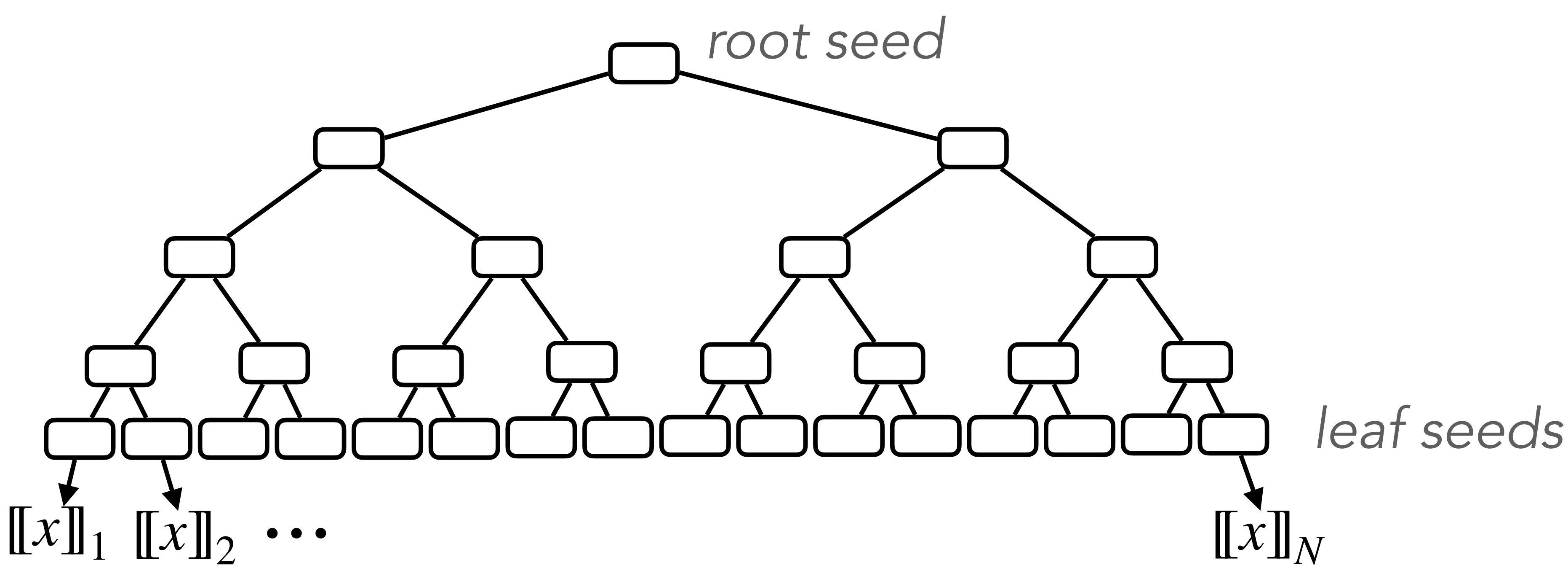
② Run MPC in their head

Generated using a GGM seed tree [KKW18]:



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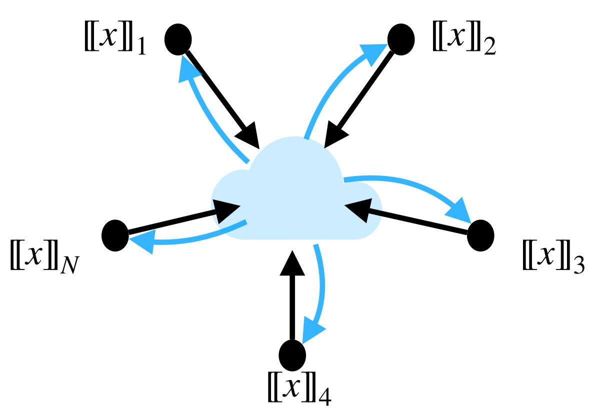

④ Open parties in I

Sharing / MPC protocol

$(N-1)$-*private*

parties

ℓ .

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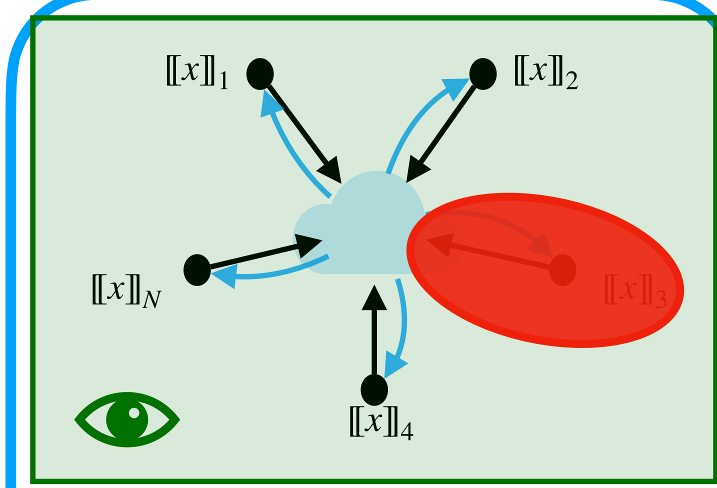

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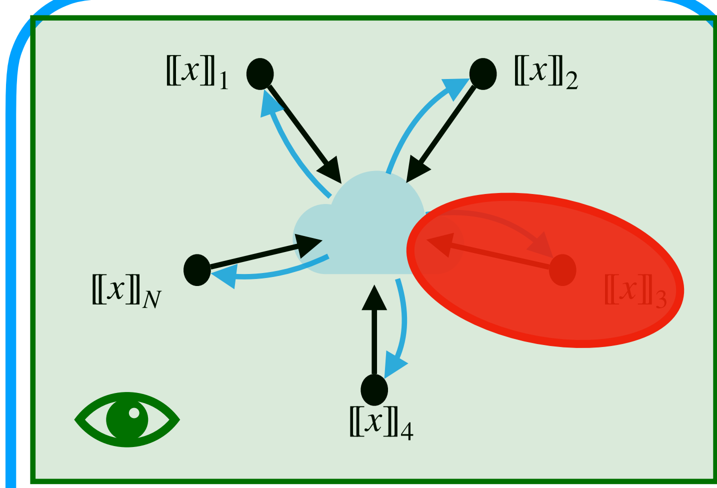

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$\Rightarrow$ soundness error $\approx \dfrac{1}{N}$

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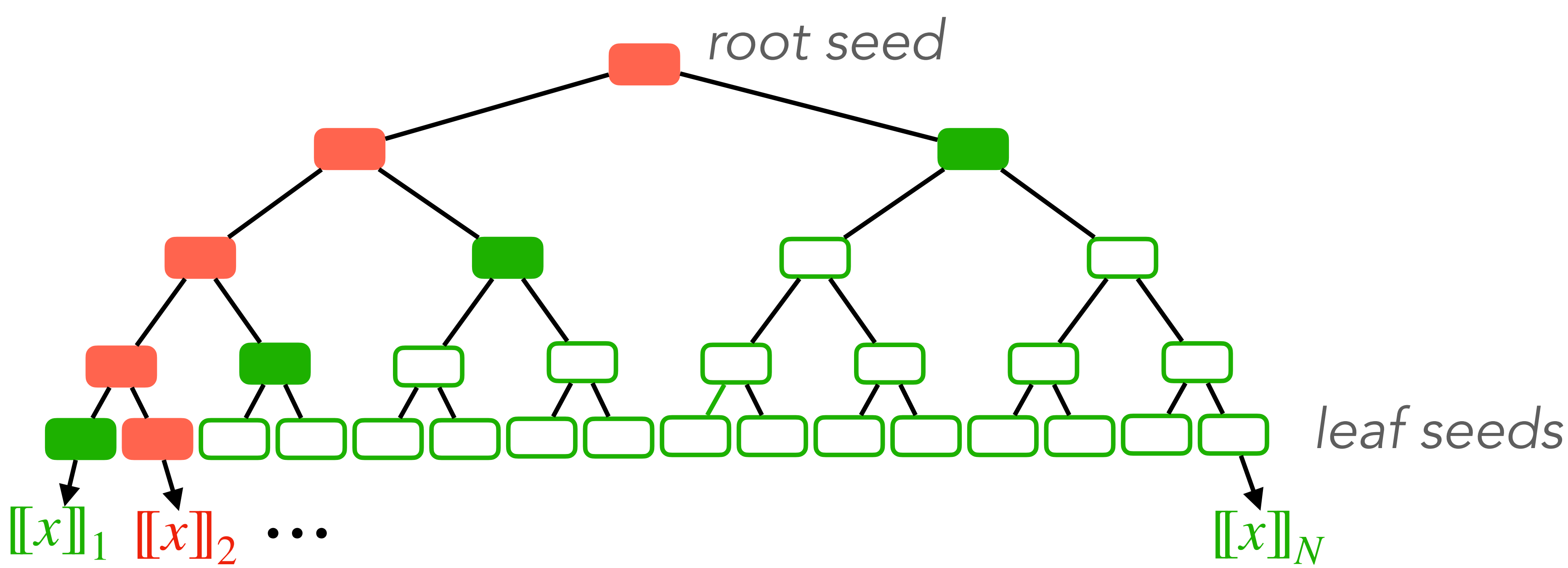

MPCitH transform: with threshold sharing

a.k.a. Threshold Computation in the Head (TCitH)

MPCitH transform: with threshold sharing

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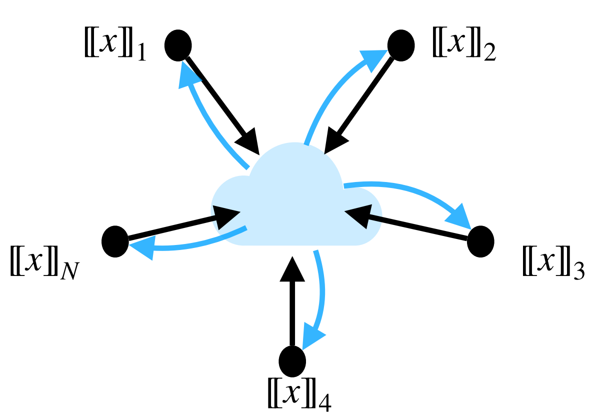

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Prover

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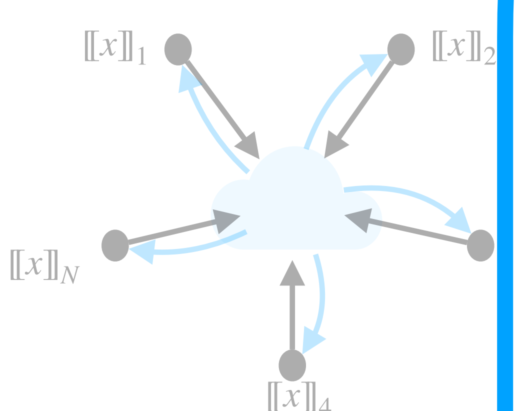

④ Open parties in I

Shamir secret sharing:

$$[\![x]\!]_i := P(e_i) \quad \forall i$$

$$\text{for } \ P(X) := x + r_1 \cdot X + \cdots + r_\ell \cdot X^\ell$$

m set of parties
t. $|I| = \ell$.

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Verifier

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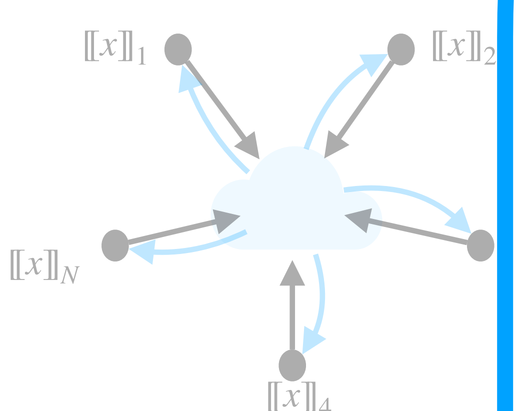

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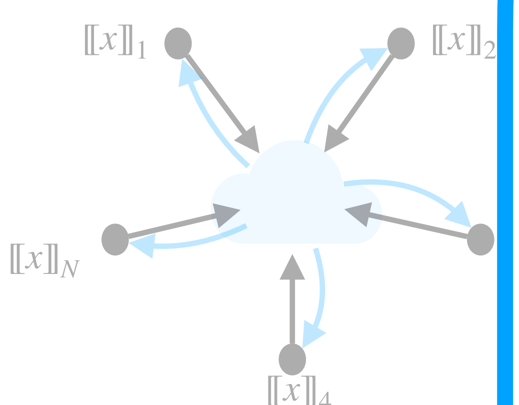

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We use $\ell \ll N$ (e.g. $\ell = 1$)

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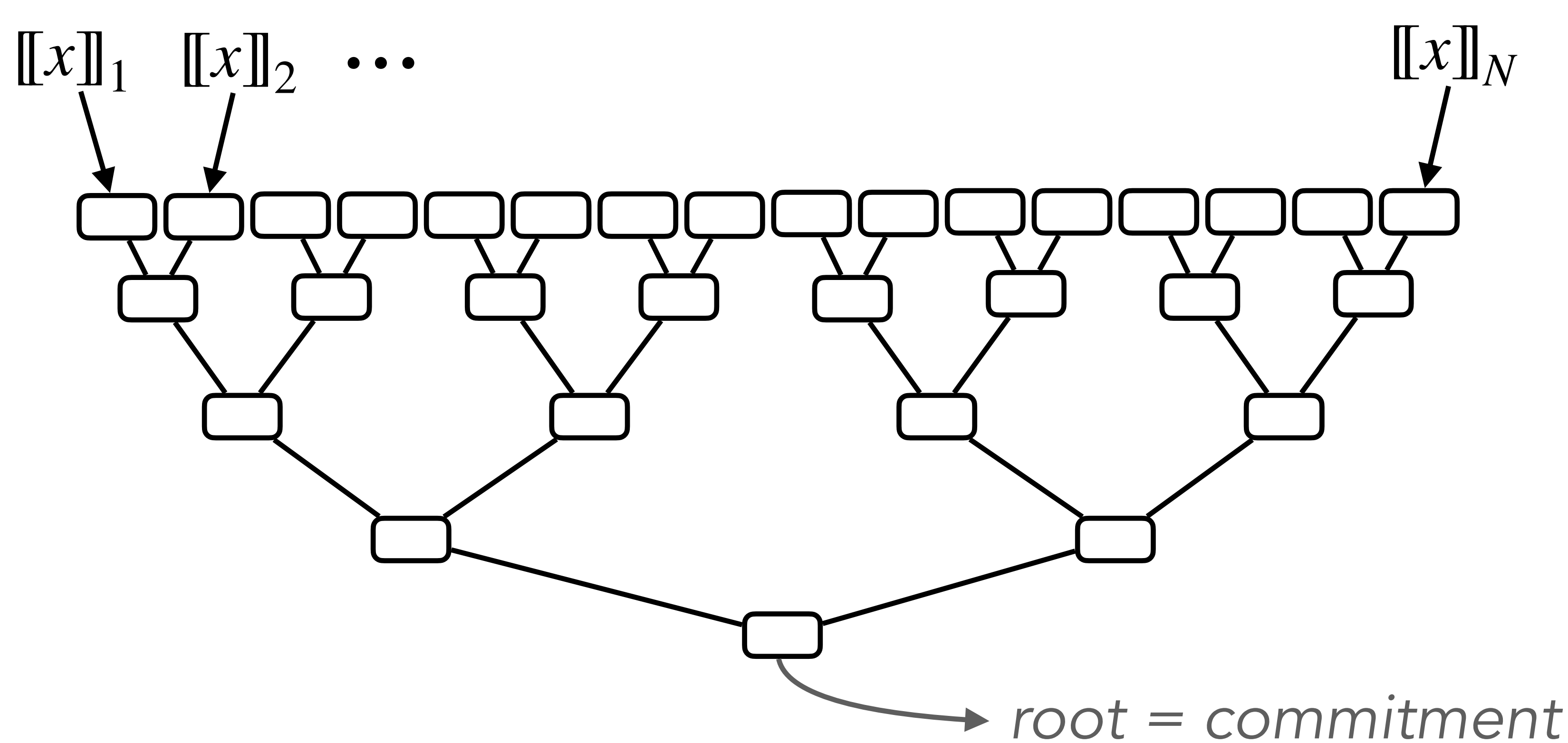
② Run MPC in their head

Committed using a Merkle tree:



es

$([\![x]\!]_i)$

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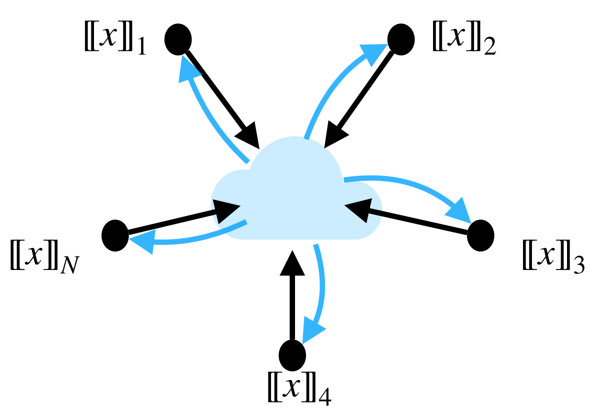

④ Open parties in I

Prover

Sharing / MPC protocol *ℓ-private*

MPCitH transform: with threshold sharing

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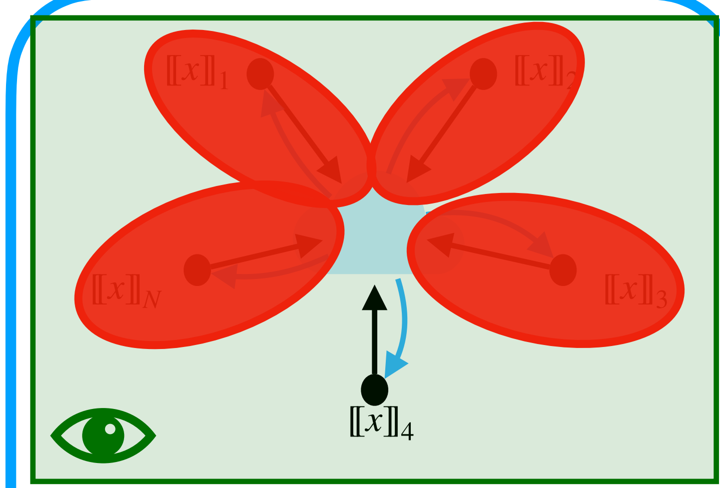

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Sharing / MPC protocol *ℓ-private*

parties

ℓ .

$]\!]_i)$

$= \varphi([\![x]\!]_i)$

Prover

Verifier

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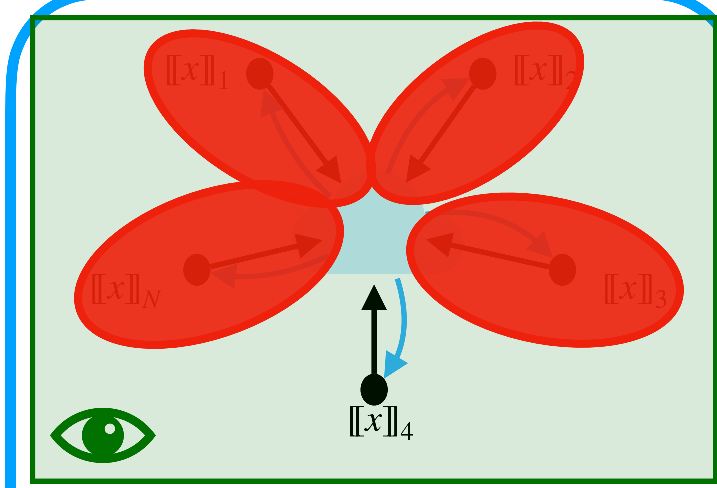

④ Open parties in I

Sharing / MPC protocol *ℓ-private*

$\Rightarrow$ soundness error $\approx (N - \ell)/N$ 🤔

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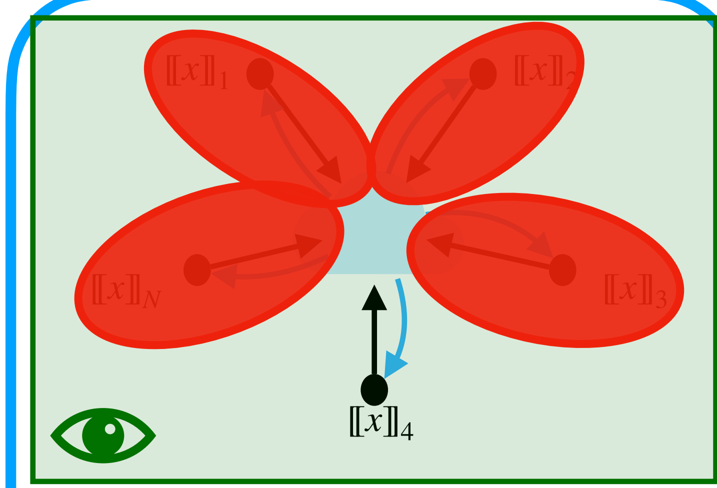

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Much better! $\approx \dfrac{1}{\binom{N}{\ell}}$ 🤩

Prover

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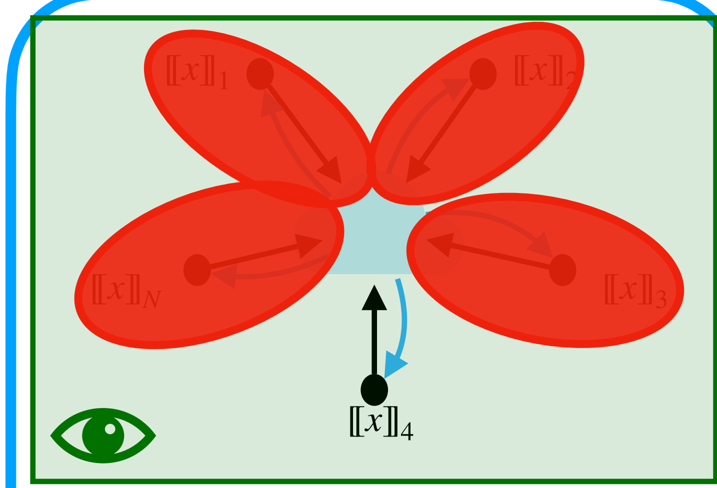

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$\Rightarrow$ soundness error $\approx (N-\ell)/N$ 🤔

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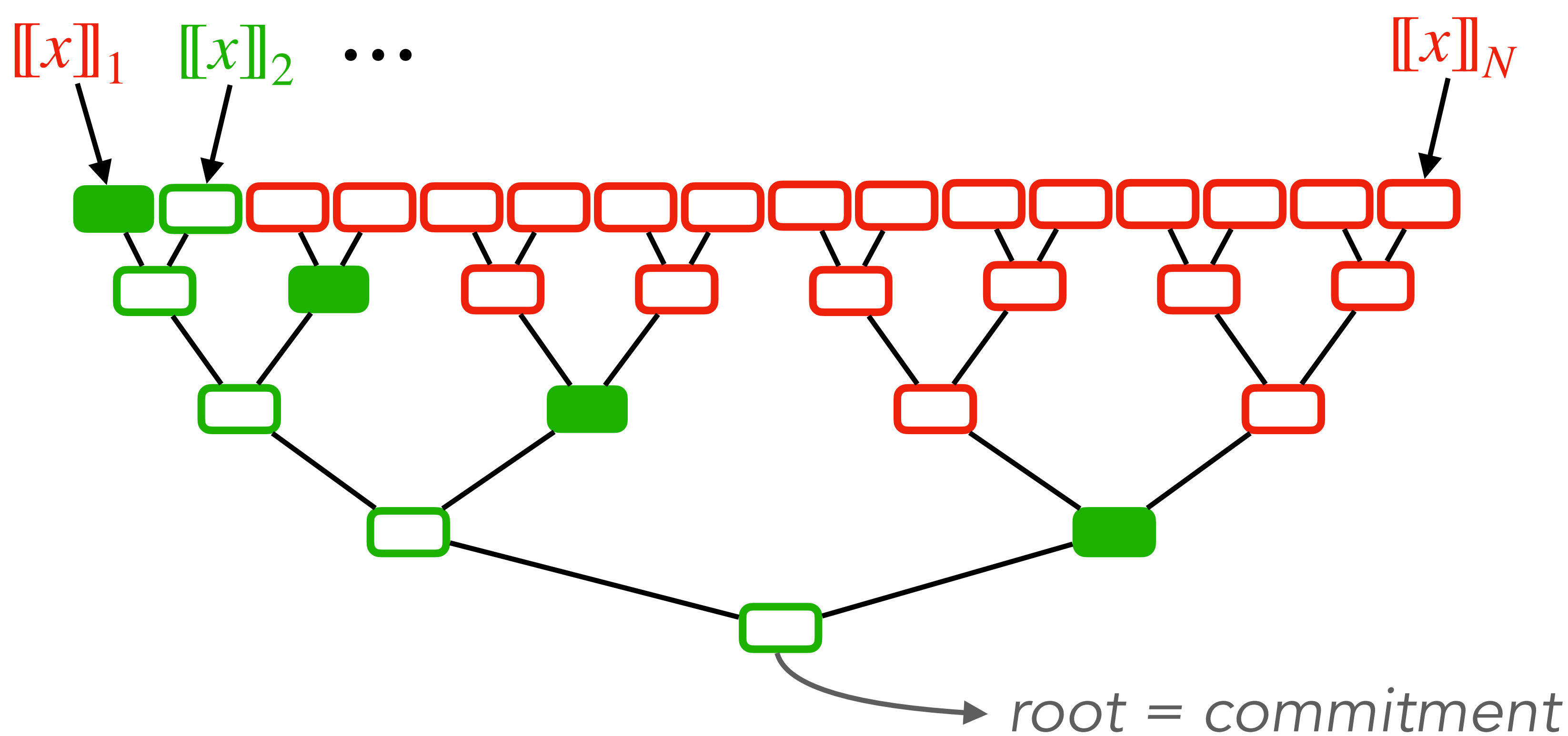
💡 broadcasted sharings = Reed-Solomon codewords

Verifier

MPCitH transform: with threshold sharing

① Generate and commit shares $\mathrm{Com}^{\rho_1}([\![x]\!]_1)$

Only $\log_2 N$ labels to be revealed:



TCitH vs. (additive-sharing) MPCitH

	MPCitH + seed trees + hypercube	**TCitH** (original framework) $\ell = 1$
Soundness error	$\approx \frac{1}{N} + p$	$\approx \frac{1}{N} + p \cdot \binom{N}{2}$
Prover runtime		
Verifier runtime		
Size of tree		

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Soundness error	$\approx \frac{1}{N} + p$	$\approx \frac{1}{N} + p \cdot \binom{N}{2}$
Prover runtime		
Verifier runtime		
Size of tree		

false positive probability

⇒ must be smaller in TCitH

TCitH vs. (additive-sharing) MPCitH

	MPCitH + seed trees **+ hypercube** [AGHHJY]	**TCitH** (original framework) $\ell = 1$
Soundness error	$\approx \frac{1}{N} + p$	$\approx \frac{1}{N} + p \cdot \binom{N}{2}$
Prover runtime	Party emulations: $\log N + 1$ Symmetric crypto: $O(N)$	Party emulations: 2 Symmetric crypto: $O(N)$
Verifier runtime		
Size of tree		

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fewer party emulations

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Size of tree		

TCitH vs. (additive-sharing) MPCitH

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Size of tree		*fewer party emulations*

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Size of tree		🚀 *much less symmetric crypto*

TCitH vs. (additive-sharing) MPCitH

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Soundness error	$\approx \frac{1}{N} + p$	$\approx \frac{1}{N} + p \cdot \binom{N}{2}$
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Size of tree	128-bit security: ~2KB 256-bit security: ~8KB	128-bit security: ~4KB 256-bit security: ~16KB

TCitH vs. (additive-sharing) MPCitH

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factor 2

TCitH vs. (additive-sharing) MPCitH

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Number of parties		$N \leq \|\mathbb{F}\|$

TCitH vs. (additive-sharing) MPCitH

	MPCitH + seed trees	TCitH (original framework)
Soundn		N / 2)
Prover		2 : $O(N)$
Verifier		1 : $O(\log N)$
Size c		4KB 16KB
Number of parties		$N \leq \lvert\mathbb{F}\rvert$

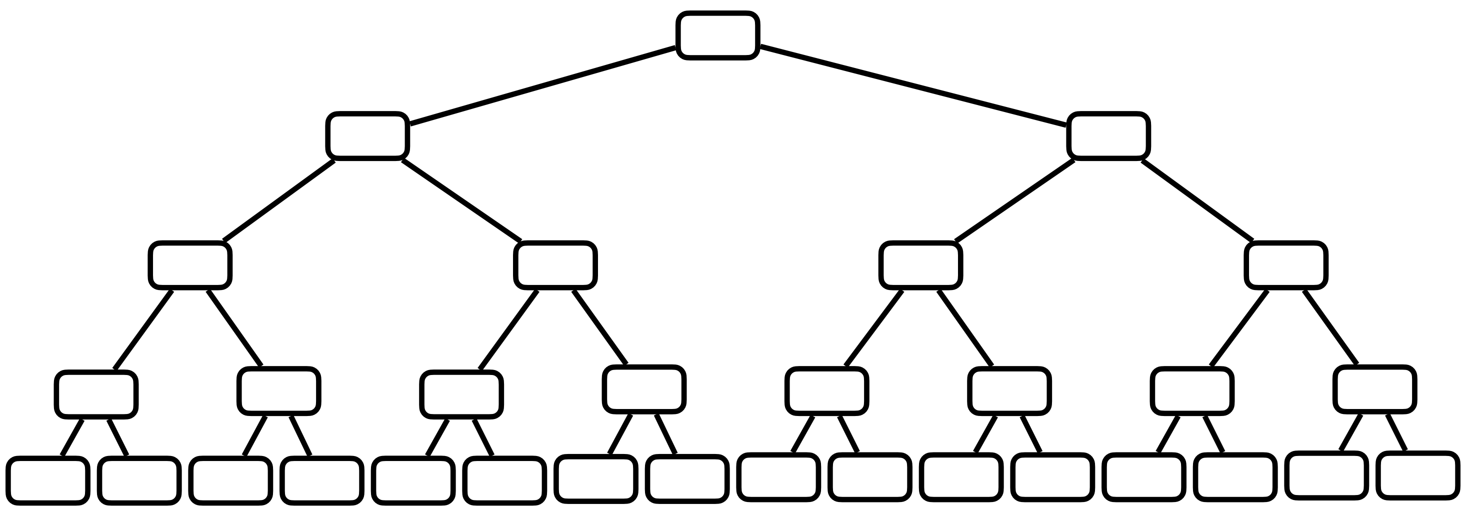
🤔 What can we do to avoid these limitations?

☝️ TCitH with GGM trees

TCitH with GGM trees

Step 1: Generate a replicated secret sharing [ISN89]

$x = r_1 + r_2 + \cdots + r_N$



$r_1 + r_2 + \cdots \cdots + r_N = x$

TCitH with GGM trees

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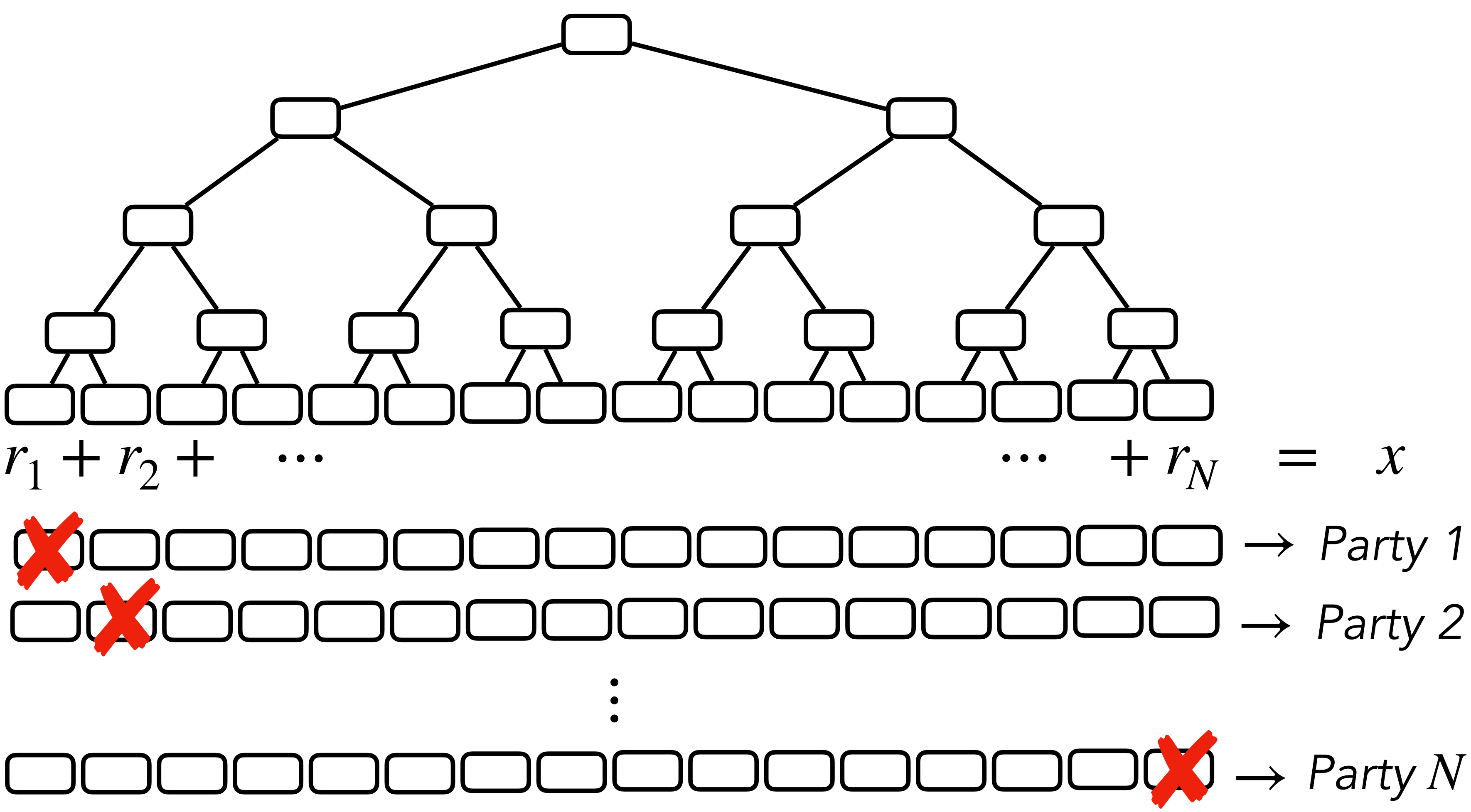

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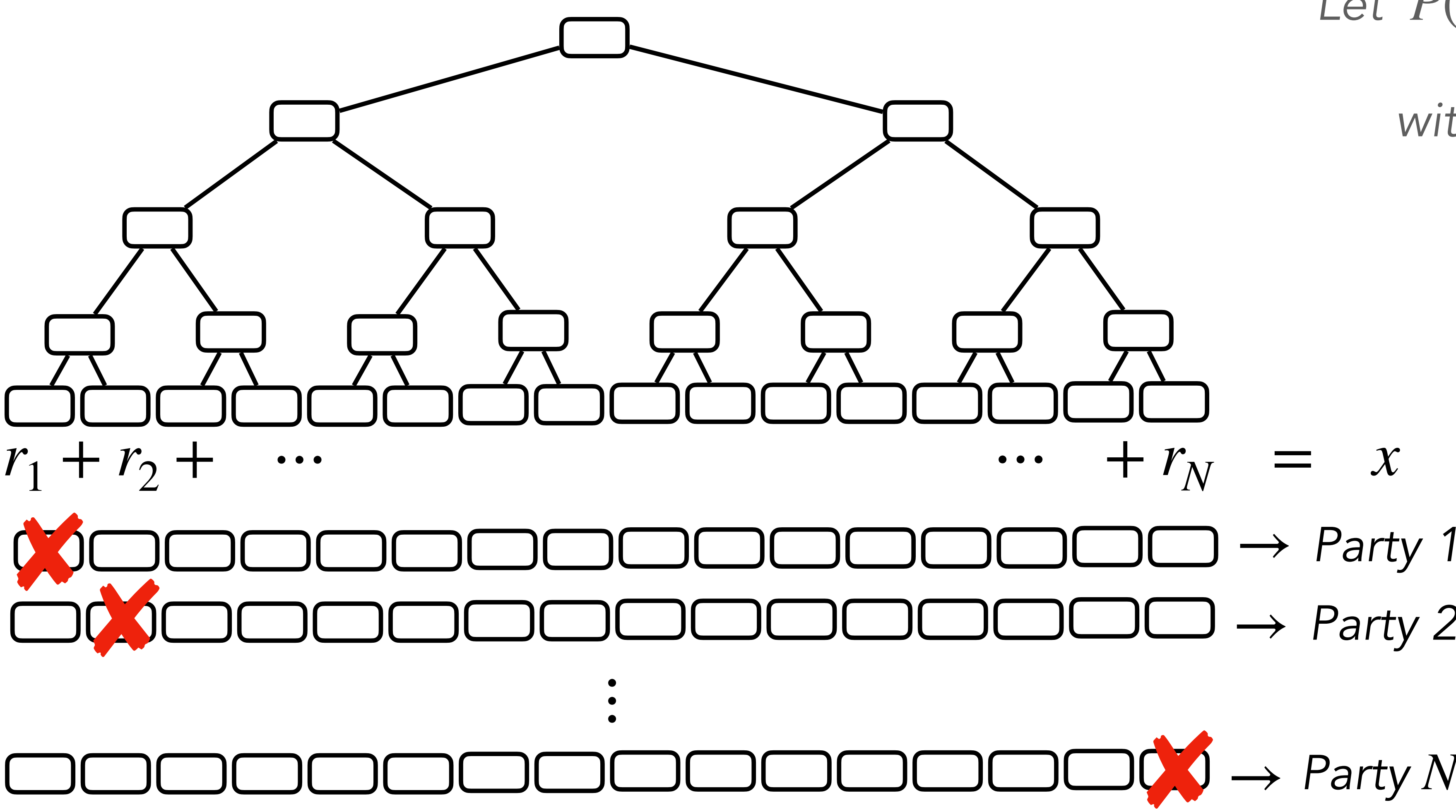

Step 2: Convert it into a Shamir's secret sharing [CDI05]

Let $P(X) = \sum_j r_j P_j(X)$

with $P_j(X) = 1 - (1/e_j) \cdot X$

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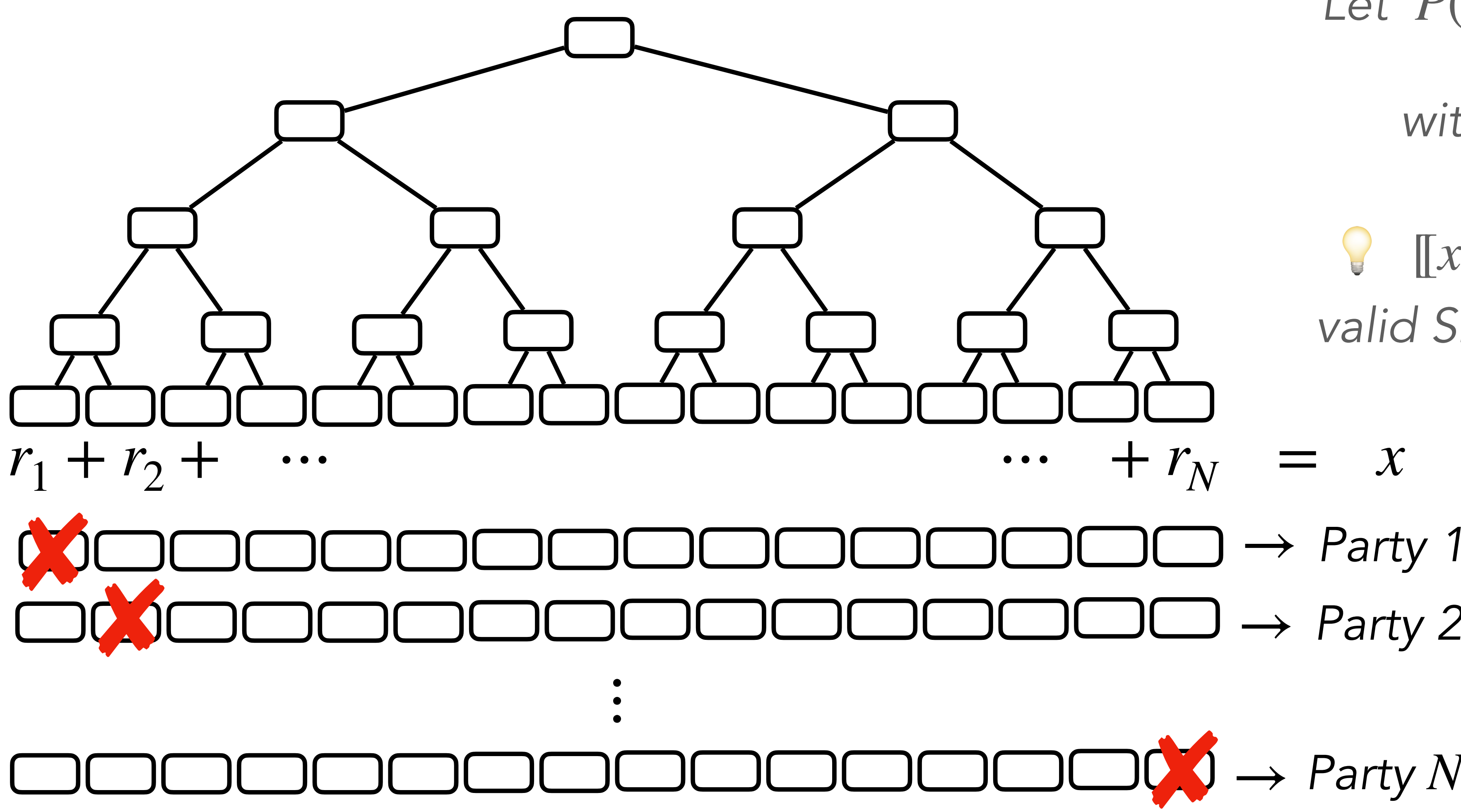

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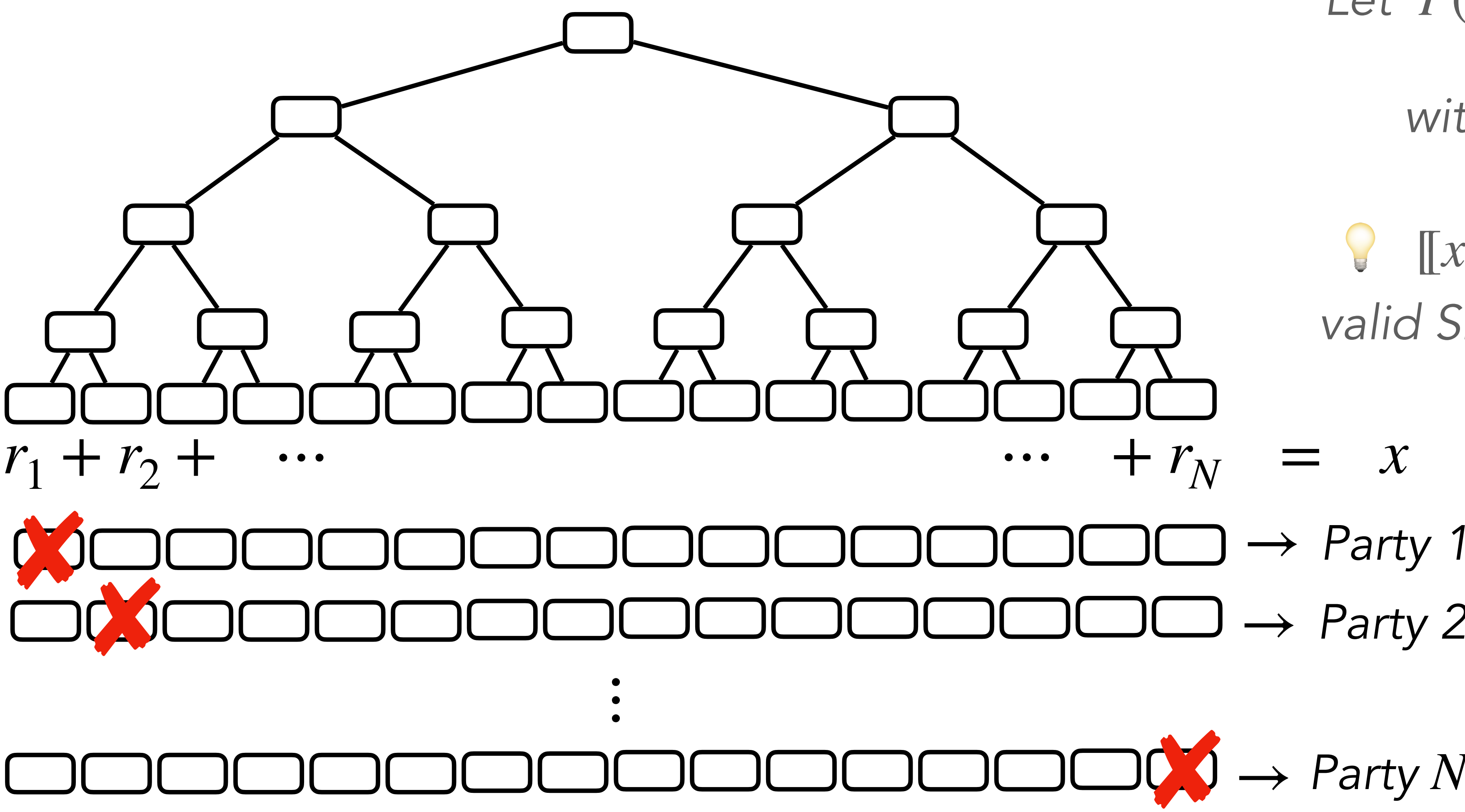

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(since $P_i(e_i) = 0$)

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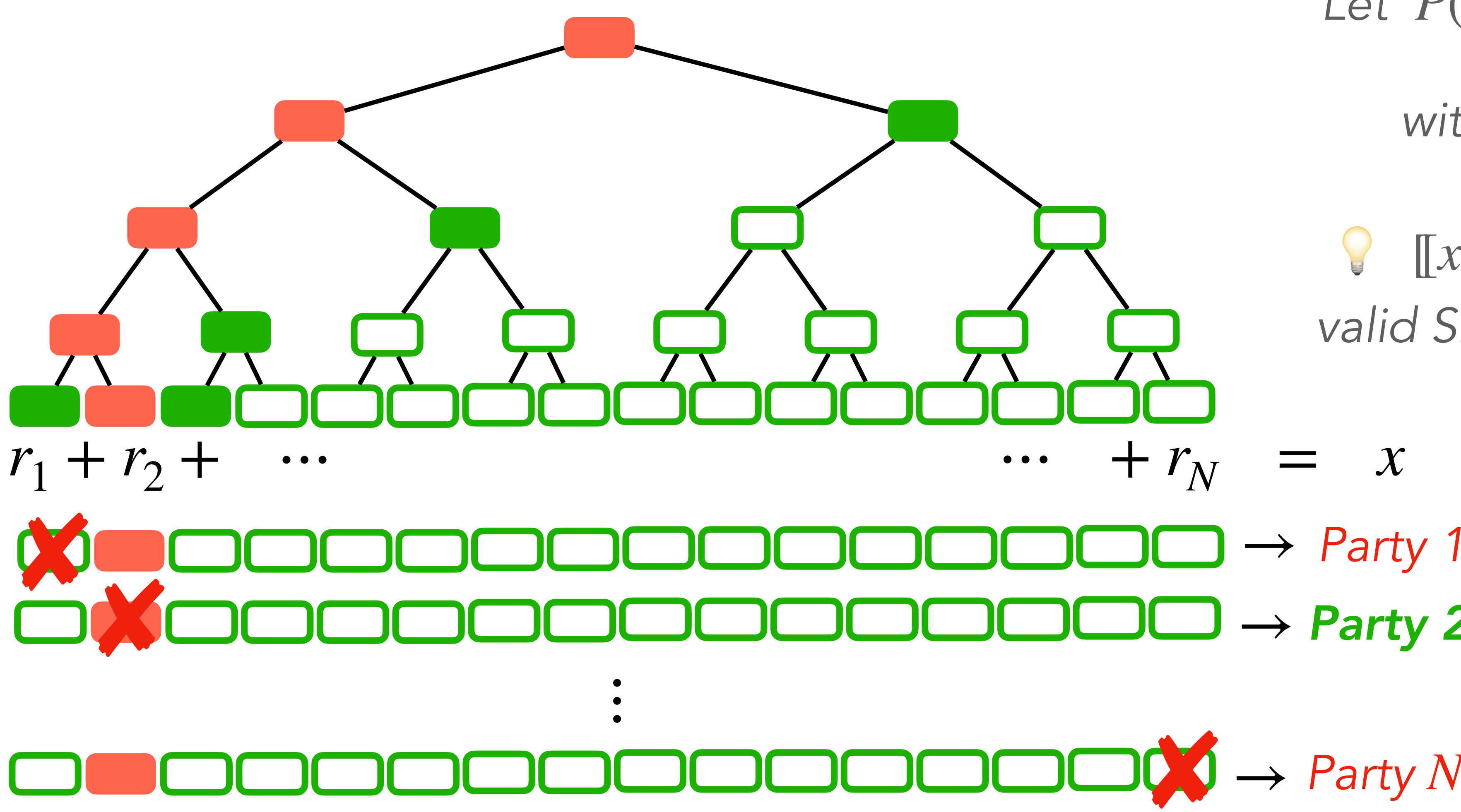

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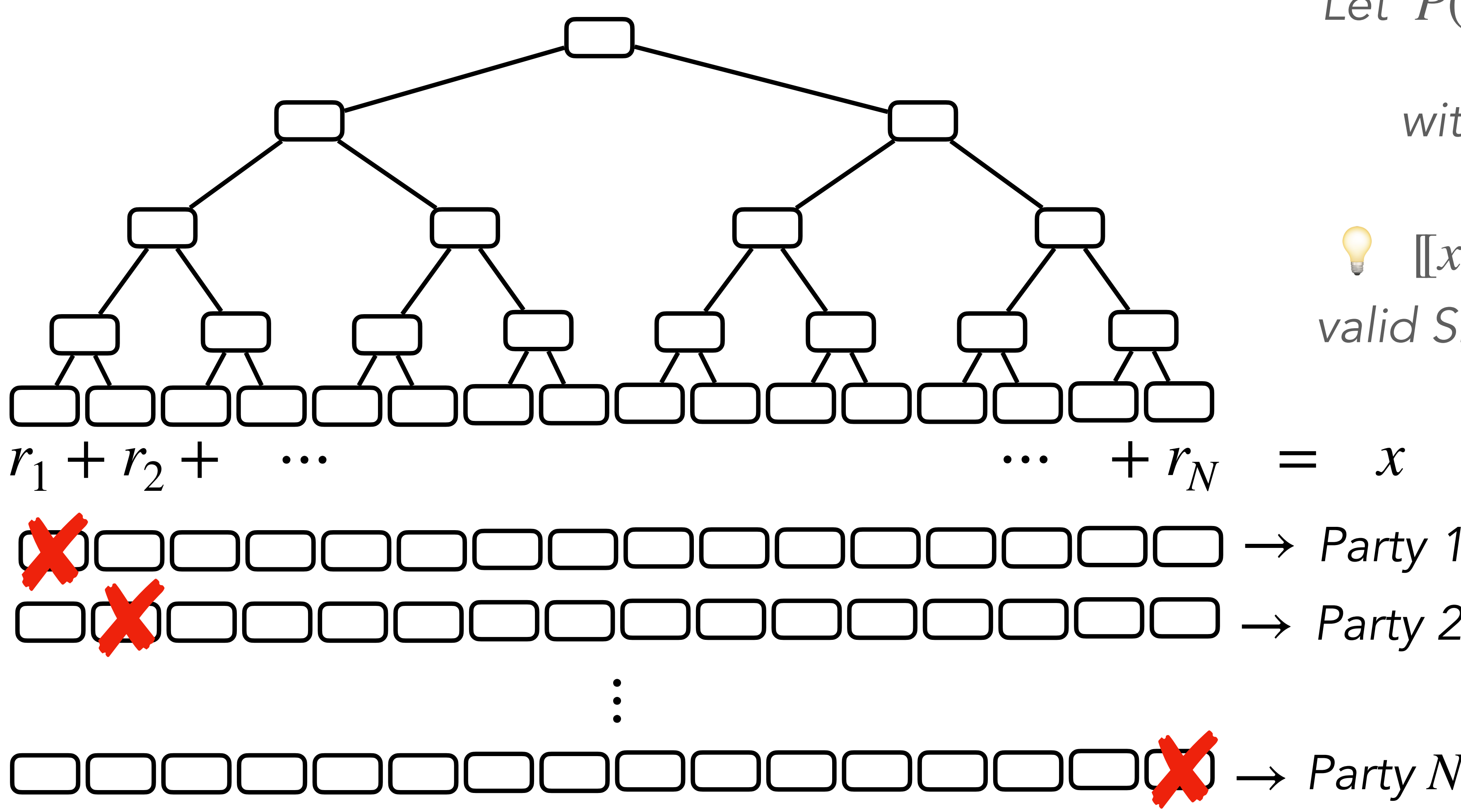

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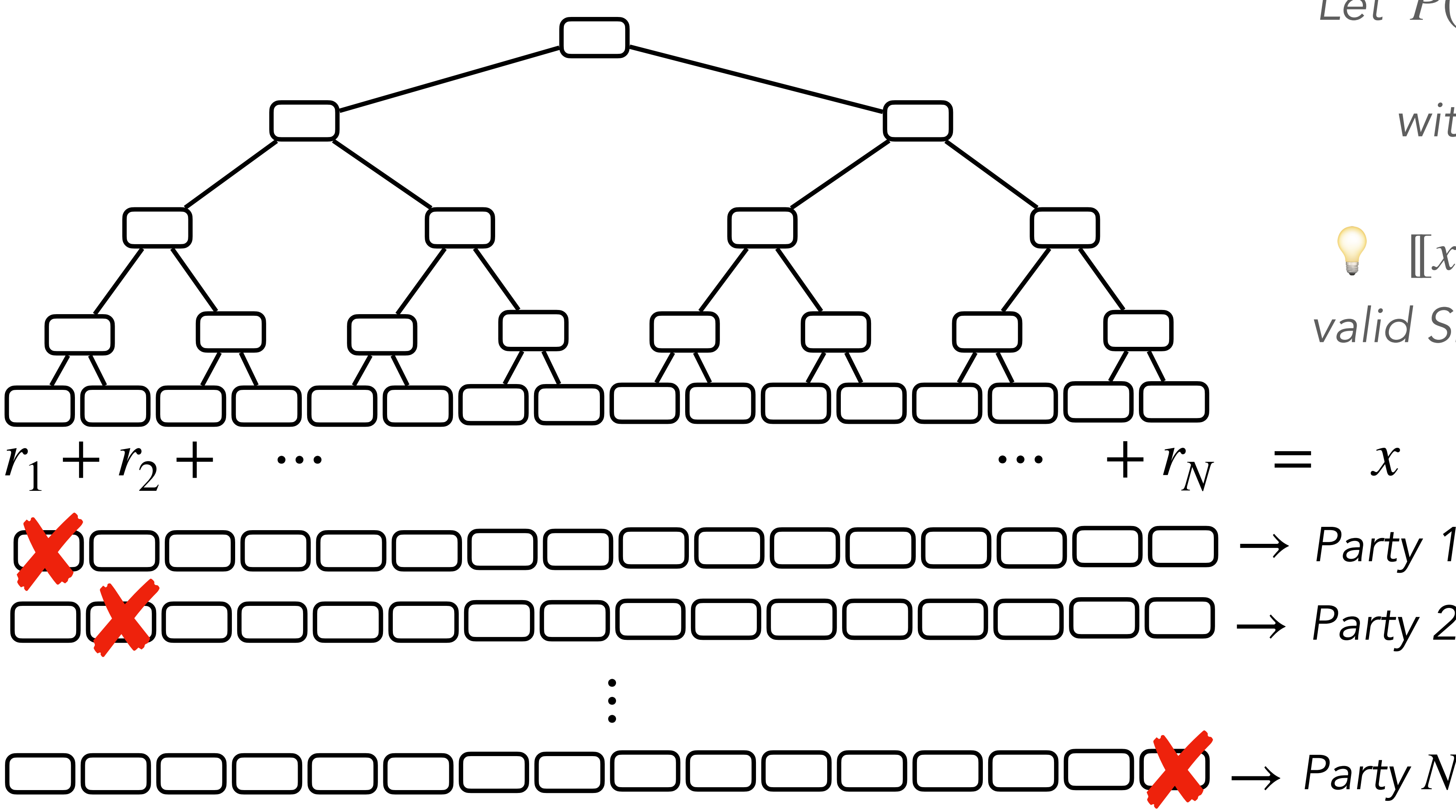

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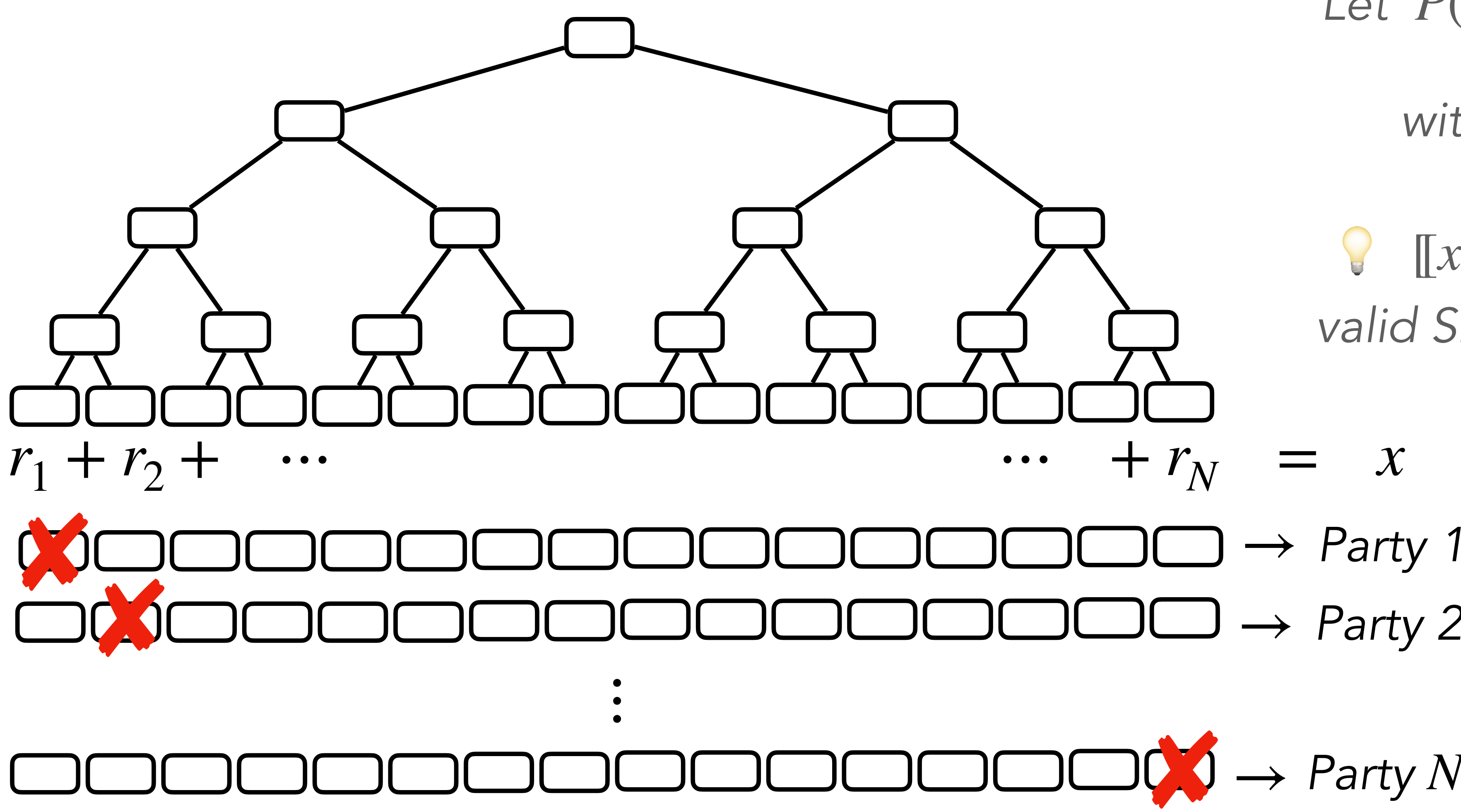

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😇 Good soundness (only valid sharings)

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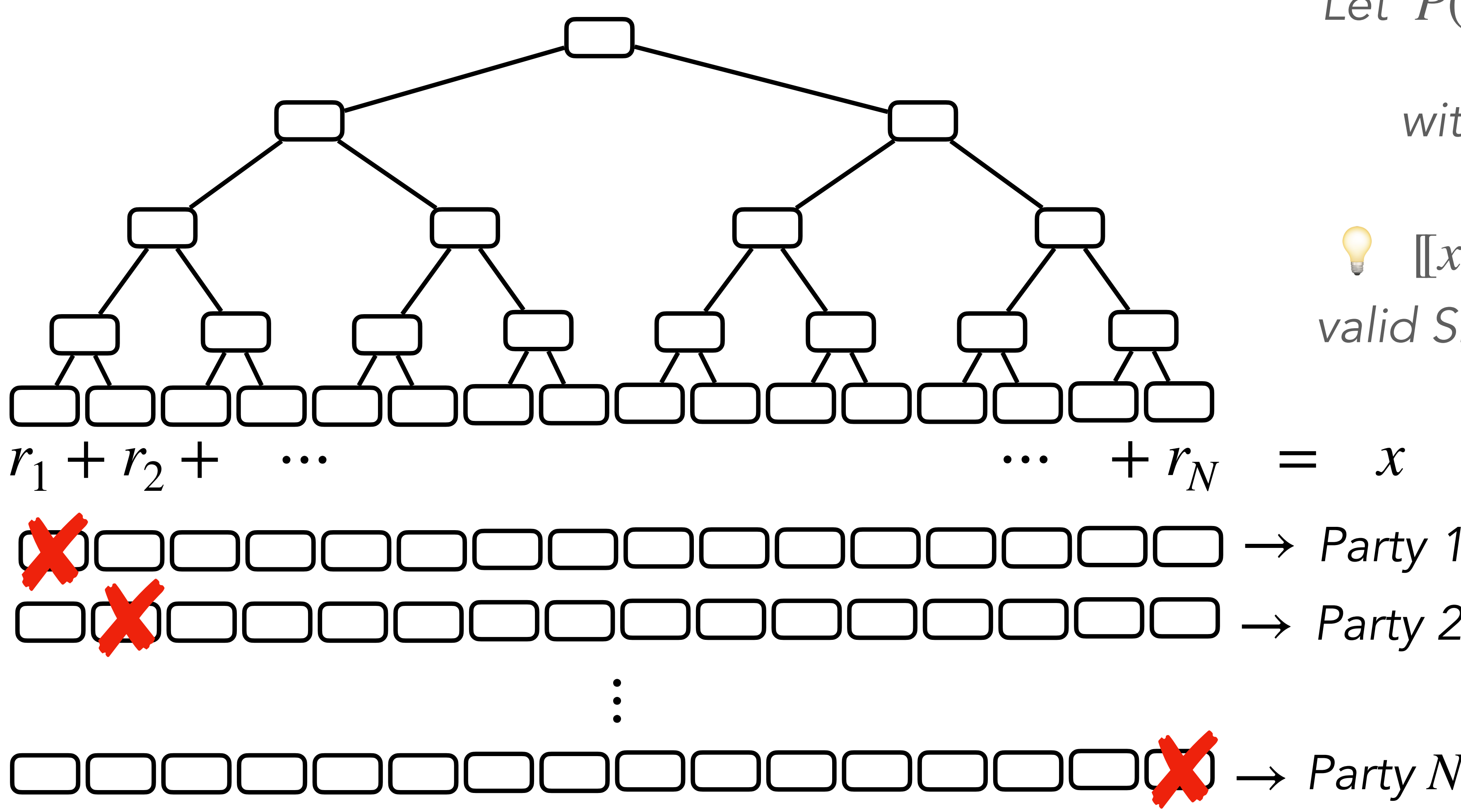

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🌲 Size of GGM tree

😇 Good soundness (only valid sharings)

🐌 Loose fast verification

Speedups for MPCitH candidates

	Additive MPCitH		TCitH (GGM tree)	
	Traditional (ms)	Hypercube (ms)	TCitH (ms)	Saving
Party emulations / repetition	N	$1 + \log_2 N$	2	

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🛠 Party emulations = $1 + \left\lceil \frac{\log_2 N}{\log_2 |\mathbb{F}|} \right\rceil$

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🛠️ Party emulations $= 1 + \left\lceil \frac{\log_2 N}{\log_2 |\mathbb{F}|} \right\rceil = \begin{cases} 2 & \text{if } |\mathbb{F}| \geq N \\ \vdots & \\ 1 + \log_2 N & \text{if } |\mathbb{F}| = 2 \end{cases}$

Speedups for MPCitH candidates

	Additive MPCitH		TCitH (GGM tree)	
	Traditional (ms)	Hypercube (ms)	TCitH (ms)	Saving
Party emulations / repetition	N	$1 + \log_2 N$	$1 + \left\lceil \frac{\log_2 N}{\log_2 \lvert \mathbb{F} \rvert} \right\rceil$	
AIMer	4.53	3.22	3.22	-0 %
Biscuit	17.71	4.65	4.24	-16 %
MIRA	384.26	20.11	9.89	-51 %
MiRitH-Ia	54.15	6.60	5.42	-18 %
MiRitH-Ib	89.50	8.66	6.66	-23 %
MQOM-31	96.41	11.27	8.74	-21 %
MQOM-251	44.11	7.56	5.97	-21 %
RYDE	12.41	4.65	4.65	-0 %
SDitH-256	78.37	7.23	5.31	-27 %
SDitH-251	19.15	7.53	6.44	-14 %

- Comparison based on a generic MPCitH library (libmpcith)
- Code for MPC protocols fetched from the submission packages

Speedups for MPCitH candidates

	Additive MPCitH		TCitH (GGM tree)	
Party em… / repe…				…ving
AIM…				… %
Bisc…				…6 %
MIR…				…1 %
MiRit…				…3 %
MiRit…				…3 %
MQOM…				…1 %
MQOM…				…1 %
RYD…				… %
SDitH-…				…7 %
SDitH-251	19.15	7.53	6.44	-14 %

🤔 Nice speedups, but could we also improve the sizes?

☝️ Let's use the multiplication homomorphism

- Comparison based on a generic MPCitH library (libmpcith)
- Code for MPC protocols fetched from the submission packages

Using multiplication homomorphism

- Shamir's secret sharing satisfies:

$$[\![x]\!]^{(d)} \cdot [\![y]\!]^{(d)} = [\![x \cdot y]\!]^{(2d)}$$

Using multiplication homomorphism

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- Simple protocol to verify polynomial constraints
  - w valid $\Leftrightarrow f_1(w) = 0, \ \ldots, f_m(w) = 0$
  - parties locally compute

$$[\![\alpha]\!] = [\![v]\!] + \sum_{j=1}^{m} \gamma_j \cdot f_j([\![w]\!])$$

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randomness
from the verifier

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pre-committed sharing of 0

randomness from the verifier

Using multiplication homomorphism

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  - w valid $\Leftrightarrow f_1(w) = 0, \; \ldots, \; f_m(w)$
  - parties locally compute

$$[\![\alpha]\!] = [\![v]\!] + \sum_{j=1}^{m} \gamma_j \cdot f_j([\![w]\!])$$

check $\alpha = 0$
false positive proba $1/|\mathbb{F}|$

pre-committed
sharing of 0

randomness
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Using multiplication homomorphism

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$$[\![x]\!]^{(d)} \cdot [\![y]\!]^{(d)} = [\![x \cdot y]\!]^{(2d)}$$

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  - w valid $\Leftrightarrow f_1(w) = 0, \ \dots, f_m(w)$
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$$[\![\alpha]\!] = [\![v]\!] + \sum_{j=1}^{m} \gamma_j \cdot f_j([\![w]\!])$$

- Tweaking MPCitH-based candidates $\Rightarrow$ smaller signatures

Shorter signatures for MPCitH-based candidates

	Original Size	Our Variant	Saving
Biscuit	4 758 B	4 048 B	-15 %
MIRA	5 640 B	5 340 B	-5 %
MiRitH-Ia	5 665 B	4 694 B	-17 %
MiRitH-Ib	6 298 B	5 245 B	-17 %
MQOM-31	6 328 B	4 027 B	-37 %
MQOM-251	6 575 B	4 257 B	-35 %
RYDE	5 956 B	5 281 B	-11 %
SDitH	8 241 B	7 335 B	-27 %
MQ over GF(4)	8 609 B	3 858 B	-55 %
SD over GF(2)	11 160 B	7 354 B	-34 %
SD over GF(2)	12 066 B	6 974 B	-42 %

* $N = 256$

Shorter signatures for MPCitH-based candidates

	Original Size	*Our Variant*	*Saving*
Biscuit	4 758 B	3 431 B	
MIRA	5 640 B	4 314 B	
MiRitH-Ia	5 665 B	3 873 B	
MiRitH-Ib	6 298 B	4 250 B	
MQOM-31	6 328 B	3 567 B	
MQOM-251	6 575 B	3 418 B	
RYDE	5 956 B	4 274 B	
SDitH	8 241 B	5 673 B	
MQ over GF(4)	8 609 B	3 301 B	
SD over GF(2)	11 160 B	7 354 B	-34 %
SD over GF(2)	12 066 B	6 974 B	-42 %

* $N = 256$ * $N = 2048$

Shorter signatures for MPCitH-based candidates

Two very recent works :

- Baum, Beullens, Mukherjee, Orsini, Ramacher, Rechberger, Roy, Scholl. *One Tree to Rule Them All: Optimizing GGM Trees and OWFs for Post-Quantum Signatures.* https://ia.cr/2024/490
  - General techniques to reduce the size of GGM trees
  - **Apply to TCitH-GGM** (gain of ~500 B at 128-bit security)

- Bidoux, Feneuil, Gaborit, Neveu, Rivain. *Dual Support Decomposition in the Head: Shorter Signatures from Rank SD and MinRank.* https://ia.cr/2024/541
  - New MPC protocols for TCitH / VOLEitH signatures based on **MinRank & Rank SD**

Other results

- Improvements for TCitH-MT
  - Degree-enforcing commitment scheme
  - Packed secret sharing
- Other applications
  - Post-quantum ring signatures
    - For any one-way function
    - $|\sigma| \leq$ 10 kB (~ 5 kB with MQ) for $|\text{ring}| = 2^{20}$
  - ZKP for lattices
    - Smallest with MPCitH paradigm
    - Competitive to lattice-based ZKP
  - Improvement of Ligero for general arithmetic circuits
- Connections to VOLEitH and Ligero proof systems

Thank you for listening 🙏

Original TCitH framework (Asiacrypt'23)

Improved TCitH framework (preprint)

References

[AGHJY23] Aguilar Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH" (EUROCRYPT 2023)

[BBMORRRS24] Baum, Beullens, Mukherjee, Orsini, Ramacher, Rechberger, Roy, Scholl: "One Tree to Rule Them All: Optimizing GGM Trees and OWFs for Post-Quantum Signatures" https://ia.cr/2024/490

[BFGNR24] Bidoux, Feneuil, Gaborit, Neveu, Rivain. "Dual Support Decomposition in the Head: Shorter Signatures from Rank SD and MinRank" https://ia.cr/2024/541

[CDI05] Cramer, Damgard, Ishai: "Share conversion, pseudorandom secret-sharing and applications to secure computation" (TCC 2005)

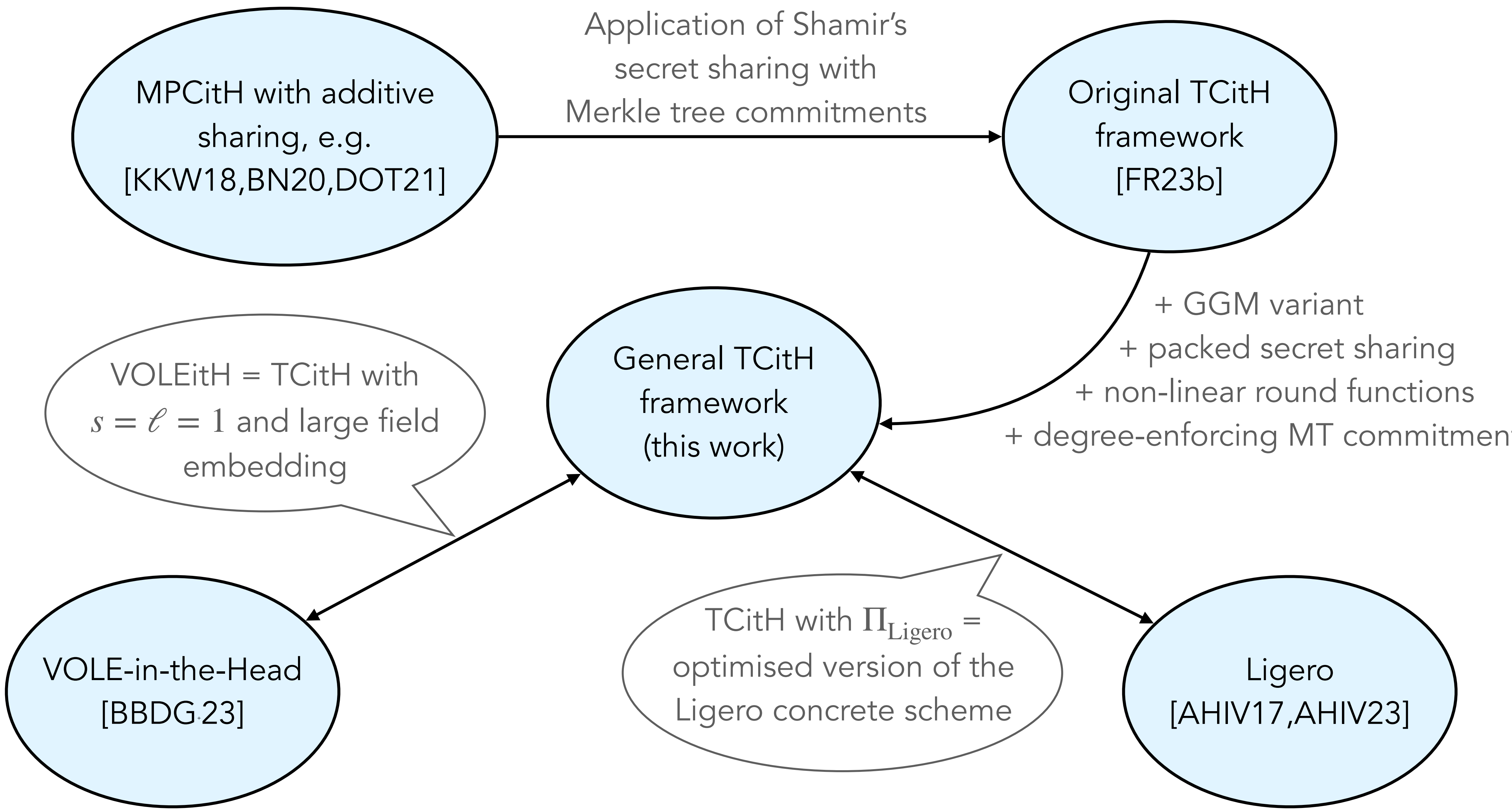
[FR22] Thibauld Feneuil, Matthieu Rivain: "Threshold Linear Secret Sharing to the Rescue of MPC-in-the-Head" https://ia.cr/2022/1407 (ASIACRYPT 2023)

[FR23] Thibauld Feneuil, Matthieu Rivain: "Threshold Computation in the Head: Improved Framework for Post-Quantum Signatures and Zero-Knowledge Arguments" https://ia.cr/2023/1573

[ISN89] Ito, Saito, Nishizeki: "Secret sharing scheme realizing general access structure" (Electronics and Communications in Japan 1989)

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

Connections to other proof systems



$N = 256$

	TCitH-GGM		VOLEitH	
	Size	Comput. Field	Size	Computat. Field
AIMer [CCH+23]	4 352 B	$19\times\ GF(2^8)$	3 938 B	$GF(2^{128})$
Biscuit [BKPV23]	4 048 B	$19\times\ GF(16^2)$	3 682 B	$GF(16^{2\times16})$
MIRA [ABB+23d]	5 340 B	$19\times\ GF(16^2)$	4 770 B	$GF(16^{2\times16})$
MiRitH-Ia [ABB+23b]	4 694 B	$19\times\ GF(16^2)$	4 226 B	$GF(16^{2\times16})$
MiRitH-Ib [ABB+23b]	5 245 B	$19\times\ GF(16^2)$	4 690 B	$GF(16^{2\times16})$
MQOM (over $\mathbb{F}_{251}$) [FR23a]	4 257 B	$19\times\ GF(251)$	3 858 B	$GF(251^{16})$
MQOM (over $\mathbb{F}_{31}$) [FR23a]	4 027 B	$19\times\ GF(31^2)$	3 660 B	$GF(31^{2\times16})$
RYDE [ABB+23c]	5 281 B	$19\times\ GF(2^8)$ $19\times\ GF(2^{31})$	4 720 B	$GF(2^{128})$
SDitH (over $\mathbb{F}_{251}$) [AFG+23]	7 335 B	$19\times\ GF(251)$	6 450 B	$GF(251^{16})$
SDitH (over $\mathbb{F}_{256}$) [AFG+23]	7 335 B	$19\times\ GF(256)$	6 450 B	$GF(256^{16})$

$N = 2048$

	TCitH-GGM		VOLEitH	
	Size	Comput. Field	Size	Computat. Field
AIMer [CCH+23]	3 639 B	$13\times\ GF(2^{11})$	3 546 B	$GF(2^{128})$
Biscuit [BKPV23]	3 431 B	$13\times\ GF(16^3)$	3 354 B	$GF(16^{3\times12})$
MIRA [ABB+23d]	4 314 B	$13\times\ GF(16^3)$	4 170 B	$GF(16^{3\times12})$
MiRitH-Ia [ABB+23b]	3 873 B	$13\times\ GF(16^3)$	3 762 B	$GF(16^{3\times12})$
MiRitH-Ib [ABB+23b]	4 250 B	$13\times\ GF(16^3)$	4 110 B	$GF(16^{3\times12})$
MQOM (over $\mathbb{F}_{251}$) [FR23a]	3 567 B	$13\times\ GF(251^2)$	3 486 B	$GF(251^{2\times12})$
MQOM (over $\mathbb{F}_{31}$) [FR23a]	3 418 B	$13\times\ GF(31^3)$	3 338 B	$GF(31^{3\times12})$
RYDE [ABB+23c]	4 274 B	$13\times\ GF(2^{11})$ $13\times\ GF(2^{31})$	4 133 B	$GF(2^{128})$
SDitH (over $\mathbb{F}_{251}$) [AFG+23]	5 673 B	$13\times\ GF(251^2)$	5 430 B	$GF(251^{2\times12})$
SDitH (over $\mathbb{F}_{256}$) [AFG+23]	5 673 B	$13\times\ GF(256^2)$	5 430 B	$GF(256^{2\times12})$